Kentucky Academic Standards

Mathematics
Adopted 2019

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INTRODUCTION

Background

In order to create, support and sustain a culture of equity and access across Kentucky, teachers must ensure the diverse needs of all learners are met. Educational objectives must take into consideration students' backgrounds, experiences, cultural perspectives, traditions and knowledge. Acknowledging and addressing factors that contribute to different outcomes among students are critical to ensuring all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the necessary support to be successful. Addressing equity and access includes both ensuring all students attain mathematics proficiency and achieving an equitable percentage of all students attaining the highest levels of mathematics achievement (Adapted from the National Council of Teachers of Mathematics Equity and Access Position, 2018).

Kentucky's Vision for Students

Knowledge about mathematics and the ability to apply mathematics to solve problems in the real world directly align with the Kentucky Board of Education's (KBE) vision that "each and every student is empowered and equipped to pursue a successful future." To equip and empower students, the following capacity and goal statements frame instructional programs in Kentucky schools. They were established by the Kentucky Education Reform Act (KERA) of 1990, as found in Kentucky Revised Statute (KRS) 158.645 and KRS 158.6451. All students shall have the opportunity to acquire the following capacities and learning goals:

- Communication skills necessary to function in a complex and changing civilization;
- Knowledge to make economic, social and political choices;
- Core values and qualities of good character to make moral and ethical decisions throughout life;
- Understanding of governmental processes as they affect the community, the state and the nation;
- Sufficient self-knowledge and knowledge of their mental health and physical wellness;
- Sufficient grounding in the arts to enable each student to appreciate their cultural and historical heritage;
- Sufficient preparation to choose and pursue their life's work intelligently; and
- Skills to enable students to compete favorably with students in other states

Furthermore, schools shall:

- Expect a high level of achievement from all students.
- Develop their students' ability to:
    - Use basic communication and mathematics skills for purposes and situations they will encounter throughout their lives;

    - Apply core concepts and principles from mathematics, the sciences, the arts, the humanities, social studies, English/language arts, health, practical living, including physical education, to situations they will encounter throughout their lives;
    - Become self-sufficient individuals;
    - Become responsible members of a family, work group or community as well as an effective participant in community service;
    - Think and solve problems in school situations and in a variety of situations they will encounter in life;
    - Connect and integrate experiences and new knowledge from all subject matter fields with what students have previously learned and build on past learning experiences to acquire new information through various media sources;
    - Express their creative talents and interests in visual arts, music, dance, and dramatic arts.
- Increase student attendance rates.
- Reduce dropout and retention rates.
- Reduce physical and mental health barriers to learning.
- Be measured on the proportion of students who make a successful transition to work, postsecondary education and the military.

To ensure legal requirements of these courses are met, the Kentucky Department of Education (KDE) encourages schools to use the *Model Curriculum Framework* to inform development of curricula related to these courses. The *Model Curriculum Framework* encourages putting the student at the center of planning to ensure that

> *…the goal of such a curriculum is to produce students that are ethical citizens in a democratic global society and to help them become self-sufficient individuals who are prepared to succeed in an ever-changing and diverse world. Design and implementation requires professionals to accommodate the needs of each student and focus on supporting the development of the whole child so that all students have equitable access to opportunities and support for maximum academic, emotional, social and physical development.*
>
> *(Model Curriculum Framework,* page 19*)*

Legal Basis

The following Kentucky Administrative Regulations (KAR) provide a legal basis for this publication:

704 KAR 8:040 Kentucky Academic Standards for Mathematics

Senate Bill 1 (2017) calls for the KDE to implement a process for establishing new, as well as reviewing all approved academic standards and aligned assessments beginning in the 2017-18 school year. The current schedule calls for content areas to be reviewed each year and every six years thereafter on a rotating basis.

The KDE collects public comment and input on all of the draft standards for 30 days prior to finalization.

Senate Bill 1 (2017) called for content standards that

- focus on critical knowledge, skills and capacities needed for success in the global economy;
- result in fewer but more in-depth standards to facilitate mastery learning;

- communicate expectations more clearly and concisely to teachers, parents, students and citizens;
- are based on evidence-based research;
- consider international benchmarks; and
- ensure the standards are aligned from elementary to high school to postsecondary education so students can be successful at each education level.

704 KAR 8:040 adopts into law the *Kentucky Academic Standards for Mathematics*.

Standards Creation Process

The standards creation process focused heavily on educator involvement. Kentucky's teachers understand elementary and secondary academic standards must align with postsecondary readiness standards and with state career and technical education standards. This process helped to ensure students are prepared for the jobs of the future and can compete with those students from other states and nations.

The Mathematics Advisory Panel was composed of twenty-four teachers, three public post-secondary professors from institutions of higher education and two community members. The function of the Advisory Panel was to review the standards and make recommendations for changes to a Review Development Committee. The Mathematics Standards Review and Development Committee was composed of eight teachers, two public post-secondary professors from institutions of higher education and two community members. The function of the Review and Development Committee was to review findings and make recommendations to revise or replace existing standards.

Members of the Advisory Panels and Review and Development Committee were selected based on their expertise in the area of mathematics, as well as being a practicing teacher in the field of mathematics. The selection committee considered statewide representation, as well as both public secondary and higher education instruction, when choosing writers (Appendix B).

Writers' Vision Statement

The Kentucky Mathematics Advisory Panel and the Review and Development Committee shared a vision for Kentucky's students. In order to equip students with the knowledge and skills necessary to succeed beyond K-12 education, the writers consistently placed students at the forefront of the Mathematics standards revision and development work. The driving question was simple, "What is best for Kentucky students?" The writers believed the proposed revisions will lead to a more coherent, rigorous set of *Kentucky Academic Standards for Mathematics*. These standards differ from previous standards in that they intentionally integrate content and practices in such a way that every Kentucky student will benefit mathematically. Each committee member strived to enhance the standards' clarity and function so Kentucky teachers would be better equipped to provide high quality mathematics for each and every student. The resulting document is the culmination of the standards revision process: the production of a high quality set of mathematics standards to enable graduates to live, compete and succeed in life beyond K-12 education.

The KDE provided the following foundational documents to inform the writing team's work:

- Review of state academic standards documents (Arizona, California, Indiana, Iowa, Kansas, Massachusetts, New York, North Carolina and other content standards).

Additionally, participants brought their own knowledge to the process, along with documents and information from the following:

- Clements, D. (2018). *Learning and teaching with learning trajectories.* Retrieved from: http:www.learningtrajectories.org/.
- Van De Walle, J., Karp, K., & Bay Williams, J. (2019). *Elementary and middle school mathematics teaching developmentally tenth edition*. New York, NY: Pearson.
- Achieve. (2017). *Strong standards: A review of changes to state standards since the Common Core*. Washington, DC. Achieve.

The standards also were informed by feedback from the public and mathematics community. When these standards were open for public feedback, 2,704 comments were provided through two surveys. Furthermore, these standards received feedback from Kentucky higher education members and current mathematics teachers. At each stage of the feedback process, data-informed changes were made to ensure the standards would focus on critical knowledge, skills and capacities needed for success in the global economy.

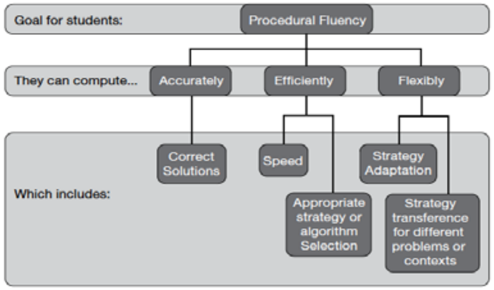
Design Considerations

The K-12 mathematics standards were designed for students to become mathematically proficient. By aligning to early numeracy trajectories which are levels that follow a developmental progressions based on research, focusing on conceptual understanding and building from procedural skill and fluency, students will perform at the highest cognitive demand-solving mathematical situations using the modeling cycle.

- Early numeracy trajectories provide mathematical goals for students based on research through problem solving, reasoning, representing and communicating mathematical ideas. Students move through these progressions in order to view mathematics as sensible, useful and worthwhile to view themselves as capable of thinking mathematically. (Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development [National Science Foundation, grant number ESI-9730804; see www.gse.buffalo.edu/org/buildingblocks/).
- Conceptual understanding refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. (Adapted from National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.)
- Procedural skill and fluency is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency (National Council Teachers of Mathematics, 2014).

Fluency in Mathematics

Wherever the word fluently appears in a content standard, the meaning denotes efficiency, accuracy, flexibility and appropriateness. Being fluent means students flexibly choose among methods and strategies to solve contextual and mathematical problems, understand and explain their approaches and produce accurate answers efficiently.



Efficiency—carries out easily, keeps track of sub-problems and makes use of intermediate results to solve the problem.

Accuracy—produces the correct answer reliably.

Flexibility—knows more than one approach, chooses a viable strategy and uses one method to solve and another method to double check.

Appropriately—knows when to apply a particular procedure.

- Application provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.

- The Modeling Cycle is essential in providing opportunities for students to reason and problem solve. In the course of a student's mathematics education, the word "model" is used in a variety of ways. Several of these, such as manipulatives, demonstration, role modeling and conceptual models of mathematics, are valuable tools for teaching and learning; however, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions using real-world context. Within the standards document, the mathematical modeling process could be used with standards that include the phrase "solve real-world problems." (*GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education,* Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2016. View the entire report, available freely online, at https://siam.org/Publications/Reports/Detail/Guidelines-for-Assessment-and-Instruction-in-Mathematical-Modeling-Education).

The Modeling Process

The *Kentucky Academic Standards for Mathematics* declare Mathematical Modeling is a process made up of the following components:

Identify the problem: Students identify something in the real world they want to know, do or understand. The result is a question in the real world.

Make assumptions and identify variables: Students select information important in the question and identify relations between them. They decide what information and relationships are relevant, resulting in an idealized version of the original question.

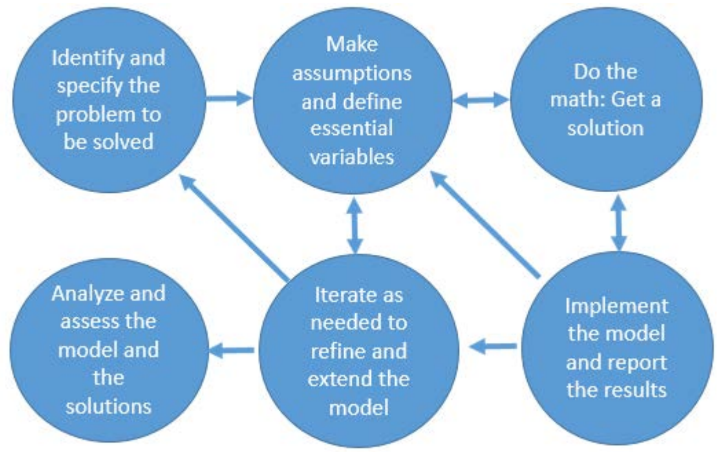
Do the math: Students translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. They do the math to derive insights and results.

Analyze and assess the solution: Students consider the following questions: Does it address the problem? Does it make sense when applied in the real world? Are the results practical? Are the answers reasonable? Are the consequences acceptable?

Iterate: Students iterate the process as needed to refine and extend a model.

Implement the model: Students report results to others and implement the solution as part of real-world, practical applications.

Mathematical modeling often is pictured as a cycle, with a need to come back frequently to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



STANDARDS USE AND DEVELOPMENT

<u>**The Kentucky Academic Standards (KAS) are Standards, not Curriculum**</u>

The *Kentucky Academic Standards for Mathematics* do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information. The order in which the standards are presented is not the order in which the standards need to be taught. Standards from various domains are connected and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards.

A standard represents a goal or outcome of an educational program. The standards do not dictate the design of a lesson or how units should be organized. The standards establish what students should know and be able to do at the conclusion of a course. The instructional program should emphasize the development of students' abilities to acquire and apply the standards. The curriculum must assure appropriate accommodations are made for diverse populations of students found within Kentucky schools.

These standards are not a set of instructional or assessment tasks, rather statements of what students should be able to do after instruction. Decisions on how best to help students meet these program goals are left to local school districts and teachers.

Translating the Standards into Curriculum

The KDE does not require specific curriculum or strategies to be used to teach the *Kentucky Academic Standards (KAS)*. Local schools and districts choose to meet those minimum required standards using a locally adopted curriculum. As educators implement academic standards, they, along with community members, must guarantee 21st-century readiness to ensure all learners are transition-ready. To achieve this, Kentucky students need a curriculum designed and structured for a rigorous, relevant and personalized learning experience, including a wide variety of learning opportunities. The *Kentucky Model Curriculum Framework* serves as a resource to help an instructional supervisor, principal and/or teacher leader revisit curriculum planning, offering background information and exercises to generate “future-oriented” thinking while suggesting a process for designing and reviewing the local curriculum.

Organization of the Standards

The *Kentucky Academic Standards for Mathematics* reflect revisions, additions, coherence/vertical alignment and clarifications to ensure student proficiency in mathematics. The architecture of the K-12 standards has an overall structure that emphasizes essential ideas or conceptual categories in mathematics. The standards emphasize the importance of the mathematical practices; whereby, equipping students to reason and problem solve. To encourage the relationship between the standards for mathematical practice and content standards, both the Advisory Panel and the Review and Assessment Development Committee intentionally highlighted possible connections, as well as provided cluster level examples of what this relationship may look like for Kentucky students. The use of mathematical practices demonstrates various applications of the standards and encourages a deeper understanding of the content.

The standards also emphasize procedural skill and fluency, building from conceptual understandings to application and modeling with mathematics, in order to solve real world problems. Therefore, both committees decided to incorporate the clarifications section to communicate expectations more clearly and concisely to teachers, parents, students and stakeholders through examples and illustrations. The standards are sequenced in a way that make mathematical sense and are based on the progressions for how students learn. To emphasize the cohesiveness of the K-12 standards, both committees decided to include Coherence/Vertical Alignment indicating a mathematics connection within and across grade levels.

- The K-5 standards maintain a focus on arithmetic, providing a solid foundation for later mathematical studies and expect students to know single-digit sums and products from memory, not memorization.
- The 6-8 standards serve as the foundation for much of everyday mathematics, which serve as the connection between earlier work in arithmetic and the future work of the mathematical demands in high school.

- The high school standards are complex and based on conceptual categories with a special emphasis on modeling (indicated with a star) which encompasses the process by which mathematics is used to describe the real world.

How to Read the Standards for Mathematical Content and the Standards for Mathematical Practice

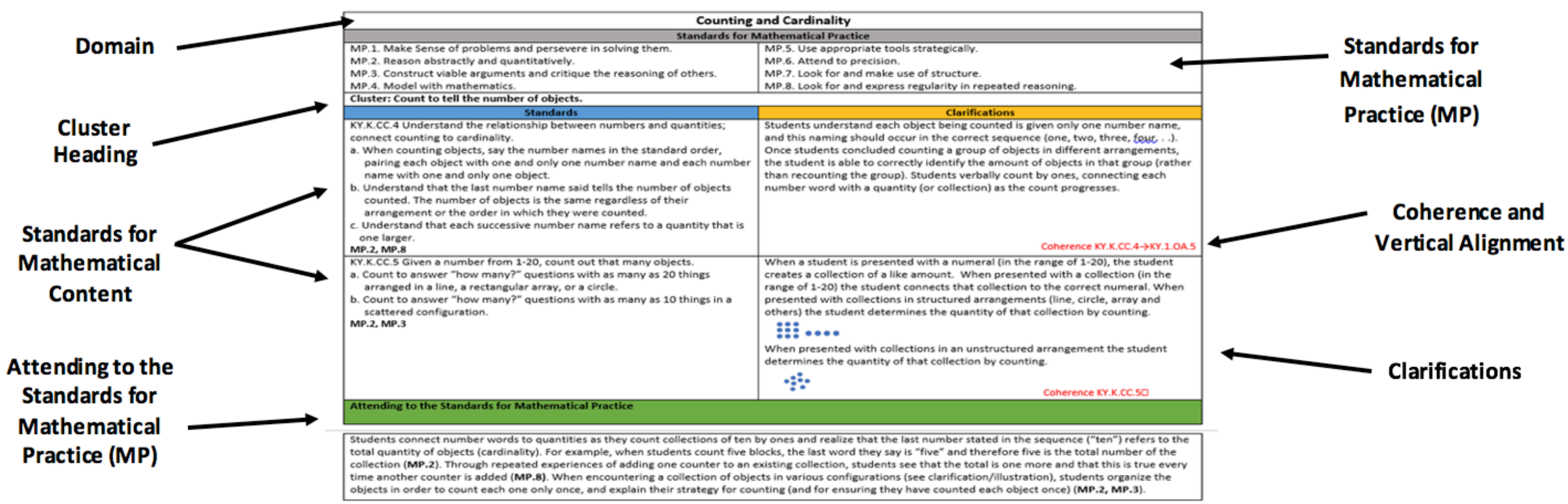
Domains are large groups of related standards. Standards from different domains sometimes may be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters sometimes may be closely related, because mathematics is a connected subject.

Standards for Mathematical Content define what students should understand and be able to do.

Standards for Mathematical Practice define how students engage in mathematical thinking.

The standards for mathematical content and the standards for mathematical practice are the sections of the document that identify the critical knowledge and skills for which students must demonstrate mastery by the end of each grade level.



