Number Operations in Base Ten: Division and Interpreting Remainders Grade 5 Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at:
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Revised 2019

Division & Interpreting Remainders 5^{th} Grade

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This concept-based lesson is intended to help you assess how well students are able to use a variety of strategies to multiply. In particular, this unit aims to identify and help students who have difficulties with:

- A division algorithm.
- Representing division in multiple ways
- Interpreting Remainders

Kentucky Academic Standards

This lesson involves mathematical content standards from across the grades, with emphasis on:

Grade 5 Number and Operations in Base Ten
Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Grade 6 Number System

Cluster: Multiply and divide multi-digit numbers and find common factors and multiples.

This lesson involves a range of standards of mathematical practices, with emphasis on:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.
MP8. Look for and make use of repeated reasoning.

Introduction

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work in small groups (pairs or threes) on a collaborative discussion task to match the word problem, model, and multiple strategies of the same multiplication or division problem. As they do this, they interpret the cards' meanings and begin to link them together. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.

- Finally, students revisit their original work or a similar task, and try to improve their individual responses

Materials required

Each individual student will need:

- Two copies of the worksheet *Division Representations & Interpreting Remainders.*
- Each small group of students will need a packet of Card Set A - E copied in **color** cut up before the lesson. *{Note: you may want to make color copies, and laminate these for use in multiple classes over multiple years.}*
- White boards/markers

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Division Representations & Interpreting Remainders*. Introduce the task briefly and help the class to understand the problem and its context.

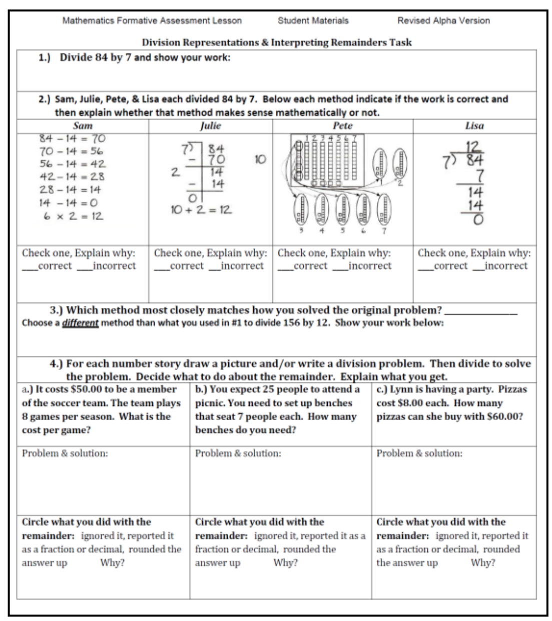
Frame the Task

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.


Mathematics Formative Assessment Lesson   Student Materials   Revised Alpha Version

Division Representations & Interpreting Remainders Task

1.) Divide 84 by 7 and show your work:

2.) Sam, Julie, Pete, & Lisa each divided 84 by 7. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

Sam	*Julie*	*Pete*	*Lisa*
84 – 14 = 70 70 – 14 = 56 56 – 14 = 42 42 – 14 = 28 28 – 14 = 14 14 – 14 = 0 6 × 2 = 12	7) 84 – 70 10 14 – 14 2 0 10 + 2 = 12	1 2 3 4 5 6 7 1 2 3 4 5 6 7	12 7) 84 7 14 14 0
Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect

3.) Which method most closely matches how you solved the original problem? ___________
Choose a *different* method than what you used in #1 to divide 156 by 12. Show your work below:

4.) For each number story draw a picture and/or write a division problem. Then divide to solve the problem. Decide what to do about the remainder. Explain what you get.

a.) It costs $50.00 to be a member of the soccer team. The team plays 8 games per season. What is the cost per game?	**b.) You expect 25 people to attend a picnic. You need to set up benches that seat 7 people each. How many benches do you need?**	**c.) Lynn is having a party. Pizzas cost $8.00 each. How many pizzas can she buy with $60.00?**
Problem & solution:	Problem & solution:	Problem & solution:
Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	**Circle what you did with the remainder:** ignored it, reported it as a fraction or decimal, rounded the answer up Why?	**Circle what you did with the remainder:** ignored it, reported it as a fraction or decimal, rounded the answer up Why?