How to Read the Coding of the Standards

Additional High School Coding

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Plus Plus (++) Standards: Indicate a standard that is optional even for calculus.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation and connections. The second are the strands of mathematical proficiency specified in the National Research Council's 2001 report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand other approaches to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students provide carefully formulated explanations to each other. By the time they reach high school, they can examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also are able to shift perspectives. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

<u>Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content</u>

The Standards for Mathematical Practice describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure, understanding and application. Expectations that begin with the word "understand" are often good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources and innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

<u>Supplementary Materials to the Standards</u>

The *Kentucky Academic Standards for Mathematics* are the result of educator involvement and public feedback. Short summaries of each of the appendices are listed below.

<u>Appendix A: Tables</u>

Mathematic tables are used throughout the *Kentucky Academic Standards for Mathematics* to provide clarity to the standards.

<u>Appendix B: Writing and Review Teams</u>

Background information on the writing team for the *Kentucky Academic Standards for Mathematics*.

Kentucky Academic Standards for Mathematics: Kindergarten Overview

Counting/Cardinality (CC)	Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Measurement and Data (MD)	Geometry (G)
• Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers.	• Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.	• Work with numbers 11-19 to gain foundations for place value.	• Describe and compare measurable attributes. • Classify objects and count the number of objects in each category. • Identify coins by name.	• Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres). • Analyze, compare, create and compose shapes.

In grade K, instructional time should focus on two critical areas:

1. In the Counting and Cardinality and Operations and Algebraic Thinking domains, students will:

- develop a more formal sense of numbers;
- use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. Note: Kindergarten students should see addition and subtraction equations and student writing of equations in kindergarten is encouraged, but it is not required; and
- choose, combine and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. In the Geometry and Measurement and Data domains, students will:

- describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and appropriate vocabulary;
- identify, name and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders and spheres; and
- use basic shapes and spatial reasoning to model objects in their everyday environment to create and compose more complex shapes.

Note: More learning time in Kindergarten should be devoted to number than to other topics.

Counting and Cardinality

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Know number names and the count sequence.

Standards	Clarifications
KY.K.CC.1 Count a. Count to 100 by ones and by tens. b. Count backwards from 30 by ones. **MP.7**, **MP.8**	Students verbally count forward by ones (1,2,3,4. . .) to 100 Students verbally count forward by tens (10, 20, 30. . .) to 100. Students verbally count backwards by ones (30, 29, 28, 27. . .) from 30. Coherence KY.K.CC.1→KY.1.NBT.1
KY.K.CC.2 Count forward beginning from a given number within the known sequence within 100 (instead of having to begin at 1). **MP.7**	Students verbally count forward starting at a number other than one (58, 59, 60, 61, 62. . .) within 100. Coherence KY.K.CC.2→KY.1.NBT.1
KY.K.CC.3 Represent numbers. a. Write numbers from 0 to 20. b. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **MP.2**, **MP.7**, **MP.8**	Students write all numerals in the range of 0-20 (1, 2, 3, 4, 5…) When students are given a written numeral, represent with objects within 20 (4… ★ ★ ★ ★). Coherence KY.K.CC.3→KY.1.NBT.1

Attending to the Standards for Mathematical Practice

Students notice repetition inherent in the counting sequence as they count to one hundred by ones and tens. For example, students notice “seven” follows “six,” and “twenty-seven” follows “twenty-six” (**MP.8**). They describe how this pattern exists into new decade families. For example, thirty-seven follows thirty-six and so on. Students use this general pattern about how numbers are structured to count forward from any given number within the range of 0-100 (counting on) without the benefit of starting at “one” (**MP.7**). When counting objects within the range of 0-20, they understand they can communicate this total using words, for example “ten” and the numeral 10. (**MP.2**)

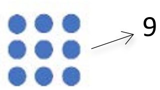
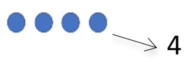
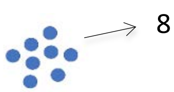
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Counting and Cardinality

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Count to tell the number of objects.

Standards	Clarifications
KY.K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **MP.2**, **MP.8**	Students understand each object being counted is given only one number name and this naming occurs in the correct sequence (one, two, three, four. . .). Once students concluded counting a group of objects in different arrangements, the student correctly identifies the amount of objects in that group (rather than recounting the group). Students verbally count by ones, connecting each number word with a quantity (or collection) as the count progresses. Coherence KY.K.CC.4→KY.1.OA.5
KY.K.CC.5 Given a number from 1-20, count out that many objects. a. Count to answer "how many?" questions with as many as 20 things arranged in a line, a rectangular array, or a circle. b. Count to answer "how many?" questions with as many as 10 things in a scattered configuration. **MP.2**, **MP.3**	When presented with a numeral (in the range of 1-20), the student creates a collection of a like amount. When presented with a collection (in the range of 1-20) the student connects that collection to the correct numeral. When presented with collections in structured arrangements (line, circle, array and others) the student determines the quantity of that collection by counting.   When presented with collections in an unstructured arrangement the student determines the quantity of that collection by counting.  Coherence KY.K.CC.5→KY.1.NBT.1

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students connect number words to quantities as they count collections of ten by ones and realize the last number stated in the sequence (“ten”) refers to the total quantity of objects (cardinality). For example, when students count five blocks, the last word they say is “five” and therefore five is the total number of the collection (**MP.2**). Through repeated experiences of adding one counter to an existing collection, students see that the total is one more and know this is true every time another counter is added (**MP.8**). When encountering a collection of objects in various configurations (see clarification), students organize the objects in order to count each one only once and explain their strategy for counting and for ensuring they have counted each object once (**MP.2**, **MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Counting and Cardinality

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Compare numbers.**

Standards	Clarifications
KY.K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. **MP.1**, **MP.3**, **MP.6**	Compare two collections (each containing up to 10 objects) to determine whether one collection is greater than, less than, or equal to the other. Students use matching strategies (pairing items from the collections) or counting strategies (counting one collection and then the other). Note: Students do not need to use the relation symbols greater than (>), less than (<) and equal to (=) to compare groups of objects. Coherence KY.K.CC.6→KY.K.MD.3
KY.K.CC.7 Compare two numbers between 1 and 10 presented as written numerals. **MP.2**	When presented with two numerals (between 1 and 10), students determine which numeral is greater than, less than, or equal to the other. Students express some mathematical reasoning regarding their determination (5 is larger than 3 because it has two more). Note: Students do not need to use the relation symbols greater than (>), less than (<) and equal to (=) to compare numbers between 1 and 10. Coherence KY.K.CC.7→KY.1.NBT.3

Attending to the Standards for Mathematical Practice

Students know different strategies for comparing groups and choose a strategy such as counting, matching and pairing to compare two groups (**MP.1**). For example, when comparing a collection of red counters to a collection of blue counters, students count each group finding which has the greater number, pair off blues and reds to see which group has extras, or make two rows and line them up to see which is longer (**MP.2**). Once a determination has been made, students articulate their ideas using precise mathematical language such as “greater than,” “less than,” and “equal to” (**MP.6**, **MP.3**). When comparing two numerals, students move flexibly between symbols and their corresponding quantities, using objects or situations to help them reason about the relative size of each quantity (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestion; however, they are not the only pathways.

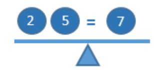
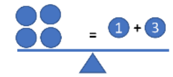
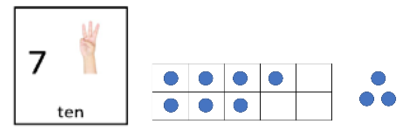
Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

Standards	Clarifications
KY.K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. **MP.2**, **MP.4**	Students flexibly model or represent addition and subtraction tasks across a range of contexts rather than just becoming proficient with a single model or representation. **See Table 1 in Appendix A.** Note: Drawings need not show detail but should accurately represent the quantities involved in the task. Coherence KY.K.OA.1→KY.K.OA.2
KY.K.OA.2 Solve addition and subtraction word problems and add and subtract within 10 by using objects or drawings to represent the problem. **MP.5**	Students flexibly model or represent addition and subtraction situations or context problems (involving sums and differences up to 10). **See Table 1 in Appendix A.** Note: Drawings need not show detail but accurately represent the quantities involved in the task. Coherence KY.K.OA.2→KY.1.OA.1
KY.K.OA.3 Decompose numbers less than or equal to 10. a. Decompose numbers into two groups in more than one way by using objects or drawings and record each decomposition by a drawing or equation. b. Use objects or drawings to demonstrate equality as the balancing of quantities. **MP.2**, **MP.4**	When presented with a numeral or collection (10 or less), the student separates that amount into two groups or collections via drawings or objects. Note: Drawings need not show detail, but accurately represent the quantities involved in the task.  Students represent an equation as the balance of quantities. Note: Drawings need not show detail, but accurately represent the quantities involved in the task.

Standards	Clarifications
	  Coherence KY.K.OA.3→KY.1.OA.6
KY.K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings and record the answer with a drawing or equation. **MP.7**, **MP.8**	When presented with a numeral or collection of objects between 1-9, represent the corresponding number that makes 10 with objects or drawings. Students record these combinations using either drawings or numbers. Drawings need not show detail, but accurately represent the quantities involved in the task.  Coherence KY.K.OA.4→KY.1.OA.6
KY.K.OA.5 Fluently add and subtract within 5. **MP.2**, **MP.7**	Students solve addition and subtraction tasks (with sums and differences within 5) efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Students express mathematical reasoning regarding their responses ("5-3 equals 2 because when you move three back, you land on two"). Note: Reaching fluency is an ongoing process that will take much of the year. Coherence KY.K.OA.5→KY.1.OA.6

Attending to the Standards for Mathematical Practice

Students use tools and models to interpret, represent and solve word problems. They make sense of addition and subtraction situations by selecting objects to represent the situation (**MP.1**) and represent the situations using an expression or equations (see clarifications) (**MP.4**). For example, students act out a story problem involving the eating of apples using cubes to represent each apple (**MP.5**). Students decomposed numbers into two subgroups in different ways and understand the subgroups do not need to be the same size, but combined they equal to original value (7) (**MP.2**). Students decompose a group of 7 objects into 3 and 4, 6 and 1, and 5 and 2. They write the related expressions (**MP.4**) and explain or show (using a balance or moving objects) these different arrangements are equal to each other and equal to 7 (**MP.2**). Students connect breaking apart 5 into 2 and 3, means 2 + 3 = 5. Beyond counting, students use visuals (dot patterns, five and ten frames) and tools such as counters and Rekenreks to determine sums within 5 and combinations of 10 (**MP.5**, **MP.7**). For example, students view a ten frame displaying 7 counters and see 3 more counters are needed to equal 10, or in seeing the sum 3 + 2 may visualize a dot pattern or notice 3 + 2 is 1 more than 2 + 2, a sum they know (**MP.2**).

Numbers and Operations in Base Ten	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Working with numbers 11-19 to gain foundations for place value.**	
Standards	**Clarifications**
KY.K.NBT.1 Compose and decompose numbers from 11 to 19 using quantities (numbers with units) of ten ones and some further ones. Understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **MP.3**, **MP.4**, **MP.7**	Using numbers or representations, students use 10 units as an anchor to compose and decompose quantities (up to 19). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. 16 triangles = 10 triangles + ΔΔΔΔΔΔ; 18 beans = 10 beans + 8 beans Coherence KY.K.NBT.1→KY.1.NBT.2
Attending to the Standards for Mathematical Practice	
Students explain a teen number can be broken apart into ten ones and some more ones (**MP.3**). They express this relationship using objects, drawings and corresponding equations (**MP.4**). For example, a student working with 16 counters places ten counters in a cup and leaves 6 counters on the table and represents this idea using the equation 16=10+6. Note the language of the standard does not require students to actually create the ten unit (that is in grade 1), but they recognize and break apart a teen number into ten ones and some more ones (**MP.7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Describe and compare measurable attributes.	
Standards	**Clarifications**
KY.K.MD.1 Describe measurable attributes (length, height, weight, width, depth) of an object or a set of objects using appropriate vocabulary. **MP.3**, **MP.6**	For a single object, students verbally identify more than one attribute measured (wooden block - height, weight). Coherence KY.K.MD.1→KY.1.MD.2
KY.K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/ "less of" the attribute and describe the difference. **MP.2**, **MP.6**	Students consider and compare a common measurable attribute shared by two objects (Which cup is taller and which is shorter? Which bucket of sand is heavier and which is lighter?). Coherence KY.K.MD.1→KY.1.MD.1

Attending to the Standards for Mathematical Practice
Students notice objects in the world around them have attributes and some of those attributes are measurable attributes. They describe measurable attributes using measuring language such as "heavy" and/or "long/short" (**MP.3**, **MP.6**). As students compare objects, they focus on a selected attribute, for example, length and then determine which object has more or less of that attribute, saying, this footprint is longer (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Classify objects and count the number of objects in each category.	
Standards	**Clarifications**
KY.K.MD.3 Classify and sort objects or people by attributes. Limit objects or people in each category to be less than or equal to 10. **MP.3**, **MP.6**	For a group of 10 (or less) objects/people, students compare and order objects according to a common measurable attribute (height, weight, length, width, depth) shared by the objects (arranging 4 blocks from heaviest to lightest; arranging classmates from tallest to shortest). Coherence KY.K.MD.3→KY.1.MD.4
Attending to the Standards for Mathematical Practice	
Students use their understanding of attributes to sort objects in different ways. They justify their rules for sorting, listen to the ideas of others and when they are unsure or disagree, they question or challenge the observations (**MP.3**). As they describe attributes, students use precise shape or measurement language such as “has all straight sides” or “is shorter than a new pencil” (**MP.6**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Identify coins by name.	
Standards	**Clarifications**
KY.K.MD.4 Recognize and identify coins by name (penny, nickel, dime, quarter). **MP.6**	Students identify coins (penny, nickel, dime, quarter) when presented. When shown a nickel, name the coin as a nickel; select a nickel when presented with a group of different coins. Note: Students need not identify the value of these coins. Coherence KY.K.MD.4→KY.1.MD.3b
Attending to the Standards for Mathematical Practice	
Students recognize the need for consistent, common language to identify coins (**MP.6**). For example, students understand that "nickel" is the name of a specific coin with a specific appearance and cannot be used to describe other coins of different appearances. Note the standard does not require students to identify values, only names.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Identify and describe shapes (squares**, **circles**, **triangles**, **rectangles**, **hexagons**, **cubes**, **cones**, **cylinders and spheres)**.	
Standards	**Clarifications**
KY.K.G.1 Name and describe shapes in the environment. a. Describe objects in the environment using names of shapes. b. Describe the relative positions of these objects using terms *above, below, in front of, behind and next to*. **MP.6**	For objects in student's environment, the student accurately provides a shape name (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres). ("The clock on the wall is a circle." "The desktop is a rectangle.") Students use positional language to describe the relationships between objects ("The clock is above the bulletin board." "My desk is next to the computer table.") Coherence KY.K.G.1 →KY.K.G.4
KY.K.G.2 Correctly name shapes regardless of orientations or overall size. **MP.7**	Students identify and name shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres) regardless of size, orientation, or positioning. (The classroom window is a rectangle and this paper is a rectangle, too.) Coherence KY.K.G.2→KY.K.G.4
KY.K.G.3 Identify shapes as two-dimensional or three-dimensional. **MP.3**, **MP.6**	When presented with a shape or object, students determine whether it is two-dimensional (square, circle, triangle, rectangle, or hexagon) or three-dimensional (cube, cone, cylinder, sphere). Students express mathematical reasoning regarding their responses. (The block is three-dimensional because it's thick and not flat like paper.) Coherence KY.K.G.3→KY.1.G.1
Attending to the Standards for Mathematical Practice	
Students use precise language to describe objects they encounter in their world and describe the locations of objects such as "up," "down," "above" and "below", as well as use language to describe characteristics of two- and three-dimensional shapes (**MP.6**). Students explain the location or position of an object does not change its attributes (**MP.7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Analyze, compare, create and compose shapes.**	
Standards	**Clarifications**
KY.K.G.4 Describe the similarities, differences and attributes of two and three dimensional shapes using different sizes and orientations. **MP.3**, **MP.7**	When considering two-dimensional shapes (square, circle, triangle, rectangle, hexagon) or objects and three dimensional shapes (cube, cone, cylinder, sphere) or objects, students describe similarities, differences and attributes. ("The window and paper are both rectangles, but the window sits sideways and my paper is long ways." "My book and my paper both look like rectangles, but my book is three-dimensional because it is thicker.") Coherence KY.K.G.4→KY.1.G.1
KY.K.G.5 Model shapes in the world by building figures from components and drawing shapes. **MP.1**, **MP.5**	Students construct and draw models of shapes (square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere) in the world around them. Students create shapes with materials that include but are not limited to straws, pipe cleaners, popsicle sticks or clay and describe the shape they create. (Students use sticks and a ball to replicate an ice cream cone.) Coherence KY.K.G.5→KY.1.G.1
KY.K.G.6 Compose simple shapes to form larger shapes. **MP.3**, **MP.5**	Students explore by using simple shapes to construct a larger shape. (Students arrange paper triangles to form a rectangle. Students arrange triangle pattern blocks to form a hexagon.) Coherence KY.K.G.6→KY.1.G.2
Attending to the Standards for Mathematical Practice	
Students use informal language as they compare objects; for example, sorting polygons by their relative size, or by a rule, such as "have three corners" (**MP.6**). Students analyze attributes of three-dimensional shapes; for example, noticing some have sides that all look like squares or rectangles, while others have sides that look like triangles (**MP.3**). Using a variety of tools, students construct objects that resemble items in their world (**MP.5**). As they construct and draw shapes, they recognize they are putting together shapes to form new larger shapes, just as they combine objects to have more objects (**MP.5**). Students analyze and describe shapes they form by combining shapes; for example, using pattern blocks or tangrams to build a design (**MP.3**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Grade 1 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Measurement and Data (MD)	Geometry (G)
• Represent and solve problems involving addition and subtraction. • Understand an apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations.	• Extend the counting sequence. • Understand place value. • Use place value understanding and properties of operations to add and subtract.	• Measure lengths indirectly and by iterating length in units. • Work with time and money. • Understand and apply the statistics process.	• Reason with shapes and their attributes.

In grade 1, instructional time should focus on four critical areas:

1. **In the Operations and Algebraic Thinking domain, students will:**
   - develop strategies for adding and subtracting whole numbers based on their prior work with small numbers;
   - use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take apart and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations;
   - understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two);
   - use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20; and
   - build their understanding of the relationship between addition and subtraction by comparing a variety of solution strategies.

2. **In the Number and Operations in Base Ten domain, students will:**
   - develop, discuss and use efficient, accurate and generalizable methods to add within 100 and subtract multiples of 10;
   - compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes;
   - think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones); and
   - understand the order of the counting numbers and their relative magnitudes through activities that build number sense.

3. **In the Measurement and Data domain, students will:**
   - develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.*

4. **In the Geometry domain, students will:**

- compose and decompose plane or solid figures and build understanding of part-whole relationships as well as the properties of the original and composite shapes;
- recognize them from different perspectives and orientations;
- describe their geometric attributes;
- determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

* Note: students should apply the principle of transitivity of measurement to make direct comparisons, but they need not use this technical term.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and solve problems using addition and subtraction.	
Standards	**Clarifications**
KY.1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions. **MP. 1**, **MP.2**	Students flexibly model or represent addition and subtraction situations or context problems (involving sums and differences up to 20). **See Table 1 in Appendix A**. Note: Drawings need not show detail, but accurately represent the quantities involved in the task. KY.1.MD.4 Coherence KY.K.OA.2→ KY.1.OA.1→KY.2.OA.1
KY.1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, by using objects, drawings and equations with a symbol for one unknown number to represent the problem. **MP. 1**, **MP.4**, **MP.5**	Students flexibly model or represent addition situations or context problems (involving adding three quantities and have a sum less than or equal to 20). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. KY.1.MD.4 Coherence KY.1.OA.2.→KY.2.NBT.6

Attending to the Standards for Mathematical Practice
Students realize mathematics involves interpreting the meaning of problems and endeavoring to solve problems by selecting useful and appropriate tools and manipulatives (**MP.1**, **MP.5**). When reading/interpreting word problems, students recognize a number (seven or 17) represents a quantity (7 dots or 17 people) and consider what is happening to these quantities in the context of the problem (**MP.2**). Students represent situations using numbers and symbols. For example, students translate “There are ten apples. Some were eaten. Three remain. How many were eaten?” into an equation such as 10 – __ = 3? (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Understand and apply properties of operations and the relationship between addition and subtraction.**

Standards	Clarifications
KY.1.OA.3 Apply properties of operations as strategies to add and subtract. **MP. 2**, **MP.7**	Students are not responsible for knowing the formal language of the different properties, but have the conceptual understanding of each property (commutative and associative property). Coherence KY.K.OA.2→KY.1.OA.3→KY.2.NBT.9
KY.1.OA.4 Understand subtraction as an unknown-addend problem. **MP. 2**, **MP.7**	Students connect addition and subtraction as operations. (I can solve 10 - 8 by thinking about what adds to 8 to make 10 [__+ 8 = 10].) Coherence KY.K.OA.2→KY.1.OA.4→KY.2.NBT.9

Attending to the Standards for Mathematical Practice

Students understand an equation such as 8 + 3 = 11, the numerals “8” and 3” represent two quantities combine to form a combined quantity of 11. Students explain the order in which the addends are combined does not affect the resulting sum (**MP.3**). Students generalize this idea (the commutative property) to all addition situations, for example, explaining that switching two piles of counters doesn’t change how many are there (**MP.7**). Similarly, students notice the order and manner in which multiple addends are combined does not affect the sum (the associative property). Students reason 10 - 8 = ? also means 8 + ? = 10; therefore, they solve the problem by asking themselves what is the number added to 8 to make 10 (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

<table>
<tr><th colspan="2">Operations and Algebraic Thinking</th></tr>
<tr><th colspan="2">Standards for Mathematical Practice</th></tr>
<tr><td>MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.</td><td>MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Cluster: Add and subtract within 20.</td></tr>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.1.OA.5 Relate counting to addition and subtraction.
MP.5, MP.8</td><td>Strategies used when relating addition to subtraction:
counting all (addition); counting on (addition); counting all (subtraction); counting back (subtraction); counting on (subtraction).
<table>
<tr><td>Counting all (addition)</td><td>Start with 1 and count to find the total number of objects</td><td>5 + 3
Start from 1, count up to 5, and count up 3 more: 1, 2, 3, 4, 5 . . . 6, 7, 8</td></tr>
<tr><td>Counting on (addition)</td><td>Count from the start number rather than starting at 1</td><td>5 + 3
Start at 5 . . . 6, 7, 8</td></tr>
<tr><td>Counting all (subtraction)</td><td>Remove the appropriate number of items and count the remaining items starting with 1</td><td>8 – 5
Start with 8 objects. Remove 5 and count the remaining items: 1, 2, 3</td></tr>
<tr><td>Counting back (subtraction)</td><td>Start with the total, count back the number being subtracted</td><td>8 – 5
Start at 8 and count back 5, one number at a time: 7, 6, 5, 4, 3</td></tr>
<tr><td>Count on (subtraction)</td><td>Start with the change number and count on to reach the total</td><td>8 – 5
Start with 5 and count up to 8 by ones (and later by larger numbers): 5 . . . 6, 7, 8</td></tr>
</table>
KY.K.CC.4→KY.1.OA.5→KY.1.OA.6</td></tr>
<tr><td>KY.1.OA.6 Add and subtract within 20.
a. Fluently add and subtract within 10.
b. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making 10; decomposing a number leading to a 10; using the relationship between addition and subtraction; creating equivalent but easier or known sums.
MP.2, MP.7, MP.8</td><td>Students solve addition and subtraction tasks (with sums and differences within 10) efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches, and they produce accurate answers efficiently.

Students make 10 (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decompose a number leading to a ten (13 - 4 = 13 – 3 - 1 = 10 – 1 = 9); know 8 + 4 = 12 and know 12 - 8 = 4 using the relationship between addition and subtraction; create equivalent, but easier or known sums, adding 6 + 7 by creating 6 + 6 + 1 = 12 + 1 = 13.

Note: Reaching fluency is an ongoing process that will take much of the year.
KY.1.NBT.4
Coherence KY.K.OA.2→KY.1.OA.6→KY.2.OA.2
KY.K.OA.3</td></tr>
</table>

Standards	Clarifications
	KY.K.OA.4 KY.K.OA.5

Attending to the Standards for Mathematical Practice
Students use tools to show sums and differences (**MP.5**). Students notice when they count two groups and count the total number of items, the total count is the sum (**MP.8**). Students employ counting strategies (forward and/or back) as strategies for adding and subtracting (**MP.2**). As students count on, they count on from the larger addend (solving 9 + 3 instead of 3 + 9) recognizing this is more efficient and addition is commutative (**MP.7**). Students recognize sums such as 8 + 9 are not efficiently solved by counting on and number relationships can be used to determine the sum. With repeated experiences, students notice relationships such as 9 + 8 = 10 + 7 (**MP.8**).

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Work with addition and subtraction equations.**	
Standards	**Clarifications**
KY.1.OA.7 Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. **MP. 2**, **MP.3**	Students determine which of the following equations are true and which are false: 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. Coherence KY.1.OA.7→KY.2.OA.4
KY.1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. **MP. 1**, **MP.2**	Students determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = __. KY.1.OA.7 Coherence KY.1.OA.8

Attending to the Standards for Mathematical Practice
Students make sense of equations such as 4 + 6 = 7 + 3, interpreting the equal sign to mean expressions on each side represent the same quantity (**MP.1**). Students justify whether an equation is true or false, not just by solving both sides, but by using relational thinking. For example, in the equation 10 + 5 = 6 + 11 students recognize both addends on the right are larger than the ones on the left, so the equation is false (**MP.3**). This reasoning is used to solve missing-value problems such as 8 + 5 = __ + 6. Students reason that because 6 is one more than 5, the missing addend must be one less than 8 (**MP.2**).

Numbers and Operations in Base Ten	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Extend the counting sequence.	
Standards	**Clarifications**
KY.1.NBT.1 Count and represent numbers. a. Count forward to and backward from 120, starting at any number less than 120. b. In this range, read and write numerals and represent a number of objects with a written numeral. **MP.2**, **MP.5**, **MP.8**	Students use strategies based on place value, properties of operations and the relationship between addition and subtraction; however, when solving any problem, students choose any strategy. A written representation shows a strategy using words, pictures and/or numbers. Coherence KY.K.CC.2→KY.1.NBT.1→KY.2.NBT.2
Attending to the Standards for Mathematical Practice	
Students recognize repeated sequences emerge as they cross into decade families and use those patterns to start a count from anywhere between 0 and 120 (**MP.8**). For example, counting within the 20s decade family involves repeated counting by ones in the range of 0-9 (20, 21, 22, 23…) and this pattern holds even as they go over 100 (100, 101, 102, 103…) (**MP.8**). In creating a representation of a number, students select a tool or picture that can be grouped to show tens and ones (**MP.5**). For example, students bundle sticks into 2 bundles of 10 and 3 remaining sticks, connect this to the numeral "23."	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand place value.**	
Standards	**Clarifications**
KY.1.NBT.2 Understand the two-digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones). **MP.5**, **MP.7**	Students use concrete models and drawings, as well as strategies based on place value, properties of operations, and the relationship between addition and subtraction. When solving any problem, students choose to use a concrete model or a drawing. Their strategy is based on place value, properties of operations or the relationship between addition and subtraction. A written representation shows a strategy using words, pictures and/or numbers. Coherence KY.K.NBT.1→KY.1.NBT.2→KY.2.NBT.1
KY.1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **MP. 2**	Students use tools such as objects on place value charts, tens frames, hundreds charts and number lines to compare two two-digit numbers. Students describe the comparisons using terms such as greater than, more than, less than, fewer than, equal to and same as. Students justify their reasoning. Students compare two two-digit numbers written as numerals. Coherence KY.K.CC.7→KY.1.NBT.3→KY.2.NBT.4
Attending to the Standards for Mathematical Practice	
Students understand the individual digits in a two-digit numeral each represent units of ten and one respectively. Students use tools to represent numbers, selecting tools such as popsicle sticks, linking cubes and straws that can physically be grouped in tens (**MP.5**). In representing numbers with concrete tools, students see one ten unit (a bundle) can be thought of as "10, two as twenty and so forth (**MP.7**). When comparing two two-digit numbers, students interpret the inherent value of each digit (22 is two tens with two remaining ones) and determine which number is larger (**MP.2**). For example, students realize that 32 is greater than 23 because of the value of its digits.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Use place value understanding and properties of operations to add and subtract.**	
Standards	**Clarifications**
KY.1.NBT.4 Add within 100 including adding a two-digit number and a one-digit number. Add a two-digit number and a multiple of 10. a. Add within 100 using… • concrete models or drawings; • strategies based on place value; • properties of operations; • the relationship between addition and subtraction. b. Relate the addition strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **MP.7**, **MP.2**, **MP.3**	Students model addition examples with sums to 100 using concrete materials, pictures and numerals. Students use mental computation strategies to develop conceptual understanding and number sense around adding one- and two-digit numbers. KY.2.NBT.7 Coherence KY.1.NBT.4→KY.2.NBT.5
KY.1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **MP.2**, **MP.8**	Students use materials and strategies to add or subtract 10 from any given number in the range 1 to 100. Coherence KY.1.NBT.5→KY.2.NBT.8
KY.1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences). a. Subtract using: • concrete models or drawings; • strategies based on place value; • properties of operations; • the relationship between addition and subtraction b. Relate the subtraction strategy to a written method and explain the reasoning used.	Students use strategies to subtract groups of ten from more tens. 80 – 30 can be expressed at 8 tens with 3 tens taken away which leaves 5 tens. Students explore using hundreds chart, base ten blocks, number lines and other tools. Coherence KY.1.NBT.6→KY.2.NBT.8

Standards	Clarifications
MP.3, **MP.5**	

Attending to the Standards for Mathematical Practice
Students recognize when solving a problem such as 33 + 20 that the 3 in the ones place will not change, but the 3 in the tens place will; additionally, they will reason this is because they are adding two tens (**MP.7**, **MP.8**). Students generalize this idea, explaining units of tens can be added or subtracted and apply this idea to adding multiples of 10 (**MP.2**). Students select a strategy for adding or subtracting, including the following: using tools, drawing pictures, jumps on a number line and/or jumps on a hundred chart. They explain which tool or model they selected, how they reasoned about the problem and how they know their answer is correct (**MP.1**, **MP.3**). Students apply strategies used to solve single-digit addition/subtraction situations in the range of 1-9 to solve addition/subtraction situations in the range of 10-90. For example, extending the Make 10 Strategy to a Make 40 strategy for adding 38 + 9 (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Measure lengths indirectly and by iterating length units.**	
Standards	**Clarifications**
KY.1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. **MP.6**	Students use nonstandard tools to estimate and measure objects. They compare lengths of three different objects. Coherence KY.K.MD.1→KY.1.MD.1→KY.2.MD.4
KY.1.MD.2 Express the length of an object as a whole number of same-size length units, by laying multiple copies of a shorter object (the length unit) end to end with no gaps or overlaps. **MP.2**, **MP.5**	Students measure numerous items with different sizes of nonstandard units. The smaller the unit, the more units needed to measure the object. Coherence KY.1.MD.2→KY.2.MD.2
Attending to the Standards for Mathematical Practice	
Students compare and order objects by analyzing their lengths. For example, they wonder which desk is taller and use their leg or a piece of string to compare each desk and determine its relative height (**MP.2**). Students describe the objects' length in relation to one another using precise language, understanding "bigger" and "smaller" are not as specific as "longer" and "shorter" for describing the attribute of length (**MP.6**). Students understand they use an object as a unit of measure. For example, a paperclip can be used to see how long a pencil is (**MP.5**). Students use a pencil to measure the length of a book and a desk. If it takes two pencils for the length of the book and four pencils for the length of the desk, students can determine the desk is longer than the book (**MP.2**).	

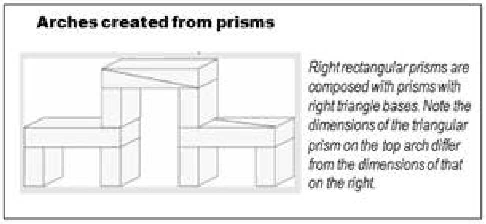
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Work with time and money.**	
Standards	**Clarifications**
KY.1.MD.3 Assign values to time and money. a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify the coins by values (penny, nickel, dime, quarter). **MP.6**, **MP.8**	a. Students understand 60 minutes = 1 hour. b. A penny has a value of one cent; a nickel has a value of five cents; a dime has a value of 10 cents; a quarter has a value of 25 cents. Note: This standard requires students to identify coins by name along with their corresponding value only (e.g. a quarter is worth twenty five cents). In grade one, coins should not be used as models or manipulatives for the purposes of teaching place value, counting (by ones or skip counting), or addition and subtraction. KY.2.MD.7 Coherence KY.K.MD.4→KY.1.MD.3→KY.2.MD.8
Attending to the Standards for Mathematical Practice	
Students realize the specific logic of an analog clock, recognizing the shorter moving part on an analog clock is called the “hour hand” and its position (relative to the encircling numerals) indicates what hour it is (**MP.6**). Students recognize patterns in how the hour and minute hands operate. For example, they notice at 4:30, the minute hand is halfway around the clock (at the six) and the hour hand is halfway between the four and the five (**MP.8**). Students understand four-thirty is expressed numerically using a digital clock (**MP.2**). With money, students use appropriate terms to describe coins and connect the coin names to their values (**MP.2**, **MP.6**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply the statistics process.	
Standards	**Clarifications**
KY.1.MD.4 Investigate questions involving categorical data. a. Pose a question that can be answered by gathering data. b. Determine strategy for gathering data from peers. c. Organize and represent data in a table/chart with up to three categories. d. Interpret data to answer questions about the table/chart that connects to the question posed, including total number of data points, how many in each category and how many more or less are in one category than in another. **MP.1**, **MP.3**, **MP.4**, **MP.6**	Students create a table or chart to organize data. KY.2.MD.9 Coherence KY.1.MD.4→KY.2.MD.10
Attending to the Standards for Mathematical Practice	
Students create carefully worded questions to be answered by their peers and gather data (**MP.6**). For example, a student may wonder about the way each classmate gets to school (walk, ride bus, car-rider). In both gathering data and creating a representation of data, students design what makes sense to them and helps them to answer the question posed (**MP.1**). Students create a table/chart representing the data collected, knowing the table/chart provides insights to answer their question (**MP.4**). Students make observations from the data and listen and critique other student observations, ultimately explaining what they learned about the question they posed (**MP.3**). For example, students observe most students take a bus to school using the data in the table/chart.	