It is important that students answer the question without assistance, as far as possible.
If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses: **Remind students they should know most of the content and this is a review for me to see what we need to review on.**

- Collect students' responses to the task. We suggest that you do not actually score student's work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.
- Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions in the *Common Misconceptions* chart may serve as examples. These questions have been drawn from commonly identified student misconceptions.
- Make notes about what their work reveals about their current levels of understanding and their different problem solving approaches.
- Strategically partner students based on pre-assessment data. Partner students with others who display similar errors/misconceptions on the pre-assessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

We recommend you:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions and highlight the questions for each individual student or
- display a small list of questions on the board that will be of help to the majority of students
- The solution to all these difficulties is not to teach one particular way of solving a problem, but to help students to find a variety of ways that work in different situations and make sense to them.

Below is a list of common misconceptions and questions/prompts that may be written on individual tasks, on the board or asked during the collaborative activity to help students clarify and extend their thinking.

Common issues:	**Suggested questions and prompts:**
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of Division.	• If you are dividing 27 ÷ 4, what does the 2 represent? the 7? • What would happen if you divided 20 ÷ 4 and 7 ÷ 4? Could you use those answers and to get the answer to 27÷4?
Student doesn't understand Grouping arrangements	• How can these number(s) we are divided be broken apart?

	• What could you do with <u>those</u> numbers to solve this problem?
Student doesn't understand the model and how it represents a division problem with remainders.	• In the problem 27÷14 let's look at the number 27. How many 10s are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14? • Is there a way to take those two models and fit them on a rectangle to discover 27÷14 without doing any calculations?
Students don't know what to do with the remainder nor how to interpret it.	• What does this remainder represent?

Suggested lesson outline

Whole class introduction (10 minutes)

Each student needs a white board and pen.

Teacher says: *Today we are going to do more work with solving problems. Pose the following problem.*

The fifth grade is taking a field trip. We need to rent minivans to get to our destination. There are 125 people going. Each minivan can only hold 8 people. How many minivans does the school need to rent?

Turn to a shoulder partner and discuss two ways you might solve this problem. Use your white boards to solve this problem.

Listen and observe as students attempt to solve the problem. Ask two or three students with different strategies to share their ideas with the class.

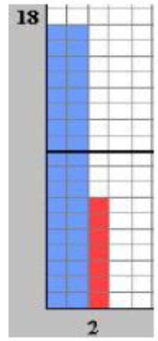
Be careful to not reteach content, instead, allow students to share their thoughts and strategies.

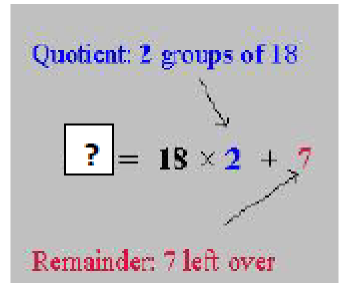
Collaborative Activity: matching *Card Sets Models A, B, C, D and E* (30 minutes)
Strategically partner students based on pre-assessment data. Partner students with others who display similar errors/misconceptions on the pre-assessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

Give each group *Card Sets A* and B – Models & Groupings.
Introduce the lesson carefully:

I want you to work as a team. Take it in turns to match a Model card a Grouping card. Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.





As the teacher, your tasks during the partner work are to make a note of student approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes.

Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors once the other sets of cards are added.

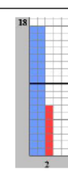
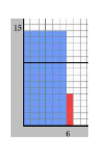
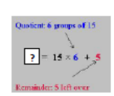
For students struggling to get started:

There is more than one way to tackle this task.

Can you think what one of them might be?

How can you calculate quotients with the model? with the groupings?

This Groupings card shows 18 x 2 + 7 ? What would the original division problem be for this model? Does that division problem match any of the other cards on the table?

Model Set A	Groupings Set B	Problem Set C	Remainder Set D	Interpreting Remainders Set E
		Sally scooped out forty-three pieces of hard candy to buy at the store. She wants to divide the candy evenly among eighteen people. How many pieces of candy will each person get?	2	Ignore the remainder.
		You are organizing a trolley ride for ninety-five total students, teachers & parents. If each trolley can seat fifteen people, how many trolleys do you need?	7	Round up the remainder.

If one student has placed a particular card, challenge their partner to provide an explanation.