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.1.G.1 Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes. **MP.7**	Defining attributes include, but are not limited to, number of sides or open/closed shapes. Non-defining attributes include, but are not limited to, color, orientation or overall size. Coherence KY.K.G.4→KY.1.G.1→KY.2.G.1
KY.1.G.2 Compose shapes. a. Compose two-dimensional shapes to create rectangles, squares, trapezoids, triangles, half-circles, quarter-circles and composite shapes to compose new shapes from the composite shapes. b. Use three-dimensional shapes (cubes, right rectangular prisms, right circular cones and right circular cylinders) to create a composite shape and compose new shapes from the composite shapes. **MP.1**, **MP.4**	Students do not need to learn formal names such as “right rectangular prisms.” b.  Coherence KY.K.G.6→KY.1.G.2
KY.1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths* and *quarters*, and use the phrases *half of*, *fourth of* and *quarter of*. Describe the whole as two of or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **MP.3**, **MP.6**	Students see the relationship of taking the same shape and partitioning it into equal pieces. For example, they compare the size of the pieces when it’s half of a shape or a fourth of the shape.  Coherence KY.K.G.6→KY.1.G.3→KY.2.G.3

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Through analyzing many shapes and making sense of what they have in common, students determine what attributes define a shape versus attributes that do not define a shape (**MP.7**). For example, students describe defining characteristics of a triangle such as straight sides, three sides, three angles or three points and describe non-defining characteristics such as blue, big or heavy (**MP.3**, **MP.7**). Students use knowledge of defining attributes to build and/or draw examples and non-examples of these shapes, attending to those attributes which define the shape (**MP.6**). Students may compare their drawings and discover a square is a square regardless of its color, size or orientation (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Grade 2 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Measurement and Data (MD)	Geometry (G)
• Represent and solve problems involving addition and subtraction. • Add and subtract within 20. • Work with equal groups of objects to gain foundations for multiplication.	• Understand place value. • Use place value understanding and properties of operations to add and subtract.	• Measure and estimate lengths in standard units. • Relate addition and subtraction to length. • Work with time and money. • Understand and apply the statistics process.	• Reason with shapes and their attributes.

In grade 2, instructional time should focus on four critical areas:

1. In the Number and Operations in Base Ten domain, students will:

- extend their understanding of the base-ten system. This includes ideas of counting in fives, tens and multiples of hundreds, tens and ones, as well as number relationships involving these units, including comparing; and
- understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. In the Operations and Algebraic Thinking and Numbers and Operations in Base Ten domains, students will:

- use their understanding of addition to develop fluency with addition and subtraction within 100;
- solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss and use efficient, accurate and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations; and
- select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. In the Measurement and Data domain, students will:

- recognize the need for standard units of measure (centimeter and inch) and use rulers and other measurement tools with the understanding that linear measure involves an iteration of units; and
- recognize that the smaller the unit, the more iterations needed to cover a given length.

4. In the Geometry domain, students will:

- describe and classify shapes as polygons or non-polygons;
- investigate, describe and reason about decomposing and combining shapes to make other shapes; and
- draw, partition and analyze two-dimensional shapes to develop a foundation for understanding area, congruence, similarity and fractions in later grades.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Represent and solve problems involving addition and subtraction.

Standards	Clarifications
KY.2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, by using drawings and equations with a symbol for the unknown number to represent the problem. **MP.1**, **MP.2 and MP.4**	Students flexibly model or represent addition and subtraction situations or context problems (involving sums and differences within 100). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. **See Table 1 in Appendix A**. Students master all word problem subtypes including the four difficult ones: ● add to-start unknown ● take from-start unknown ● put together/take apart-addend unknown ● compare-bigger unknown/smaller unknown Coherence KY.1.OA.1→KY.2.OA.1→KY.3.OA.8

Attending to the Standards for Mathematical Practice

When reading/interpreting word problems, students recognize a number (eight or 8) represents a quantity (eight buttons) and consider what is happening to these quantities in the context of the problem (**MP.2**). Students experiment in different ways to solve the problem (**MP.4**). Students think of questions to ask themselves, such as “Which diagram could help me?” Students work in groups to make addition and subtraction stories using concrete objects/pictures to demonstrate different situations and write an addition or subtraction equation to match their stories (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Add and subtract within 20.	
Standards	**Clarifications**
KY.2.OA.2 Fluently add and subtract within 20 using mental strategies. **MP.2**, **MP.7**, **MP.8**	Students determine addition and subtraction strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently and appropriately use mental strategies that include: ● counting on ● making ten ● decomposing a number leading to a ten ● using the relationship between addition and subtraction ● creating equivalent but easier or known sums. Note: Reaching fluency is an ongoing process that will take much of the year. KY.2.NBT.5 Coherence KY.1.OA.6→KY.2.OA.2
Attending to the Standards for Mathematical Practice	
Students select and use reasoning strategies to solve addition and subtraction problems efficiently. For example, for 8 + 7, a student decides to use a make 10 strategy, while another student notices the answer is one more than 7 + 7 (a known double fact). Students notice these patterns and through experiences such as games, become more efficient at applying the strategies eventually reaching automaticity (**MP.8**). Students use 10 as a benchmark in solving problems and recognize the relationship between addition and subtraction, recognizing these relationships lead to more efficient ways to add and subtract than counting. For example, to solve 16 – 9, a student counts up to 10 (1) and up to 16 (6) to get the answer of 7 (**MP.7**).	

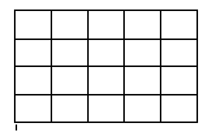
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Work with equal groups of objects to gain foundation for multiplication.

Standards	Clarifications
KY.2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends. **MP.2**, **MP.7**	Students understand a number can be broken apart by pairing objects to see if there are leftovers (odd) or not (even).  Coherence KY.1.OA.7→KY.2.OA.3→KY.3.OA.9
KY.2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **MP.2**, **MP.4**	Students model using rectangular arrays to determine the number of objects and discuss their reasoning. For example the array shows 4 + 4 + 4 + 4 + 4 =20 or 5 + 5 + 5 + 5 = 20  Coherence KY.1.OA.7→KY.2.OA.4→KY.3.OA.1

Attending to the Standards for Mathematical Practice

Students use contexts and visuals to reason about whether numbers are even or odd (**MP.2**). They notice if a number can be decomposed (broken apart) into two equal addends (16 = 8+8), then it is even, or if they group the number in twos it is even (**MP. 7**). They build on the idea of two equal sized groups to adding more equal sized groups. Students use concrete objects (counters) and pictorial representations (arrays) to explore repeated addition of equal sized groups (**MP. 5**). Students recognize in a rectangular array there are two ways to have same sized groups (rows or columns) and they can choose either way to find the total (**MP.2**).

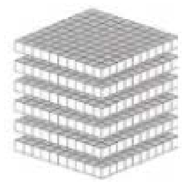
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

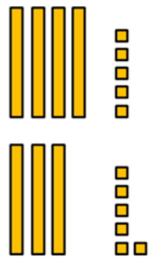
Cluster: Understand place value.

Standards	Clarifications
KY.2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **MP.2**, **MP.7**	Students unitize or understand 10 tens as a group or unit called 1 hundred.  is the same as   6 hundreds are the same as 600 Coherence KY.1.NBT.2→KY.2.NBT.1→KY.3.NBT.1
KY.2.NBT.2 Count forwards and backwards within 1000; skip-count by 5s, 10s and 100s. **MP.8**, **MP. 1**, **MP. 6**	Students start at various numbers to skip-count. Some use tools such as base ten blocks, hundreds charts, number lines and money. Coherence KY.1.NBT.1→KY.2.NBT.2
KY.2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names and expanded form. **MP.7**	739, seven hundred thirty-nine, 700 + 30 + 9 Coherence KY.1.NBT.1→KY.2.NBT.3
KY.2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, =, and < symbols to record the results of comparisons. **MP.2**, **MP.6**	Students use base ten blocks, hundred charts and/or number lines when comparing two three-digit numbers using the symbols <, >, and =. Coherence KY.1.NBT.3→KY.2.NBT.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students use concrete, groupable objects (counters in cups, unifix cubes in stacks) to show that 10 tens make one hundred and 10 hundreds make one thousand (**MP.5**, **MP.7**). Using place value structure, students build a physical model of a number and then practice saying it, eventually moving to written form (**MP.7**). When comparing 2 three-digit numbers, students interpret the inherent value of each digit (234 is two hundreds, three tens and 4 ones) and determine which number is larger (**MP.7**). In building numbers, students see the equivalence of numbers written in standard form and expanded form (**MP.7**). In addition, they reason about which number is greater using their place value understanding (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use place value understanding and properties of operations to add and subtract.	
Standards	**Clarifications**
KY.2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction. **MP.2**, **MP.8**	Students solve addition and subtraction tasks (with sums and differences within 100) efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Note: Reaching fluency is an ongoing process that will take much of the year. Students are not expected to use an algorithm for addition and subtraction until grade 4. 45 + 36 =  Students can solve this problem many ways. Student one counted the tens first, so 10, 20, 30, 40, 50, 60, 70. Then they counted the ones, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81. So 45+36=81 Student two broke 36 into 30+1+5. Then gave 5 from 36 to the 45 to make 50 because 50 is a friendly number. Then added 30+50 to make 80. Finally added 1 to 80 to get 81. So 45+36=81. Coherence KY.1.NBT.4→KY.2.NBT.5→KY.3.NBT.2

Standards	Clarifications
KY.2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. **MP.2**, **MP.7**	Note: Students are not expected to know a standard algorithm until grade 4. Coherence KY.1.OA.2→KY.2.NBT.6
KY.2.NBT.7 Add and subtract within 1000. a. Represent and solve addition and subtraction problems using… • concrete models or drawings; • strategies based on place value; • properties of operations; • the relationship between addition and subtraction and; • relate drawings and strategies to expressions or equations. b. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **MP.1**, **MP.5**	Students model with concrete tools to build on previous place value understandings. For example, students can see the relationship of addition and subtraction by counting up from 87 to get to 243 and realize that there is a difference of 156. Coherence KY.1.NBT.4→KY.2.NBT.7→KY.3.NBT.2
KY.2.NBT.8 Mentally add 10 or 100 to a given number 100–900 and mentally subtract 10 or 100 from a given number 100–900. **MP.7**, **MP.8**	Students use a variety of tools and strategies to add or subtract 10 or 100 from a three-digit number in the range of 100-900. KY.1.NBT.6 Coherence KY.1.NBT.5→ KY.2.NBT.8→KY.3.NBT.2
KY.2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. **MP.3**, **MP.7**	Students support explanations with drawings and/or objects built on place value and properties of operations. KY.1.OA.4 Coherence KY.1.OA.3→KY.2.NBT.9

Attending to the Standards for Mathematical Practice
Students notice their knowledge of tens and ones can be used to solve addition problems. For example, decomposing 24 + 42 into tens and ones: 20 + 40 + 4 + 2) (**MP. 8**). For other problems, students choose to use a counting up/back strategy. For 57 – 18, students use an open number line and jump back 20 (to 37) and then up 2 (to 39). Students select among their repertoire of strategies based on the numbers in the problem (**MP.1**, **MP.2**). These strategies are extended to adding strings of numbers as well as larger numbers. Students explain their strategies, critique the strategies shared by others and reflect on which strategies are efficient for the problem posed (**MP.3**). Students notice when numbers are added or subtracted in the base-ten system, like units are added or subtracted (ones are added to ones, tens to tens, hundreds to hundreds) and use this pattern to solve problems mentally (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Measure and estimate lengths in standard unit.

Standards	Clarifications
KY.2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. **MP.5**, **MP.6**	Students are exposed to different situations where they choose the appropriate tool(s) to measure. Coherence KY.1.MD.2→KY.2.MD.1→KY.3.MD.5
KY.2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **MP.3**, **MP.5**	Students measure an object using two different units and describe how the two measurements relate to the size of the unit chosen. (Students measure a door in inches and then in feet. Students relate the size and amount of each unit to discover more inches than feet are needed to measure the door.) Coherence KY.1.MD.2→KY.2.MD.2
KY.2.MD.3 Estimate lengths using units of inches, feet, yards, centimeters and meters. **MP.2**, **MP.6**	Students understand estimates are not exact answers or unreasonable guesses. Estimates are based on prior knowledge and the ability to reason about the appropriateness of their estimates. Coherence KY.1.MD.2→KY.2.MD.3
KY.2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of either a customary or metric standard length unit. **MP.5**, **MP.6**	Students measure using appropriate tools and standard unit lengths to find the difference between the lengths. Coherence KY.2.MD.3→KY.2.MD.4→KY.2.MD.5

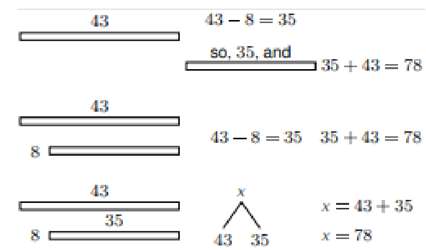
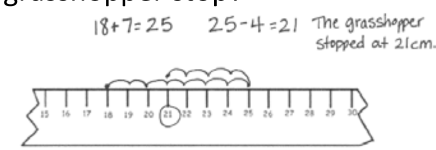
Attending to the Standards for Mathematical Practice
Students choose appropriate units and the related tools they need in order to measure (**MP.5**). For example, if asked to measure the length of the hallway, students select a meter or yard as an appropriate unit and seek out a meter stick, yardstick or trundle wheel. In addition, students measure objects using different units within the same system, such as meters and centimeters, record the measurements in a table and notice relationships (**MP.8**). Students notice it takes more of a smaller unit. For example, explaining a desk measured 2 feet because a foot is a longer unit, but measures

24 inches because an inch is smaller unit (**MP.3**). Students accurately estimate lengths and use these estimates to determine if a measurement is reasonable, as well as to compare the lengths of objects (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Relate addition and subtraction to length.

Standards	Clarifications
KY.2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units by using drawings and equations with a symbol for the unknown number to represent the problem. **MP.1**, **MP.4**	Students use concrete models and/or representations such as drawings of rulers to make sense of adding and subtracting word problems involving length. For example, a girl had a 43 cm section of a necklace and another section that was 8 cm shorter than the first. How long would the necklace be if she combined the two sections?  Coherence KY.2.MD.5→KY.3.MD.2
KY.2.MD.6 Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2, ... and represent whole-number sums and differences within 100 on a number line. **MP.3**, **MP.4**	Students show their thinking of adding and subtracting quantities using a number line. For example, a grasshopper jumped 7 cm forward and 4 cm back and then stopped. If the grasshopper started at 18 cm, where did the grasshopper stop?  Coherence KY.2.MD.6→KY.3.NF.2

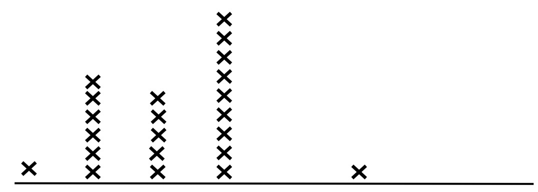
Attending to the Standards for Mathematical Practice

Students make sense of linear-focused story problems, using number lines and bar diagrams to make sense of the situation (**MP.1**, **MP.4**). Students use the number line as a reasoning strategy to add or subtract and explain their reasoning. In addition, they listen to other students' ways to use the number line to solve problems and compare strategies with a focus on which strategies are efficient for the given problem (**MP.3**).

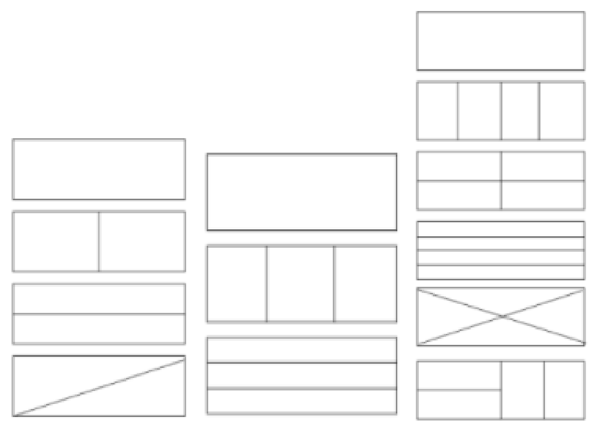
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Work with time and money.	
Standards	**Clarifications**
KY.2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **MP.5**, **MP.6**	Students orally tell and write the time from both types of clocks to the nearest five minutes. Realizing that a clock can be seen as a number line. KY.2.NBT.2 Coherence KY.1.MD.3→KY.2.MD.7→KY.3.MD.1
KY.2.MD.8 Solve word problems with adding and subtracting within 100, (not using dollars and cents simultaneously) using the $ and ¢ symbols appropriately (not including decimal notation). **MP.1**, **MP.5**	Students add or subtract two coin values or dollar values, but not both in the same problem. • For example, if you have 6 dimes and 3 pennies, how many cents do you have? Students would understand 6 dimes is equal to 60 cents and 3 pennies is equal to 3 cents. Together, they would total 63 cents. • If Mary had $31 and Tommy gave her $22 for her birthday, how much money does Mary have now? $31 + $22 = $53 Note: Students are not introduced to decimals until grade 4. KY.2.OA.1 Coherence KY.1.MD.3→KY.2.MD.8
Attending to the Standards for Mathematical Practice	
Students connect skip-counting by fives and five minute intervals on the clock (**MP.8**). Students attend to precision as they notice how minutes and hours are determined on analog and digital clocks, as well as whether to label the time as a.m. or p.m. (**MP.6**). Students makes sense of authentic problems involving money, using actual coins or representations of coins and use these coins to solve the problem (**MP.1**). As students solve such problems, they write equations to represent the situation, using units ($ or ¢) to correctly communicate the quantities (**MP.4**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply the statistics process.	
Standards	**Clarifications**
KY.2.MD.9 Investigate questions involving measurements. a. Identify a statistical question focused on measurements. b. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. c. Show the measurements by making a dot plot, where the horizontal scale is marked off in whole-number units. **MP.1**, **MP.6**	Students gather information from a statistical question, generate measurements of objects from the nearest whole-number unit and create a dot plot like the one below. For example, as a class, how long are our feet with our shoes on?  Coherence KY.2.MD.9→KY.3.MD.4
KY.2.MD.10 Create a pictograph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart and compare problems using information presented in a bar graph. **MP.2**, **MP.6**	**See Table 1 in Appendix A.** Coherence KY.1.MD.4→KY.2.MD.10→KY.3.MD.3
Attending to the Standards for Mathematical Practice	
Students understand the purpose of creating a graph is to make sense of data related to a question (**MP.1**). They look at the data they have collected and decide how to set up a graph, labeling it so anyone can understand what the data represents (**MP.6**). Students select a graph that makes sense, recognizing a dot plot is for numeric data while bar and pictographs are for categorical data (**MP.1**). Students analyze the data in their graphs, noticing relationships such as how many more fall in one category than another and relating those observations back to the original question they posed (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or sides. Identify triangles, quadrilaterals, pentagons, hexagons and cubes (identify number of faces). **MP.4**, **MP.7**	Sizes are compared directly or visually, not compared by measuring. Coherence KY.1.G.1→KY.2.G.1→KY.3.G.1
KY.2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **MP.6**, **MP.8**	The rectangle should not be divided up into anything larger than 5 rows and 5 columns to correlate with KY.2.OA.4. Coherence KY.2.G.2→KY.3.MD.6
KY.2.G.3 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc.; and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. **MP.2**, **MP.3**	Students explore rectangles and circles being partitioned in multiple ways to recognize that equal shares may be different shapes within the same whole.  halves thirds fourths Coherence KY.1.G.3→KY.2.G.3→KY.3.NF.1

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students describe attributes they notice for a group of shapes, such as sides and angles for 2-dimensional shapes and number of faces for 3-dimensional shapes (**MP.6**). They explain what characteristics are true for all shapes following in the same category (for example, attributes that are true for all triangles), as well as attributes true for some triangles, but not all triangles. Students use tiles to equally cover the rectangle and use repeated addition to determine the number of unit squares in the rectangle, noticing the pattern of equal rows (groups) (**MP. 8**). Students partition circles and rectangles into up to 4 equal parts. Students use a variety of tools to show halves, fourths and thirds (**MP.5**). They partition rectangles into thirds and fourths in different ways, showing and explaining the parts do not need to be the same shape, only the same size (**MP.2**, **MP.3**). Conversely, students identify shapes that are incorrectly partitioned, with the sections not being the same size.

Kentucky Academic Standards for Mathematics: Grade 3 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations and identify and explain patterns in arithmetic.	• Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: A range of algorithms may be used.	• Develop understanding of fractions as numbers. Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.	• Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects. • Understand and apply the statistics process. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	• Reason with shapes and their attributes.

In grade 3, instructional time should focus on four critical areas:

1. **In the Operations and Algebraic Thinking domain, students will:**
    - develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays and area models; multiplication is finding an unknown product and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size;
    - use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors; and
    - compare a variety of solution strategies to learn the relationship between multiplication and division.

2. In the Number Sense and Operations—Fractions domain, students will:
- develop an understanding of fractions, beginning with unit fractions;
- view fractions in general as being built out of unit fractions and use fractions along with visual fraction models to represent parts of a whole;
- understand that the size of a fractional part is relative to the size of the whole. Use fractions to represent numbers equal to, less than and greater than one; and
- solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. In the Measurement and Data domain, students will:

- recognize area as an attribute of two-dimensional regions;
- measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area; and
- understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

4. In the Geometry domain, students will:

- compare and classify shapes by their sides and angles; and
- relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Note: Multiplication, division and fractions are the most important developments in grade 3.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Represent and solve problems involving multiplication and division.

Standards	Clarifications
KY.3.OA.1 Interpret and demonstrate products of whole numbers. **MP.2**, **MP.5**	Students use models for multiplication situations. For example, students interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. Coherence KY.2.OA.4→KY.3.OA.1→KY.4.OA.1
KY.3.OA.2 Interpret and demonstrate whole-number quotients of whole numbers, where objects are partitioned into equal shares. **MP.2**, **MP.5**	Students use models for division situations. For example, students interpret 56 ÷ 8 as the number of 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 object each. Coherence KY.3.OA.1→KY.3.OA.2→KY.5.NF.3
KY.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, by using drawings and equations with a symbol for the unknown number to represent the problem. **MP.1**, **MP.4**	Students flexibly model or represent multiplication and division situations or context problems (involving products and quotients up to 100). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. **See Table 2 in Appendix A**. Coherence KY.3.OA.3→KY.4.OA.2
KY.3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **MP.6**, **MP.7**	Students determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = □ ÷ 3, 6 x 6 = ?. Coherence KY.3.OA.4→KY.4.MD.3

Attending to the Standards for Mathematical Practice

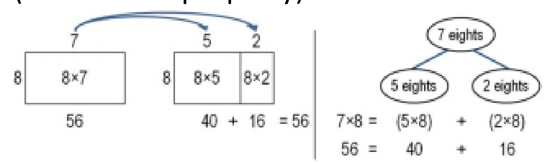
Students recognize the numbers and symbols in an equation such as 5 x 8 = 40 are related to a context using groups or arrays (**MP.2**). For example, a student analyzes this equation and tells a story about walking 8 blocks round-trip to and from school each day, connecting to the equation by saying: 5 days x 8 blocks each day is 40 total blocks walked. To represent the problem, they show 5 jumps of 8 on an open number line or show five 8-unit long Cuisenaire Rods (**MP.5**). When reading story situations, students seek to make sense of the story and its quantities (**MP.1**). They do not just lift numbers out or use keywords. To help make sense of the problem, students decide to write an equation or use a number line. In other words they 'mathematize' the situation (**MP.4**). In missing value problems, students attend to what value is unknown and what operation is represented (**MP. 6**) and use this information to determine what value will result in both sides of the equations being equal (**MP.7**).

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

Standards	Clarifications
KY.3.OA.5 Apply properties of operations as strategies to multiply and divide. **MP.3**, **MP.4**	Students need not use formal terms for these properties. If 6 x 4 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5+2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property).  KY.4.NBT.5 Coherence KY.3.OA.5→KY.4.NBT.6
KY.3.OA.6 Understand division as an unknown-factor problem. **MP.2**	Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. Coherence KY.3.OA.6→KY.4.NBT.6

Attending to the Standards for Mathematical Practice

Students use strategies beyond skip counting to solve multiplication problems. They decide how to use known facts to solve facts like 6 x 9. Students use strategies like Adding a Group, thinking 5 groups of 9 (45) plus one more group (54) and Subtracting a Group, thinking 9 x 6 and reasoning 10 groups of 6 (60) minus one group of 6 (54) (**MP.7**). Students explain their selected reasoning strategy and listen and critique other students' strategies, considering which strategies make sense and are efficient (**MP.3**). Students think about 84 ÷ 4 as, "How many sets of 4 can be made from 84 items?" or "How many in a group, if there 84 items and 4 groups?" and use this relationship to solve the problem (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Multiply and divide within 100.	
Standards	**Clarifications**
KY.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. **MP.2**, **MP.8**	Students determine multiplication and division strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Knowing 8 x 5 = 40, one knows 40 ÷ 5 = 8. Note: Reaching fluency is an ongoing process that will take much of the year. Coherence KY.3.OA.7→KY.4.OA.4
Attending to the Standards for Mathematical Practice	
By studying patterns and relationships in multiplication facts, students develop fluency for multiplication facts (**MP.8**). For example, students notice 4 x 6 is equivalent to 2 x 2 x 6 (doubling strategy). They know 9 facts can be found by thinking of the other factor x 10 and subtracting one group. For example, recognizing 9 x 8 is equivalent to 10 x 8 – 8. For each fact, the student thinks, "What reasoning strategy can I use that is more efficient than skip counting?" (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

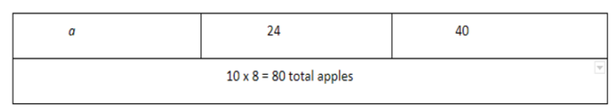
Cluster: Solve problems involving the four operations and identify and explain patterns in arithmetic.

Standards	Clarifications
KY.3.OA.8 Use various strategies to solve two-step word problems using the four operations (involving only whole numbers with whole number answers). Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **MP.1**, **MP.4**	Students solve problems using models, pictures, words and numbers. Students explain how they solved the problem using accurate mathematical vocabulary and why their answer makes sense. Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.2.OA.1→KY.3.OA.8→ KY.4.OA.3
KY.3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. **MP.3**, **MP.8**	Students observe 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends. Coherence KY.2.OA.3→KY.3.OA.9→ KY.4.OA.5

Attending to the Standards for Mathematical Practice

Given a non-straightforward story situation about gathering apples and sharing them among 8 families, students decide on ways to make sense of the problem (**MP.1**). One student decides to use a bar diagram to make sense of the situation and then use the bar diagram to write equations and solve the problem (**MP.4**).

Maggie was picking apples from her three apple trees. She picked some from the first tree and realized she should count the rest of what she was picking. She picked 24 apples from the second tree and 40 apples from the third tree. She had enough apples to give 10 to each of eight families. How many apples did she pick from the first tree?



$a + 24 + 40$ = total apples and 10×8 = total apples. There are 80 apples total.

$a + 64 = 80$

$a = 16$

Another student thinks of the situation differently and decides to figure out how many apples each family has from the known apples (**MP.1**). Other students use counters to model the problem and/or use trial and error. If their first approach doesn't work, students persevere by trying another strategy (**MP.1**). In each case, students check to see if the answer of 16 apples makes sense.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Numbers and Operations in Base Ten

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: A range of algorithms may be used.

Standards	Clarifications
KY.3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100. **MP.7**	On a number line, students determine 178 rounded to nearest 10 is 180.  Coherence KY.2.NBT.1→KY.3.NBT.1→ KY.4.NBT.3
KY.3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and/or the relationship between addition and subtraction. **MP.2**, **MP.3**	Students determine addition and subtraction strategies efficiently, accurately, flexibly and appropriately. Being fluent means students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches and they are able to produce accurate answers efficiently. Note: Reaching fluency is an ongoing process that will take much of the year. KY.2.NBT.5 Coherence KY.2.NBT.7→KY.3.NBT.2→ KY.4.NBT4
KY.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range of 10–90 using strategies based on place value and properties of operations. **MP.7**, **MP.8**	To solve 8 x 60, students interpret this as 8 groups of 6 tens, which is 480. KY.3.OA.5 Coherence KY.2.NBT.1→KY.3.NBT.3→ KY.4.NBT.5

Attending to the Standards for Mathematical Practice

Students look at the numbers in a problem and consider which strategy they will use to solve the given problem (**MP.2**). For example, for the problem 405 - 381, a student notices these values are close to each other, so rather than take away 381, they find the difference. They count up to 400 (19) and add on 5 more to equal 24. For the problem 425 - 98, the student notices 98 is close to 100, so chooses to take away 100 and add 2 more back on to equal 327. Students share the strategy they used, why it works and why they chose it (**MP.3**).

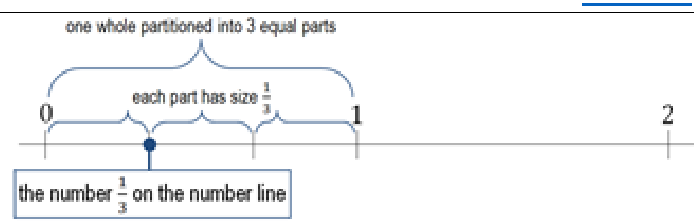
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

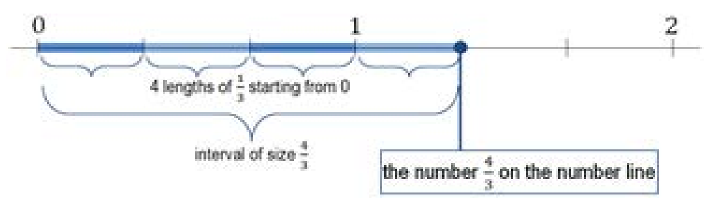
Numbers and Operations-Fractions

Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

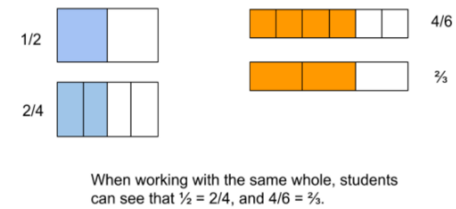
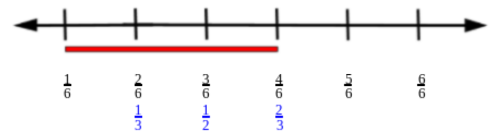
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Develop understanding of fractions as numbers. Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>
KY.3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
MP.2, MP.7
</td><td>
Students name parts of the whole using fractions and explain the fraction is made up of unit fractions. Students describe the numerator and the denominator using pictures, numbers and words.
$\frac{4}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.
Coherence KY.2.G.3→KY.3.NF.1→ KY.4.NF.3
</td></tr>
<tr><td>
KY.3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line.
a. Represent a fraction $\frac{1}{b}$ (unit fraction) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.
• Recognize each part has size $\frac{1}{b}$.
• a unit fraction, $\frac{1}{b}$ is located $\frac{1}{b}$ of a whole unit from 0 on the number line.
b. Represent a non-unit fraction $\frac{a}{b}$ on a number line by marking off a lengths of $\frac{1}{b}$ (unit fractions) from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the non-unit fraction $\frac{a}{b}$ on the number line.
MP.4
</td><td>




Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.
Coherence KY.2.MD.6→KY.3.NF.2 →KY.4.NF.3
</td></tr>
</table>

Standards	Clarifications
KY.3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or same point on a number line. b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent through writing or drawing. c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions. **MP.2**, **MP.3**	   $\frac{3}{6}$ is greater than $\frac{3}{8}$ or $\frac{3}{6} > \frac{3}{8}$ Note: grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8. KY.4.NF.1 Coherence KY.3.NF.3→ KY.4.NF.5

Attending to the Standards for Mathematical Practice
Students use the number line to reason about the relative size of a fraction (**MP.4**). They locate $\frac{5}{6}$ on a number line by accurately partitioning the line into 6 equal-length segments. They explain that $\frac{5}{6}$ means five segments that are each one-sixth of a unit in length, for example counting, "One-sixth, two-sixths, three-sixths, four-sixths, five-sixths." (**MP.7**). As they partition the line in other ways, they recognize three-sixths is half of the distance to 1 whole, as is $\frac{2}{4}$, $\frac{1}{2}$, and $\frac{4}{8}$, and reason these fractions are equivalent (**MP.2**). Similarly, they can generate other illustrations or justifications to explain why two fractions are equivalent or not (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.	
Standards	**Clarifications**
KY.3.MD.1 Tell and write time to the nearest minute and measure elapsed time intervals in minutes. Solve word problems involving addition and subtraction of time intervals within and across the hour in minutes. **MP.4**, **MP.6**, **MP.1**, **MP.4**	Students solve elapsed time problems using strategies and tools such as clock models and number lines (seeing a clock as a number line). Coherence KY.2.MD.7→KY.3.MD.1→ KY.4.MD.2
KY.3.MD.2 Measure and solve problems involving mass and liquid volume. a. Measure and estimate masses and liquid volumes of objects using standard units of grams (g), kilograms (kg) and liters (L). b. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. **MP.1**, **MP.6**	a. Students have multiple opportunities to weigh classroom objects and fill containers to help them develop a basic understanding of the size and weight of a liter, a gram and a kilogram. b. **See Table 2 in Appendix A.** Coherence KY.2.MD.5→KY.3.MD.2→KY.4.MD.1
Attending to the Standards for Mathematical Practice	
Students solve story situations using a model to support their reasoning (**MP.4**). For example, a student solves a task such as: you try to run for 15 minutes without stopping. When you look at the clock, the time is 2:52. What time will it say when you have reached 15 minutes? On an open number line, they show a jump from 2:52 to 3:00 as 8 minutes and then jump 7 minutes more to 3:07. Students estimate and then measure objects using standard units. For example, how many grams might balance with a selected item (**MP.6**)?	