Maria placed this Grouping card with this Model. Martin, why has Maria placed it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue**,** then you may want to write a couple of questions on the board and organize a whole class discussion.

Placing *Card Sets C, D & E: Word Problems, Remainder, & Interpreting Remainders*

As students finish placing the Model & Groupings cards, hand out *Card Sets C, D, & E: Word Problems, Remainder, & Interpreting Remainders*. These provide students with different ways of interpreting the situation.

Do not collect the card sets they have been using. An important part of this task is for students to make connections between all the different representations of multiplication problems.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. Groups should have 7 different clusters of cards with 5 cards in each. The original cards show the correct matches on each row of the table they are originally arranged in.

Sharing work (10 minutes)

When students get as far as they can with matching cards*,* ask one student from each pair to visit another pair's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

Teacher says: *If you are staying at your desk, be ready to explain the reasons for your partner's matches. If you are visiting another partner, make note of your card placements on a piece of paper. Go to another partner's desk and check to see which matches are different from your own. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.*

Students may now want to make changes.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Division Representations & Interpreting Remainders* as well as a second blank copy of the task.

Teacher says: *Look at your original responses and think about what you have learned this lesson. Using what you have learned, try to improve your work.*
If you have not added questions to individual pieces of work then write your list of questions on the board.

Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Whole Class Wrap up Discussion

Ask students:

- *Which representation/strategy did you find the most difficult to understand? Why?*
- *Which easiest? Why?*
- *Which problem was the most challenging? Why?*
- *How might you use these representations in other situations?*

Solutions

Assessment Task: Division Representations & Interpreting Remainders
Question 1: Each of the responses from Sam, Julie, Pete, Lisa, & Fred are all correct. Sam used repeated subtraction, Julie used a variation of partial quotients, Pete drew base-10 blocks to show equal shares and Lisa used the traditional division algorithm. Be sure that the responses have correct interpretations of each model, but answers may vary in the way each is described.

Question 2: Student should choose a strategy to solve 156 ÷ 12 = 13 correctly.

Question 3:

a.) \$50.00 ÷ 8 = \$6 R 2 (or \$6.25) Report remainder as a decimal because it is money.
b.) 25 ÷ 7 = 3 R 4 Round the remainder up because the 4 people will need a bench to sit on.
c.) \$60.00 ÷ \$8.00 = 7 R 4 Ignore the remainder because you don't have enough left over to buy a whole pizza.

<u>**Rationale for the card sets**</u>
Sally scooped out forty-three pieces of hard candy to buy at the store. She wants to divide the candy evenly among eighteen people. How many pieces of candy will each person get?
43 ÷ 18 = 2 remainder 7 but each person only gets 7 to keep the each person's share fair, each only gets 2 pieces

You are organizing a trolley ride for ninety-five total students, teachers & parents. If each trolley can seat fifteen people, how many trolleys do you need?
95 ÷ 15 = 6 remainder 5 but you need 7 trolleys, however you need another whole trolley to make sure everyone has a ride

Mr. Jones bought ninety-five new pencils to give his class of nineteen students. How many pencils will each student get?
95 ÷ 19 = 5 no remainder

The soccer team bought their coach a \$55.00 sweatshirt. The fifteen players split the bill evenly. How much did each pay?
55 ÷ 15 = 3 remainder 10 or 3.67
However since this is money the remainder can be written as a decimal so each player pays \$3.67

Compact discs are on sale for \$13.00 including tax. How many can you buy with \$84.00?

84 ÷ 13 = 6 remainder 6
So you could only by 6 CDs and you have 6 dollars left over

There are eighty-four girls in a basketball league and six girls on each team. How many teams are there?
84 ÷ 6 = 14 no remainder

The twelve cheerleaders each want a piece of pink ribbon to wear for the breast cancer march. There is eighty-seven inches of ribbon. How much ribbon should each girl get?
87 ÷ 12 = 7 remainder 3 or 7 3/12 which reduces to 7 ¼
However since the ribbon is cut in inches each girl would get 7 ¼ inches of ribbon.

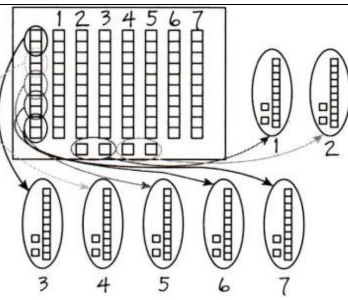
These materials were adapted from *Everyday Mathematics, Uncovering Student Misconceptions in Mathematics,* and the *National Library of Virtual Manipulatives*.