Measurement and Data	
Standards for Mathematical Practice	
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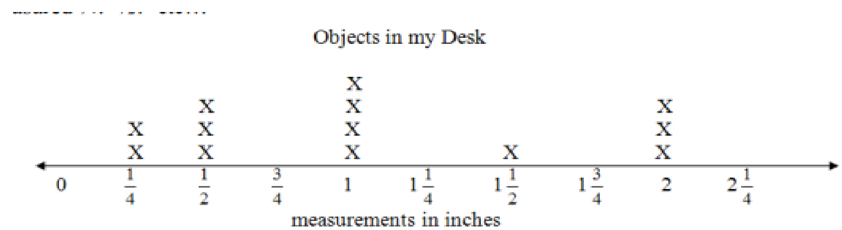
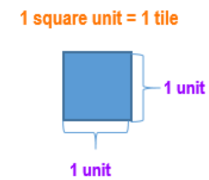
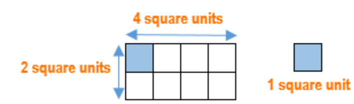
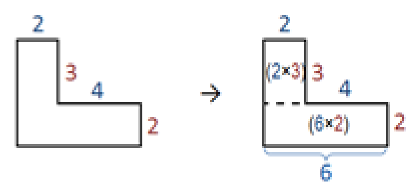
Cluster: Understand and apply the statistics process.	
Standards	**Clarifications**
KY.3.MD.3 Investigate questions involving categorical data. a. Identify a statistical question focused on categorical data and gather data; b. Create a scaled pictograph and a scaled bar graph to represent a data set (using technology or by hand); c. Make observations from the graph about the question posed, including “how many more” and “how many less” questions. **MP.3**, **MP.5**, **MP.6**	Students select a question of interest (how many pets does each classmate have), gather data and create a bar graph (each square in the bar graph might represent 2 pets). Coherence KY.2.MD.10→KY.3.MD.3
KY.3.MD.4 Investigate questions involving numerical data. a. Identify a statistical question focused on numerical data; b. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. c. Show the data by making a dot plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters. d. Make observations from the graph about the question posed, including questions about the shape of the data and compare responses. **MP.1**, **MP.3**, **MP.6**	Students measure objects in their desk to the nearest ½ or ¼ of an inch, display data collected on a dot plot and analyze the data.  Coherence KY.2.MD.9→KY.3.MD.4→KY.4.MD.4
Attending to the Standards for Mathematical Practice	
Students understand the purpose of creating a graph is to make sense of data related to a question (**MP.1**). They look at the data they have collected and decide on how to set up a graph to best communicate the data (**MP.6**). Students determine if the scale on a dot plot should be in whole numbers, halves or fourths, based on the data gathered. For example, if they measured the length of each person’s pencil to the nearest fourth inch, the related dot plot would be created using fourths (**MP.6**).	

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understanding concepts of area and relate area to multiplication and to addition.

Standards	Clarifications
KY.3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. **MP.5**	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area.  A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.  Coherence KY.3.MD.5→KY.5.MD.3
KY.3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft. and improvised units). **MP.5**, **MP.6**	Students use grid paper of varying square units to count the number of unit squares in a figure. Coherence KY.2.G.2→KY.3.MD.6→KY.5.MD.4
KY.3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b*	d.  Coherence KY.3.MD.7→KY.4.MD.3→ KY.5.MD.5

Standards	Clarifications
and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of figures that can be decomposed into non-overlapping rectangles by adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **MP.1**, **MP.8**	

Attending to the Standards for Mathematical Practice
Students use 1 inch color tiles to cover a rectangle, understanding that color tile as a square inch (**MP.5**). As students place the tiles in repeated rows to fill the rectangle, they notice each row has the same number of tiles and the number of tiles that will fill a rectangle can be written as [number of tiles in one row] x [number of rows] (**MP.8**). They solve story problems that sometimes have the area as the unknown and sometimes have the number of rows or columns as the unknown and use their knowledge of area to solve the problem (**MP.1**).

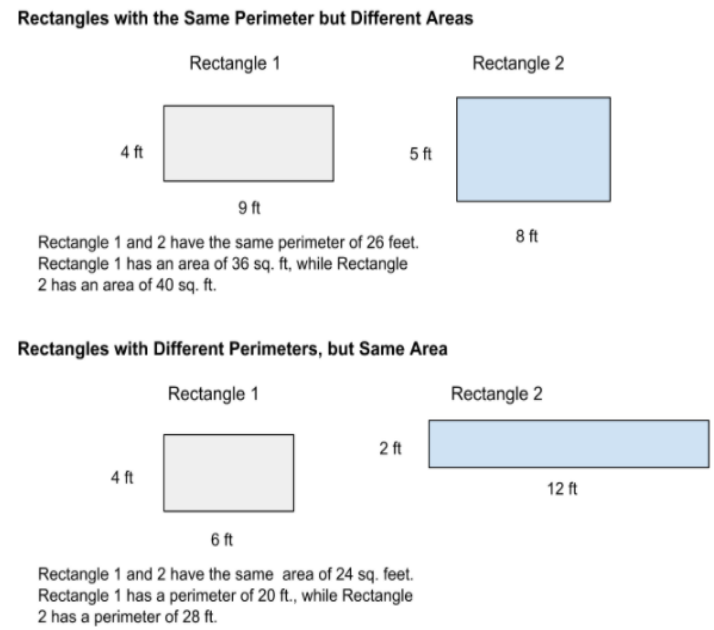
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Standards	Clarifications
KY.3.MD.8 Solve real world and mathematical problems involving perimeters of polygons. a. Find the perimeter given the side lengths of a polygon. b. Find an unknown side length, given the perimeter and some lengths. c. Draw rectangles with the same perimeter and different areas or with the same area and different perimeters. **MP.1**, **MP.4**	c.  Coherence KY.3.MD.8→KY.4.MD.3

Attending to the Standards for Mathematical Practice

Students recognize perimeter is a measure of length and see perimeters of polygons as a collection of side lengths added together to form the perimeter (**MP.1**). Therefore, they see if a side length is missing, it is like a missing addend problem and write an equation or draw a bar diagram to solve for the missing value (**MP.4**). Students recognize they can use a given perimeter (such as 16 inches) and form different rectangles (such as 4 x 4, 3 x 5, 2 x 6, 1 x 7) and that these rectangles have different areas (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.3.G.1 Classify polygons by attributes. a. Recognize and classify polygons based on the number of sides and vertices (triangles, quadrilaterals, pentagons and hexagons). b. Recognize and classify quadrilaterals (rectangles, squares, parallelograms, rhombuses, trapezoids) by side lengths and understanding shapes in different categories may share attributes and the shared attributes can define a larger category. c. Identify shapes that do not belong to a given category or subcategory. **MP.6**, **MP.7**	Students describe, analyze and compare properties of two–dimensional shapes. Coherence KY.2.G.1→KY.3.G.1→KY.4.G.2
KY.3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. **MP.2**, **M.5**	Partitioned parts should be halves, thirds, fourths, sixths, eighths. Students partition a shape into 6 parts with equal areas and describe the area of each part as $\frac{1}{6}$ of the area of the shape. KY.3.NF.1 Coherence KY.2.G.3→KY.3.G.2
Attending to the Standards for Mathematical Practice	
Students describe attributes they notice for a particular type of quadrilateral, focusing on side lengths and angles (**MP.6**). They explain what different types of quadrilaterals have in common and can distinguish between what are defining attributes (such as having four sides) and what are not defining (such as its size or color) (**MP.3**). Students use a variety of tools and drawings to show fractional parts (**MP.5**) and they reason if a shape is partitioned into four equal-sized parts (even if they are not the same shape), each part represents one-fourth of the whole shape (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Grade 4 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Use the four operations with whole numbers to solve problems. • Gain familiarity with fractions and multiples. • Generate and analyze patterns.	• Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.	• Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions and compare decimal fractions.	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Understand and apply the statistics process. • Geometric measurement: understand concepts of angle and angle measurements.	• Draw and identify lines and angles and classify shapes by properties of their lines and angles.

In grade 4, instructional time should focus on three critical areas:

1. **In the Number and Operations in Base Ten domain, students will:**
    - generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place;
    - apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value and properties of operations, in particular the distributive property, as they develop, discuss and use efficient, accurate and generalizable methods to compute products of multi-digit whole numbers;
    - determine and accurately apply appropriate methods to estimate or mentally calculate products;
    - develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems;
    - apply their understanding of models for division, place value, properties of operations and the relationship of division to multiplication as they develop, discuss and use efficient, accurate and generalizable procedures to find quotients involving multi-digit dividends;
    - select and accurately apply appropriate methods to estimate and mentally calculate quotients and interpret remainders based upon the context.

2. **In the Numbers and Operations--Fractions domain, students will:**
    - create an understanding of fraction equivalence and operations with fractions;
    - recognize that two different fractions can be equal and they develop methods for generating and recognizing equivalent fractions;
    - extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions; decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. In the Geometry domain, students will:

- describe, analyze, compare and classify two-dimensional shapes;
- strengthen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry through building, drawing and analyzing two-dimensional shapes.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
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Cluster: Use the four operations with whole numbers to solve problems.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.4.OA.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.
MP.2, MP.4</td><td>Students interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.

Coherence KY.3.OA.1→ KY.4.OA.1→KY.5.NF.5</td></tr>
<tr><td>KY.4.OA.2 Multiply or divide to solve word problems involving multiplicative comparisons by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
MP.1, MP.2, MP.3</td><td>Students solve multiplicative comparison problems using drawings and equations to determine situations like the ones below (Table 2 in Appendix A) on which quantity is being multiplied and which factor is telling how many times.
<table>
<tr><th colspan="3">Common Comparison Problems for Multiplication and Division</th></tr>
<tr><td>Unknown product</td><td>Group size unknown</td><td>Number of groups unknown</td></tr>
<tr><td>A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?

Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</td><td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?
Measurement example: A rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?</td><td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue?
Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td></tr>
<tr><td>a × b = ?</td><td>a × ? = p and p ÷ a = ?</td><td>? × b =p and p ÷ b = ?</td></tr>
</table>
Coherence KY.3.OA.3→ KY.4.OA.2→KY.5.NF.3</td></tr>
<tr><td>KY.4.OA.3 Solve multistep problems.</td><td>a. Students use their knowledge of order of operations even when</td></tr>
</table>

Standards	Clarifications
a. Perform operations in the conventional order when there are no parentheses to specify a particular order. b. Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computations and estimation strategies including rounding. **MP.1**, **MP.4**	there are no parentheses or brackets. 31 + 3 × 8 - 20 = b. For example, Mr. May's grade four class is collecting canned goods for a food drive. Their goal is to bring in 50 cans of food by Friday. So far, the students have brought in 10 on Monday and Tuesday, 14 cans on Wednesday and 13 on Thursday. How many more cans will the class need to bring in to reach their goal? $50 = 2 \times 10 + 14 + 13 + \boldsymbol{c}$ $50 = 20 + 14 + 13 + \boldsymbol{c}$ $50 = 47 + \boldsymbol{c}$ $\mathbf{3} = \boldsymbol{c}$ Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.3.OA.8→ KY.4.OA.3→KY.7.NS.3

Attending to the Standards for Mathematical Practice
Students recognize a number represents a specific quantity and connects the quantity to written symbols and creates a logical representation of the problem considering both the appropriate units involved and the meaning of quantities (**MP2**). In an equation such as 35= 5 x 7, students identify and verbalize which quantity is being multiplied and which number tells how many times, saying, "Sally is five years old. Her mom is seven times older. How old is Sally's Mom?" Students discover a pattern or structure (**MP.7**). For example, a student distinguishes an additive comparison by identifying this type of question asks, " How many more?" and a multiplicative comparison focuses on comparing two quantities by asking," How many times as much?" or "How many times as many?" Students solve contextual problems using models and equations using a symbol to represent the unknown (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Gain familiarity with factors and multiples.	
Standards	**Clarifications**
KY.4.OA.4 Find factors and multiples of numbers in the range 1-100. a. Find all factor pairs for a given whole number. b. Recognize that a whole number is a multiple of each of its factors. c. Determine whether a given whole number is a multiple of a given one-digit number. d. Determine whether a given whole number is prime or composite. **MP.5**, **MP.7**	Students extend their knowledge of multiplication and division facts by exploring patterns they have found by building conceptual understanding of prime numbers (numbers with exactly two factors) and composite numbers (numbers with more than two factors). Patterns include: • Numbers that end in 0 have 10 as a factor. These are multiples of 10. • Numbers that end in 0 or 5 as a factor. These are multiples of 5. • Even numbers have 2 as a factor. These numbers are multiples of 2. • Numbers that can be halved twice have 4 as a factor. These numbers are multiples of 4. Coherence KY.3.OA.7→ KY.4.OA.4→KY.6.NS.4
Attending to the Standards for Mathematical Practice	
Students use the structure and pattern of the counting numbers to find factor pairs, recognizing once they reach a certain point they don't have to keep looking for factors (**MP.7**). Students build arrays with a given area and look for patterns such as numbers of possible arrays to identify if the number is prime or composite. For example, noticing the number 7 has only two possible arrays, 1 x 7 and 7 x 1, therefore, it is prime. The number 4 has more than two rectangular arrays, 1 x 4, 4 x 1 and 2 x 2 and therefore, it is composite. (**MP.5**)	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Generate and analyze patterns.

Standards	Clarifications
KY.4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern not explicit in the rule itself. **MP.2**, **MP.3**	For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. Coherence KY.3.OA.9→ KY.4.OA.5→KY.5.OA.3

Attending to the Standards for Mathematical Practice

Students analyze growing patterns and determine rules to describe the pattern (**MP.2**). Students know a pattern is a sequence that repeats the same rule over and over. Students generate their own rules and create an example using that rule, for example, they write 1, 3, 9, 27, 81, 243 for the rule "times 3". Students describe features of the pattern for example, all numbers are odd, or sums of the digits equal 9 and the rule for generating the next number, for example "times 3", as well as critique the reasonableness of features and rules they hear from others (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Generalize place value understanding for multi-digit whole numbers.	
Standards	**Clarifications**
KY.4.NBT.1 Recognize in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. **MP.7**	Students recognize the relationship of same digits located in different places in a whole number. For example, in the number 435, the digit 5 in the ones place, while the digit 5 in 652 is in the tens place. The five in 652 is ten times greater than the five in 435. Coherence KY.2.NBT.1→ KY.4.NBT.1→KY.5.NBT.1
KY.4.NBT.2 Represent and compare multi-digit whole numbers. a. Read and write multi-digit whole numbers using base-ten numerals, number names and expanded form. b. Compare two multi-digit numbers based on meanings of the digit in each place, using >, =, and < symbols to record the results of comparisons. **MP.2**, **MP.7**	a. Students write numbers in three different forms. For example, 435, four hundred thirty-five, 400 + 30 + 5. b. Students use different forms of the number to help compare. For example, when students are comparing numbers, they determine that 453 is greater than 435 because the 5 is worth 50 in 453, while the tens place only has 3 worth 30 in 435. So 453 > 435. Coherence KY.4.NBT.2→KY.5.NBT.3
KY.4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. **MP.2**, **MP.6**	Students go beyond the application of a procedure when rounding. Students demonstrate a deeper understanding of number sense and place value when they explain and reason about the answers they get when rounding. KY.4.OA.3 Coherence KY.3.NBT.1→ KY.4.NBT.3→KY.5.NBT.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students use precise language, such as “ten times as much as” rather than “ten times more than” as they describe place value relationships (**MP.6**). Students make the conceptual connection between place value and multiplying and dividing by 10, noticing when any digit is multiplied by 10, the place of the digit moves one place to the left and when a digit is divided by 10, it moves to one place to the right. Beyond noticing this pattern, students understand this pattern exists because place value is structured this way (**MP.7**). For example, in solving 35 x 10 = ____, students might place 35 in a place value chart and explain 5 tens is 50, therefore, moving the 5 to the tens place and 30 tens equals 3 hundreds, therefore, moving the 3 to the hundreds place.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten

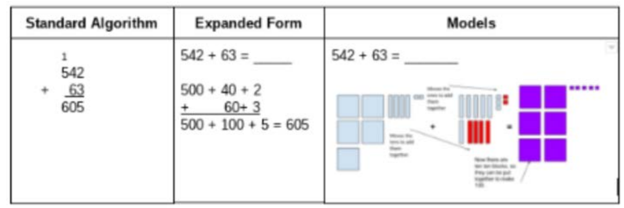
Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

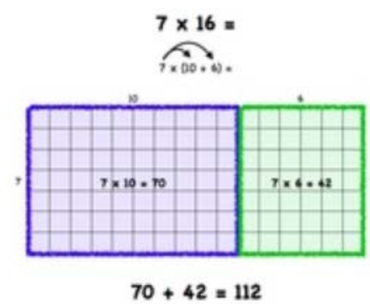
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

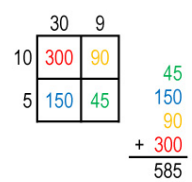
Standards	Clarifications
KY.4.NBT.4 Fluently add and subtract multi-digit whole numbers using an algorithm. **MP.2**, **MP.8**	Students make connections from previous work with addition and subtraction, using models/representations to develop an efficient algorithm to add and subtract multi-digit numbers. These are types of algorithms/strategies one could possibly use (but not limited to) to solve adding and subtracting multi-digit whole numbers.

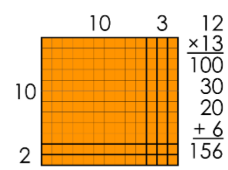


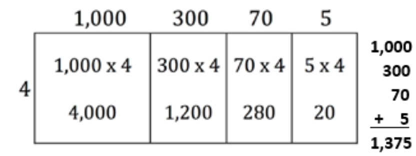
Coherence KY.3.NBT.2→ KY.4.NBT.4→KY.5.NBT.5

Standards	Clarifications
KY.4.NBT.5 Multiply whole numbers • Up to four digit number by a one-digit number • Two-digit number by two-digit number Multiply using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. **MP.3**, **MP.4**, **MP.8**	Students use a variety of models (rectangular arrays and area models) and strategies to represent multi-digit factors times a one-digit factor and a two-digit number by a two-digit number. Students also connect their reasoning to a written equation. Some examples include:







Standards	Clarifications
	KY.3.OA.5 Coherence KY.3.NBT.3→ KY.4.NBT.5→KY.5.NBT.5 KY.3.MD.7
KY.4.NBT.6 Divide up to four-digit dividends by one-digit divisors. Find whole number quotients and remainders using • strategies based on place value • the properties of operations • the relationship between multiplication and division Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. **MP.3**, **MP.7**, **MP.8**	Student use a variety of models (rectangular arrays and area models) and strategies to divide up to four-digit dividends by one-digit divisors.  5,500 ÷ 4 = ? Note: By the end of grade 4 students should be able to model, write and explain division by a one-digit divisor. KY.3.OA.5 Coherence KY.3.OA.6→ KY.4.NBT.6→KY.5.NBT.6 KY.3.MD.7

Attending to the Standards for Mathematical Practice
Students select from their repertoire of strategies to solve multi-digit whole number addition or subtraction problems. For example, for the problem 345,402 – 67,087 =□, a student might choose to stack it and subtract using an algorithm. The same student seeing 56,708 – 9,998 = __, might notice how close the subtrahend (second value) is to 10,000 and decide to subtract 10,000 and add 2 onto the answer (**MP.2**). In general, students determine their approach based on the numbers in the problem seeking an efficient strategy. For multiplication and division, students recognize the relationship between area and multiplication and take advantage of rectangular arrays to model multiplication problems (**MP.4**). In creating such models and recording them as equations, students notice repetitive actions in computation and make generalizations to solve other similar problems (**MP.8**). Students explain how and why their selected models and/or algorithms work (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

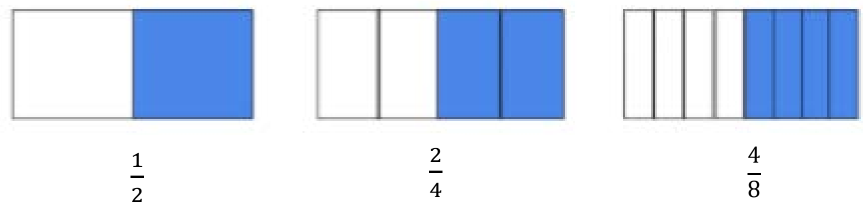
Number and Operations – Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

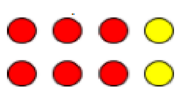
Cluster: Extend understanding of fraction equivalence and ordering.

Standards	Clarifications
KY.4.NF.1 Understand and generate equivalent fractions. a. Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size. b. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$. **MP.4**, **MP.7**, **MP.8**	Students draw fractions and see equivalent fractions.  Coherence KY.3.NF.3→ KY.4.NF.1→KY.5.NF.1
KY.4.NF.2 Compare two fractions with different numerators and different denominators using the symbols <, =, or >. Recognize comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions. **MP.2**, **MP.3**	Students use a variety of representations to compare fractions including concrete models, benchmarks, common denominators and common numerators. Note: Students determine which strategy makes the most sense to them, realizing they use different strategies for different situations. Coherence KY.3.NF.3d→ KY.4.NF.2→KY.5.NF.2

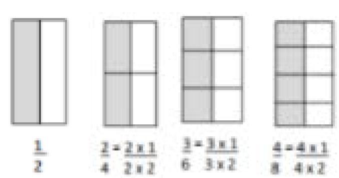
Attending to the Standards for Mathematical Practice

Work in this standard extends the work in grade 3 by using additional denominators (5, 10, 12 and 100). Students use visual models such as area models, number lines, or sets of objects to illustrate how two fractions are equivalent **(MP.4)**

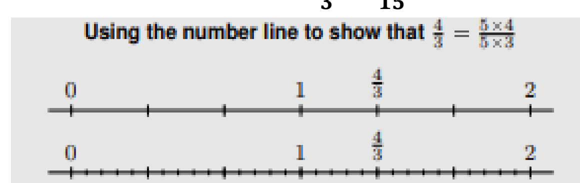
set model $\frac{6}{8} = \frac{3}{4}$



area model $\frac{1}{2} = \frac{4}{8}$



number line $\frac{4}{3} = \frac{20}{15}$



When students are asked to compare two fractions, they do not use a strategy they don't understand, such as the butterfly method, but rather employ reasoning strategies. They first consider whether they can decide which fraction is greater by observation (for example, the fractions have the same numerator or denominator or one fraction is greater than a benchmark and the other is less). If the fractions cannot be compared in this way, students decide whether to find a common denominator or a common numerator and then find the necessary fraction **equivalencies** to compare. For example, to compare $\frac{3}{8}$ and $\frac{5}{12}$, one can see $\frac{5}{12}$ is closer to $\frac{1}{2}$ (only $\frac{1}{12}$ away, while $\frac{3}{8}$ is $\frac{1}{8}$ away) and therefore know that $\frac{5}{12}$ is greater. Another student might not see this relationship, but decide that finding a common numerator is easier (being a basic fact) and multiply $\frac{3}{8}$ by $\frac{5}{5}$ to get $\frac{15}{40}$ and $\frac{5}{12}$ by $\frac{3}{3}$ to get $\frac{15}{36}$. Then recognize and explain that $\frac{15}{36}$ is greater (the pieces are larger) (**MP.2**, **MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>

KY.4.NF.3 Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions.
c. Add and subtract mixed numbers with like denominators.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

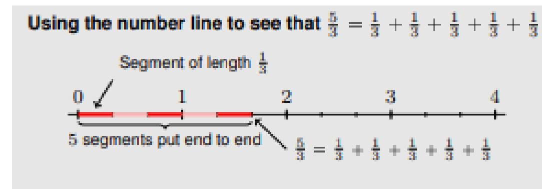
MP.1, **MP.5**, **MP.7**

</td><td>

b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ OR $\frac{3}{5} = \frac{2}{5} + \frac{1}{5}$

$3\frac{1}{4} = 1 + 1 + 1 + \frac{1}{4}$ OR $3\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$

c/d. Adding and subtracting using visual fraction models and/or equations to represent the problem.



KY.5.NF.1

Coherence KY.3.NF.1→ KY.4.NF.3→KY.5.NF.2

</td></tr>
<tr><td>

KY.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.
b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number.

MP.5, **MP.8**

</td><td>

Students refer this standard to *n* groups of a fraction (where *n* is a whole number) for example 3 groups of $\frac{1}{4}$, which can be seen as repeated addition. In grade 5 students will multiply a fraction by a whole number.

a. Students use visual fraction models to represent $\frac{7}{5} = 7 \times \frac{1}{5}$
b. Students use the same thinking to see $3 \times \frac{2}{5}$ as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5}$

KY.4.OA.2

Coherence KY.3.NF.1→ KY.4.NF.4→KY.5.NF.4

</td></tr>
</table>

Attending to the Standards for Mathematical Practice
As students begin to work with fractions greater than unit fractions such as $\frac{2}{3}+\frac{2}{3}=$ ____, they recognize, like whole numbers, they can decompose the non-unit fraction solve problems (Example: $\frac{2}{3}+\frac{2}{3}=\frac{2}{3}+\frac{1}{3}+\frac{1}{3}=1\frac{1}{3}$) (**MP.7**). Students apply this knowledge make sense of word problems and persevere in solving them (**MP.1**). By using tools and situations, students notice a pattern and generalize how to multiply a fraction by a whole number (for example, problems in the form n x $\frac{a}{b}$). For example, they use pattern blocks or Cuisenaire Rods to determine the answer to a set of tasks: 4 x $\frac{1}{2}$, 5 x $\frac{1}{3}$, 6 x $\frac{1}{3}$, 5 x $\frac{2}{3}$, 6 x $\frac{2}{3}$ and notice they multiply to find how many parts and thereby multiplying the whole number by the numerator (**MP.5**, **MP.8**). Note: Following a rote process of "putting a one under the whole number" or other rules not understood work against building understanding of 4.NF.4 and the development of mathematical practices.

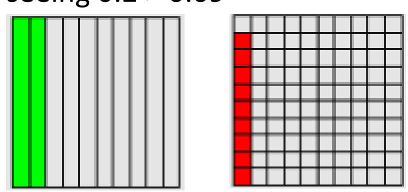
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand decimal notation for fractions and compare decimal fractions.

Standards	Clarifications
KY.4.NF.5 Convert and add fractions with denominators of 10 and 100. a. Convert a fraction with a denominator of 10 to an equivalent fraction with a denominator of 100. b. Add two fractions with respective denominators 10 and 100. **MP.5**, **MP.7**	For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$ Note: Students who generate equivalent fractions develop strategies for adding fractions with unlike denominators in general. Addition and subtraction with unlike denominators in general is not a requirement at grade 4. Coherence KY.3.NF.3→ KY.4.NF.5→KY.5.NBT.7
KY.4.NF.6 Use decimal notation for fractions with denominators 10 or 100. **MP.4**, **MP.7**	For example, students rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line. Coherence KY.4.NF.6→KY.5.NBT.3
KY.4.NF.7 Compare two decimals to hundredths. a. Compare two decimals to hundredths by reasoning about their size. b. Recognize that comparisons are valid only when the two decimals refer to the same whole. c. Record the results of comparisons with the symbols >, =, or < and justify the conclusions. **MP.2**, **MP.3**, **MP.5**	Students recognize comparisons are valid only when the two decimals refer to the same whole. For example, students use a visual model: seeing 0.2 > 0.09  Coherence KY.4.NF.7→KY.5.NBT.3

Attending to the Standards for Mathematical Practice

Students consider available tools and choose to use base ten blocks, graph paper, place value charts, number lines and other place value models to explore the relationships between fractions with denominators of 10 and denominators of 100 (**MP.5**). By using these tools, students begin to make abstract and quantitative connections to the relationship between fractions with denominators of 10 and 100 (**MP.2**). Through these experiences and work with fraction models, they build the understanding comparisons between fractions and decimals are only valid when the

whole is the same for both cases (hundredths or tenths) (**MP.7**)**.** Students use base ten blocks, 10 by 10 geoboards and 10 by 10 grids to illustrate and compare decimal fractions and justify their conclusions (**MP.3**, **MP.5**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.4.MD.1 Know relative size of measurement units (mass, weight, liquid volume, length, time) within one system of units (metric system, U.S. standard system and time).
a. Understand the relationship of measurement units within any given measurement system.
b. Within any given measurement system, express measurements in a larger unit in terms of a smaller unit.
c. Record measurement equivalents in a two-column table.
MP.5, MP.6</td><td>c. Two- column tables may include:
<table><tr><th>kg</th><th>g</th></tr><tr><td>1</td><td>1000</td></tr><tr><td>2</td><td>2000</td></tr><tr><td>3</td><td>3000</td></tr></table><table><tr><th>ft</th><th>in</th></tr><tr><td>1</td><td>12</td></tr><tr><td>2</td><td>24</td></tr><tr><td>3</td><td>36</td></tr></table><table><tr><th>lb</th><th>oz</th></tr><tr><td>1</td><td>16</td></tr><tr><td>2</td><td>32</td></tr><tr><td>3</td><td>48</td></tr></table>
Coherence KY.4.MD.1→KY.5.MD.1</td></tr>
<tr><td>KY.4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money.
a. Solve measurement problems involving whole number, simple fractions or decimals.
b. Solve problems that require converting a given measurement from a larger unit to a smaller unit within a common measurement system, such as 2 km = 2,000 m.
c. Visually display measurement quantities using representations such as number lines that feature a measurement scale.
MP.1, MP.4</td><td>Note: grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Coherence KY.3.MD.2→KY.4.MD.2</td></tr>
<tr><td>KY.4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.
MP.1, MP.3</td><td>Students apply the area and perimeter formulas to real world problems with an unknown factor:
Area = length × width ($A = l \times w$)</td></tr>
</table>

Standards	Clarifications
	perimeter = length + width + length + width ($p = l + w + l + w$ OR $p = 2l + 2w$) KY.3.MD.8 Coherence KY.3.MD.7→KY.4.MD.3→KY.5.MD.5

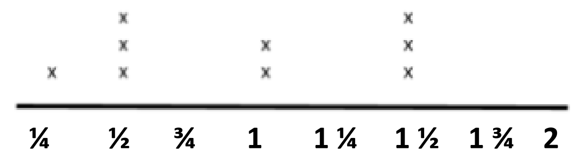
Attending to the Standards for Mathematical Practice
Students know relative sizes of measurement units by actually measuring with the units and establishing a reference to an object. For example, recognizing a centimeter is about the width of their finger (**MP.5**). Students also measure objects using different units within the same system, such as meters and in centimeters (using a meter stick). Record the measurements in a table and notice relationships (**MP.8**). They explain why this pattern is true, arguing each meter has 100 centimeters, so 3 meters will have 300 centimeters and more generally explaining the smaller the unit the more units there will be when measuring the same object (**MP.3**). As students solve problems, they attend to and explain the attribute being measured (length or area), the unit being used to measure and make sense of the problem using drawings, tools, or strategies that make sense to them (**MP.1**, **MP.3**).

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand and apply the statistics process.

Standards	Clarifications
KY.4.MD.4 Use dot plots to analyze data to a statistical question. a. Identify a statistical question focused on numerical data. b. Make a dot plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). c. Solve problems involving addition and subtraction of fractions by using information presented in dot plots. **MP.1**, **MP.6**	Students create dot plots to show a data set of objects with fractional measurements.  Coherence KY.3.MD.4→KY.4.MD.4→KY.5.MD.2

Attending to the Standards for Mathematical Practice

Students recognize a statistical question is one that has variability in the answer and create such a question of interest to them and for which there are numerical responses (**MP.1**). After gathering data on a question of interest, students recognize they have many data points and therefore creating a graph helps to analyze the data. In creating the dot plot, students create a scale from 0 to 1 and label the scale to include intervals of $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ (**MP.6**). As they solve problems related to the graph, they stay focused on the reason they created the graph - to provide insights into the question they first posed, so responses focus on the statistical question posed (**MP.1**).

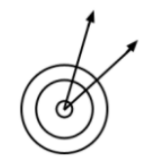
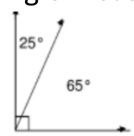
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understand concepts of angle and measure angles.

Standards	Clarifications
KY.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement. **MP.7**	An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. Angles are measured in reference to a circle with the center at that common point. 
KY.4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **MP.5**, **MP.6**	KY.4.MD.6 Coherence KY.4.MD.5→KY.4.MD.7
KY.4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. **MP.1**, **MP.4**	For example, students use an equation with a symbol for the unknown angle measure.  25° + □ = 90° Coherence KY.4.MD.7→KY.7.G.5

Attending to the Standards for Mathematical Practice

Students explore angle measures using tools (**MP.5**). For example, the white rhombus in a pattern block set or a cardboard cut-out is used as a 'unit' angle (a non-standard unit). Students use this tool to measure the size of other angles, noticing that angle measures are additive (**MP.1**). Building on concrete experiences, students explain $\frac{1}{360}$ of a circle, called a "one-degree angle," is the unit for measuring angles (**MP.7**). Students connect their concrete measuring experiences with a new tool, the protractor and use it to more precisely determine angle measures (**MP.5**, **MP.6**). When solving word problems involving angle measures, students use drawings and tools to make sense of the problem, recognizing non-overlapping angles can be added or subtracted to find missing angles (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	

Standards	Clarifications
KY.4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures. **MP.5**, **MP.6**	Coherence KY.3.G.1→KY.4.G.1
KY.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence of absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **MP.7**	Coherence KY.3.G.1→KY.4.G.2→KY.5.G.3
KY.4.G.3 Identify lines of symmetry. a. Recognize a line of symmetry for a two-dimensional figure. b. Identify line-symmetric figures and draw lines of symmetry. **MP. 5**, **MP.7**	

Attending to the Standards for Mathematical Practice

Using technology, using straightedges and/or protractors, students draw points, lines, line segments, rays, angles and perpendicular and parallel lines (**MP.5**). Students reason about the possible relationship of two lines or line segments. For example, students might use technology, uncooked spaghetti, or lines drawn on two transparency strips, to arrange two lines in different ways to determine possible events (the two lines might intersect, might intersect and be perpendicular, or may be parallel) (**MP.7**). Students analyze, compare and sort polygons based on their sides, angles and symmetry, explaining whether an attribute is a defining characteristic of that shape (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Grade 5 Overview

Operations and Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
• Write and interpret numerical expressions. • Analyze patterns and relationships.	• Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths.	• Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	• Convert like measurement units within a given measurement system. • Understand and apply the statistics process. • Geometric measurement: understand concepts of volume and relate volume to multiplications and to addition.	• Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.

In grade 5, instructional time should focus on three critical areas:

1. **In the Numbers and Operations - Fractions and Operations and Algebraic Thinking domains, students will:**
   - apply their knowledge of fractions and fraction models to illustrate the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators;
   - establish fluency in calculating sums and differences with fractions and make a reasonable estimate of those sums and differences;
   - use the meaning of fractions, of multiplication and division, and the relationship between those operations to understand and explain why the procedures for multiplying and dividing fractions make sense.