When teaching these multiplication and division strategies, *Teaching Student Centered Mathematics* by Van de Walle will be a useful resource.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

Division Representations and Interpreting Remainders Task

Name________________________

1.) Sam, Julie, Pete, & Lisa each divided 84 by 7. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.			
Sam	***Julie***	***Pete***	***Lisa***
84 − 14 = 70 70 − 14 = 56 56 − 14 = 42 42 − 14 = 28 28 − 14 = 14 14 − 14 = 0 6 × 2 = 12	7) 84 − 70 10 14 − 14 2 0 10 + 2 = 12	 	12 7) 84 7 14 14 0
Check one, Explain why: ____correct ____incorrect	Check one, Explain why: ____correct ___incorrect	Check one, Explain why: ____correct ____incorrect	Check one, Explain why: ____correct ___incorrect

2.) Choose one of the methods in #1 to divide 156 by 12. Show your work below:

3.) For each number story draw a picture and/or write a division problem. Then divide to solve the problem. Decide what to do about the remainder. Explain what you get.		
a.) It costs $50.00 to be a member of the soccer team. The team plays 8 games per season. What is the cost per game?	***b.) You expect 25 people to attend a picnic. You need to set up benches that seat 7 people each. How many benches do you need?***	***c.) Lynn is having a party. Pizzas cost $8.00 each. How many pizzas can she buy with $60.00?***
Problem & solution:	Problem & solution:	Problem & solution:

Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	**Circle what you did with the remainder:** ignored it, reported it as a fraction or decimal, rounded the answer up Why?	**Circle what you did with the remainder:** ignored it, reported it as a fraction or decimal, rounded the answer up Why?

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Set B</th><th>Problem
Set C</th><th>Solution to
Problem
Set D</th><th>Interpreting
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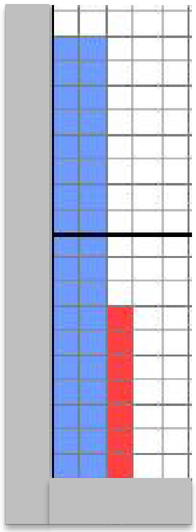
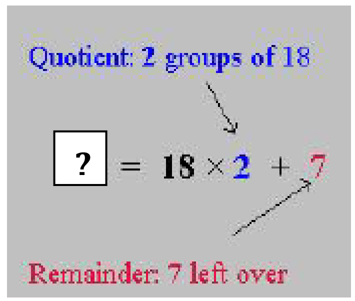
<tr><td></td><td>
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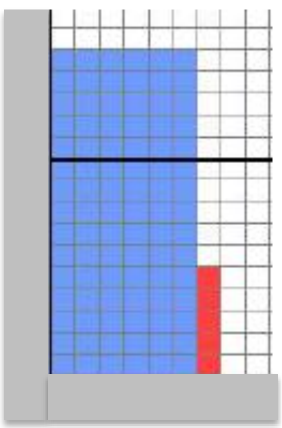
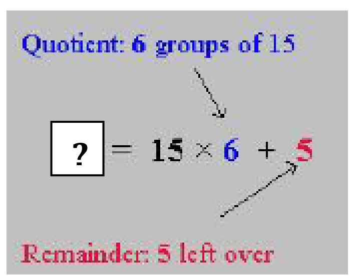
<tr><td></td><td>
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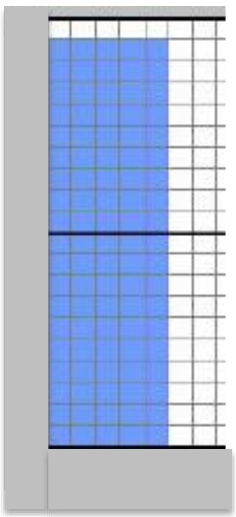
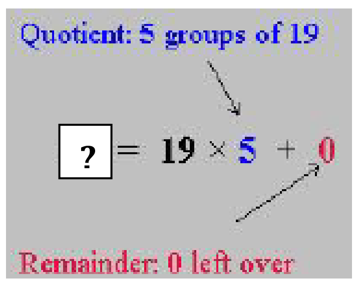
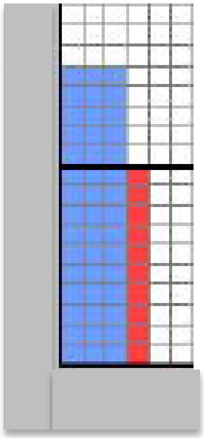
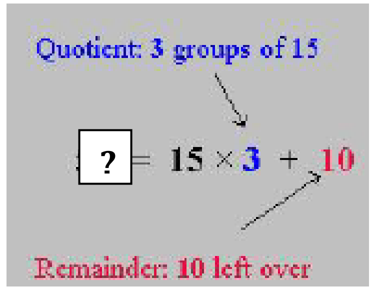
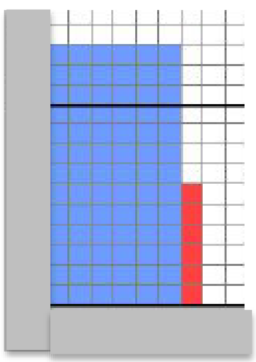
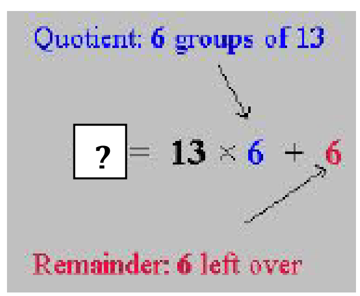
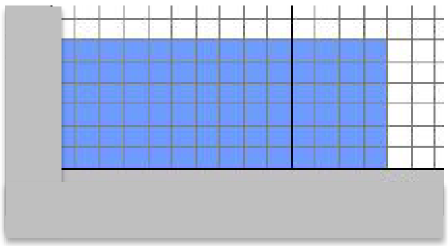
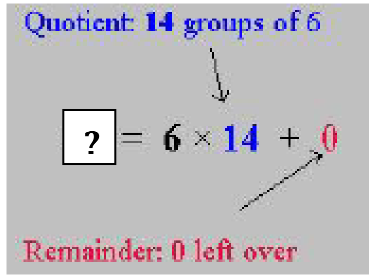
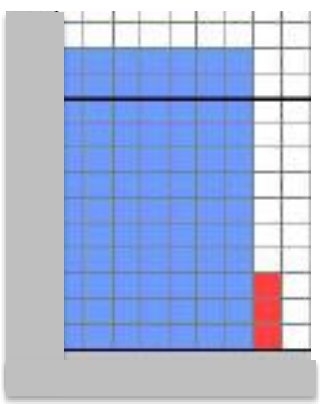
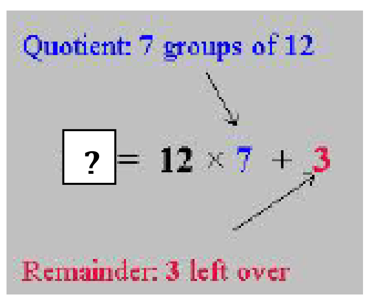
Model **Set A**	**Groupings** **Set B**	**Problem** **Set C**	**Solution to Problem** **Set D**	**Interpreting Remainders** **Set E**
		**Mr. Jones bought ninety-five new pencils to give his class of nineteen students. How many pencils will each student get?**	**5**	**There is no remainder.**

		The soccer team bought their coach a $55.00 sweatshirt. The fifteen players split the bill evenly. How much did each pay?	**3.67**	**Write the remainder as a decimal.**
Model **Set A**	**Groupings** **Set B**	**Problem** **Set C**	**Solution to Problem** **Set D**	**Interpreting Remainders** **Set E**
		Compact discs are on sale for $13.00 including tax. How many can you buy with $84.00?	**6**	**Ignore the remainder.**

		**There are eighty-four girls in a basketball league and six girls on each team. How many teams are there?**	**14**	**There is no remainder.**
Model Set A	**Groupings Set B**	**Problem Set C**	**Solution to Problem Set D**	**Interpreting Remainders Set E**
		The twelve cheerleaders each want a piece of pink ribbon to wear for the breast cancer march. There is eighty-seven inches of ribbon. How much ribbon should each girl get?	**7 ¼**	**Write the remainder as a fraction.**