   (Note: This is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. **In the Operations and Algebraic Thinking and Number and Operations in Base Ten, students will:**
   - develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations;
   - apply understandings of models for decimals, decimal notation and properties of operations to add and subtract decimals to hundredths;
   - develop fluency with decimal computations to hundredths and make reasonable estimates of their computation;
   - use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers to understand and explain why the procedures for multiplying and dividing finite decimals make sense.
3. **In the Measurement and Data and Geometry domains, students will:**
   - recognize volume as an attribute of three-dimensional space;
   - understand that a 1-unit by 1-unit cube is the standard unit for measuring volume;
   - understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps;
   - choose appropriate units, strategies and tools for solving problems which involve estimating and measuring volume;
   - decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes;
   - measure attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Write and interpret numerical expressions.	
Standards	**Clarifications**
KY.5.OA.1 Use parentheses, brackets or braces in numerical expressions and evaluate expressions that include symbols. **MP.1**, **MP.3**	Students work with the order of first evaluating terms in parentheses, then brackets, [] and then braces, {}. Coherence KY.5.OA.1→ KY.6.EE.2
KY.5.OA.2 Write simple expressions with numbers and interpret numerical expressions without evaluating them. **MP.2**, **MP.7**	Students translate from words "add 8 and 7, then multiply by 2" to 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. KY.6.EE.2 Coherence KY.4.OA.1→ KY.5.OA.2→KY.6.EE.3 KY.6.EE.4
Attending to the Standards for Mathematical Practice	
Students move between words and symbols, understanding equivalent ways to express a statement. Students interpret the statement "The sum of 347, 124 and 99, divided by 30 as, (347 + 124 + 99) ÷ 30 and as $\frac{347+124+99}{30}$ (**MP.7**). As they evaluate such expressions, they realize there are options within the order of operations. In this expression, they add the three values and then divide by 30, or divide each addend by 30 and get the same answer. They think of a context to convince themselves two options will lead to the same answer (**MP.2**). In this case, students consider the two options and see the first idea is less 'messy' and therefore, a good choice (**MP.1**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Analyze patterns and relationships.	
Standards	**Clarifications**
KY.5.OA.3 Generate numerical patterns for situations. a. Generate a rule for growing patterns, identifying the relationship between corresponding terms (x, y). b. Generate patterns using one or two given rules (x, y). c. Use tables, ordered pairs and graphs to represent the relationship between the quantities. **MP.2**, **MP.4**	Given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, students generate terms in the resulting sequences (creating ordered pairs). Students observe the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. Graph the ordered pairs on a coordinate plane. Coherence KY.4.OA.5→ KY.5.OA.3→KY.6.EE.9
Attending to the Standards for Mathematical Practice	
Students notice when they apply a rule, like add 3, several patterns emerge. The explicit pattern is the new value is 3 more than the original value. But, as they explore they notice if they pick an input that is 5 more than the last input, then the output is also 5 more. They reason about this contextually, for example thinking of people ages in three years. So, if they have a sibling that is 5 years older now, in three years, they will still be 5 years older (**MP.2**). They represent these patterns on graphs and use the graphs to make sense of the situation (**MP.4**)	

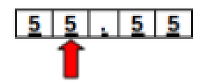
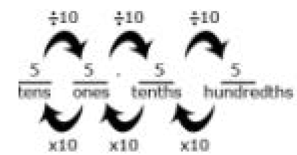
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten

Note: grade 5 expectations in this domain are limited to decimals through the thousandths place.

Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

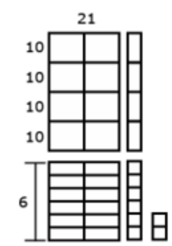
Cluster: Understand the place value system.

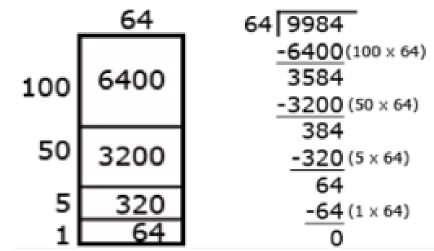
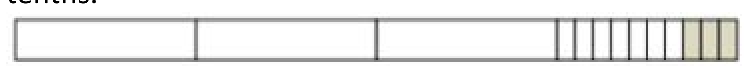
Standards	**Clarifications**
KY.5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. **MP.2**, **MP.7**	In the number 55.55, each digit is 5, but the value of each digit is different because of the placement.  The arrow points to is 1/10 of the 5 to the left and 10 times greater than the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times greater than five tenths.  Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.4.NBT.1→KY.5.NBT.1
KY.5.NBT.2 Multiply and divide by powers of 10. ● Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. ● Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. ● Use whole-number exponents to denote powers of 10. **MP.3**, **MP.8**	Students recognize when a number is multiplied by 10, a zero is added to the end because each digit's value became 10 times larger. Students use the same reasoning to explain in the problem. ● $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places. ● $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places. ● $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.5.NBT.2→ KY.6.EE.1

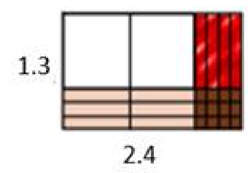
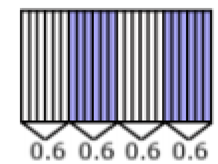
Standards	Clarifications
KY.5.NBT.3 Read, write and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **MP.2**, **MP.5**, **MP.7**	a. For the number 347.392… ● number name: three hundred forty-seven and three hundred ninety-two thousandths ● expanded form: $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$ Students relate numbers they are comparing back to common benchmarks of 0, $\frac{1}{2}$ (0.5, 0.50 and 0.500) and 1. When comparing numbers, 0.35 and 0.12, students make the connection 0.35 > 0.12, but also see the relationship of 0.12 < 0.35. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. KY.4.NBT.2 Coherence KY.4.NF.7→ KY.5.NBT.3
KY.5.NBT.4 Use place value understanding to round decimals to any place. **MP.5**, **MP.7**	Students go beyond application of an algorithm or procedure when rounding. Students demonstrate a deeper understanding of number sense and place value and explain and reason about the answers they get when they round. Note: grade 5 expectations in this domain are limited to decimals through the thousandths place. Coherence KY.4.NBT.3→ KY.5.NBT.4

Attending to the Standards for Mathematical Practice
Students compare the value of the digits based on where they are in a number (**MP.7**). They reason 10 tens equal 100, 70 tens equal 700 and this can be illustrated with base 10 blocks or other visuals (**MP.2**). Students look across series of problems to notice a pattern when multiplying by 10, 100 or 1000 (**MP.8**) and justify why patterns exist (why 36 x 100 = 3600), rather than superficially noting ‘you add zeros,’ they explain or show there are actually 36 *hundreds*, so 3600 (**MP.3**). Students use similar reasoning to compare decimal values, explaining tenths are larger than hundredths and therefore, they look to first see which values have more tenths before looking at how many hundredths it has (**MP.2**, **MP.7**). Students use tools such as number lines and base 10 blocks to see place value relationships with decimals in order to compare and to round (**MP.5**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

<table>
<tr><th colspan="2">Number and Operations in Base Ten</th></tr>
<tr><th colspan="2">Standards for Mathematical Practice</th></tr>
<tr><td>MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.</td><td>MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.</td></tr>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.5.NBT.5 Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
MP.7, MP.8</td><td>Students make connections from previous work with multiplication, using models/representations to develop an efficient algorithm to multiply multi-digit whole numbers.
<table>
<tr><td></td><td>300</td><td>70</td><td>4</td><td></td></tr>
<tr><td>50</td><td>15,000</td><td>3,500</td><td>200</td><td>= 18,700</td></tr>
<tr><td>3</td><td>900</td><td>210</td><td>12</td><td>= 1,122</td></tr>
<tr><td></td><td></td><td></td><td></td><td>19,822</td></tr>
</table>
374
× 53
12 (3 × 4)
210 (3 × 70)
900 (3 × 300)
200 (50 × 4)
3,500 (50 × 70)
15,000 (50 × 300)
19, 822
Coherence KY.4.NBT.5→ KY.5.NBT.5→KY.6.NS.3</td></tr>
<tr><td>KY.5.NBT.6 Divide up to four-digit dividends by two-digit divisors.
a. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using…
● strategies based on place value
● the properties of operations
● the relationship between multiplication and division
b. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.
MP.2, MP.3, MP.4</td><td>Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of partitive and measurement models for division to an algorithm, including partial quotients.
Some examples include:
968 ÷ 21 =
Students use base ten models by representing 962 and use the model to make an array with one dimension of 21. Student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.
</td></tr>
</table>

Standards	Clarifications
	Students use an area model for division shown below. As the student uses the area model, s/he keeps track of how much of the 9,984 is left to divide.  Students use expanded notation 2682 ÷ 25 = (2000 + 600 +80 + 2) ÷ 25. Students use his or her understanding of the relationship between 100 and 25, to think “I know 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80. Then 600 divided by 25 has to be 24. Since 3 × 25 is 75, I know that 80 divided by 25 is 3 with a remainder of 5. I can’t divide 2 by 25 so 2 plus the 5 leaves a remainder of 7. 80 + 24 + 3 =107. So the answer is 107 with a remainder of 7.” Students use an equation that relates division to multiplication, 25 × n = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 × 100 =2500. Coherence KY.4.NBT.6→ KY.5.NBT.6→KY.6.NS.2
KY.5.NBT.7 Operations with decimals to hundredths. a. Add, subtract, multiply and divide decimals to hundredths using… ● concrete models or drawings ● strategies based on place value ● properties of operations ● the relationship between addition and subtraction b. Relate the strategy to a written method and explain the reasoning used. **MP.2**, **MP.3**, **MP.5**	Students connect previous experiences with the meaning of multiplication and division of whole numbers to multiplication and division of decimals using estimation, models and place value structure. For example: 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.  The answer is 3 and $\frac{7}{10}$ or 3.7 An area model can be used for illustrating products.

Standards	Clarifications
	 2.4 × 1.3 .12 .60 .40 +2.00 3.12 Students describe the partial products displayed by the area model. For example, "$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$. $\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$. 1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$. 1 group of 2 is 2." Students dividing decimals for example could find the number in each group or share by applying the fair sharing model or separating decimals in to equal parts such as 2.4 ÷ 4 = 0.6  Coherence KY.4.NBT.6→ KY.5.NBT.7→KY.6.NS.3

Attending to the Standards for Mathematical Practice
Students understand when given a multiplication problem, they have a choice in how they solve it and select a way that makes sense for the values in the problem. For example, for 1234 x 12, they see the small numbers lend to a break apart strategy and solve the problem this way: 1234 x 10 = 12340 1234 x 2 = 2468 Then add the partial products to equal 14,808 (**MP.7**). Other students may stack the two values and use an algorithm. Students recognize a rectangle is an effective model for ensuring all partial products are calculated, for both whole numbers and decimals (**MP.4**). As students explore problems with decimal values, they reason about the problem, rather than following rules devoid of meaning (count the number of decimal places). For example, when multiplying 4 x 1.5, they use a break apart strategy, as they have for whole numbers, noticing 4 x 1 = 4 and 4 x 0.5 = 2, so therefore, 4 x 1.5 = 6 (**MP.2**). They explain why this works and when they use this strategy (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

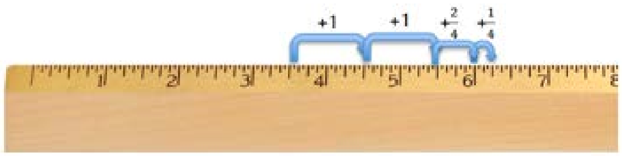
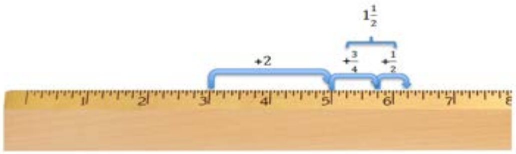
Number and Operations - Fractions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use equivalent fractions as a strategy to add and subtract fractions.

Standards	Clarifications
KY.5.NF.1 Efficiently add and subtract fractions with unlike denominators (including mixed numbers) by… ● using reasoning strategies, such as counting up on a number line or creating visual fraction models ● finding common denominators **MP.2**, **MP.3**	Using common denominator $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$ In general, $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$ KY.4.NF.1 Coherence KY.4.NF.3→ KY.5.NF.1→KY.6.EE.7
KY.5.NF.2 Solve word problems involving addition and subtraction of fractions. a. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. b. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. **MP.1**, **MP.4**	a. For example: Mary ate $\frac{1}{3}$ of the pizza. Tommy ate $\frac{2}{5}$ of the pizza. How much of the total pizza did they eat together? ● making equivalent fractions to add/subtract fractions ● using visual representations to add/subtract fractions ○ Area Model ○ Linear Model b. Recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, by observing that $\frac{3}{7}<\frac{1}{2}$. Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense. Coherence KY.4.NF.3→KY.5.NF.2

Attending to the Standards for Mathematical Practice

As students add and subtract fractions, they make sense of situations in story problems, selecting and creating representations of the situation such as partitioned rectangles or number lines (**MP.1**, **4**). Students notice if the fractions in the problem can be solved using a reasoning strategy, or if it is more efficient to find common denominators (**MP.2**). For example, for the problem $2\frac{3}{4}+3\frac{1}{2}$, students may mentally or physically refer to a ruler and use a counting up strategy:

Attending to the Standards for Mathematical Practice
$\boxed{2\frac{3}{4} + 3\frac{1}{2} = 3\frac{1}{2} + 2 + \frac{3}{4}}$  $2\frac{3}{4} + 3\frac{1}{2} = 3 + 2 + \frac{3}{4} + \frac{1}{2} = 5 + 1\frac{1}{4} = 6\frac{1}{4}$  Or, students use a break apart strategy noticing $\frac{3}{4}$ is $\frac{1}{2} + \frac{1}{4}$ and therefore, reason there are 6 wholes and $\frac{1}{4}$ more, so $6\frac{1}{4}$ is the sum. Other students rewrite the fractions as $2\frac{3}{4} + 3\frac{2}{4}$ and add the whole numbers and fractions separately and then combine them. Students explain their reasoning strategies and students listen to others who solved the problem differently than they solved it and determine if the reasoning makes sense, if it is efficient and if the answer is correct (**MP.3**).

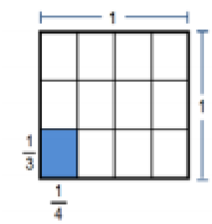
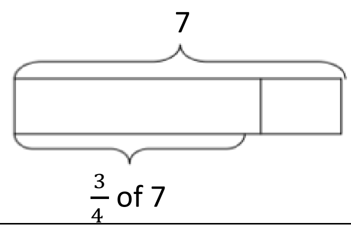
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

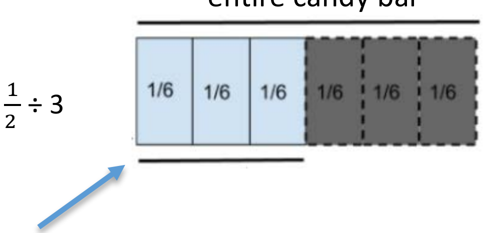
Number Operations - Fractions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standards	**Clarifications**
KY.5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. **MP.4**, **MP.8**	For example students interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3 and when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. Coherence KY.5.NF.3→KY.6.RP.2
KY.5.NF.4 Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(\frac{a}{b})$ x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x $q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. **MP.1**	a. Students use a visual fraction model to show $(\frac{2}{3})$ x 4 = $\frac{8}{3}$ and create a story context for this equation. Do the same with $(\frac{2}{3})$ x $(\frac{4}{5})$ = $\frac{8}{15}$. (In general, $(\frac{a}{b})$ x $(\frac{c}{d})$ = $\frac{ac}{bd}$.)  b. For example the shaded portion shows the rectangle with the appropriate unit fraction side lengths. Coherence KY.4.NF.4→ KY.5.NF.4→KY.6.G.1
KY.5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a	$\frac{1}{4}$ x 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7. 

Standards	Clarifications
fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1. **MP.2**, **MP.6**	Coherence KY.4.OA.1→ KY.5.NF.5→KY.6.RP.1
KY.5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers. **MP.4**, **MP.5**	KY.5.MD.2 Coherence KY.4.NF.4→KY.5.NF.6
KY.5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. b. Interpret division of a whole number by a unit fraction and compute such quotients. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. **MP.1**, **MP.4**, **MP.8**	Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of division of whole numbers to divide whole numbers by unit fractions and unit fractions by whole numbers. Division of a fraction by a fraction is not a requirement at grade 5. a. Create a story context for $(\frac{1}{3}) \div 4$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$. b. Create a story context for $4 \div (\frac{1}{5})$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$, because $20 \times (\frac{1}{5}) = 4$. c. By using visual fraction models and equations to represent the problem. entire candy bar $\frac{1}{2} \div 3$  Each child will get one piece. Half to be shared with 3 students. Coherence KY.4.NF.4→ KY.5.NF.7→KY.6.NS.1

Attending to the Standards for Mathematical Practice
Students look for repeated reasoning in order to understand the meaning of the operations (**MP.8**). Rather than memorize rules that do not make sense, students use mathematical representations to consider the relative size of their answers (**MP.4**). For example, students solve the classic "brownie sharing" problems, wherein brownies are shared equally with children. In considering how 4 children share 5 brownies. They use drawings of rectangles and partition to show each child will get $1\frac{1}{4}$ brownies. As students continue to explore brownie sharing, they notice patterns. In this case, they see $5 \div 4$ means the same as $\frac{5}{4}$ (**MP.4**). Students reason quantitatively as they work on scaling problems in context (**MP.2**). For example, in $\frac{3}{4}$ of 16, students might reason the answer is less than 16. To solve it, they begin by thinking $\frac{1}{4}$ of 16 is 4, then think 3 groups of 4 is 12. As students divide a problem such as $4 \div \frac{1}{8}$, $7 \div \frac{1}{8}$, $10 \div \frac{1}{8}$, they notice how many eighths in one whole and then multiply by how many wholes they have. This pattern leads to an understanding of why it looks like they are multiplying by the denominator (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Convert like measurement units within a given measurement system.

Standards	Clarifications
KY.5.MD.1 Convert among different size measurement units (mass, weight, liquid volume, length, time) within one system of units (metric system, U.S. standard system and time). **MP.3**, **MP.8**	Within the same system convert measurements in a larger unit in terms of a smaller unit and a smaller unit in terms of a larger unit. Use these conversions in solving multi-step, real world problems. Coherence KY.4.MD.1→ KY.5.MD.1→KY.6.RP.3

Attending to the Standards for Mathematical Practice

Students notice patterns about how units and measurements relate to each other (**MP.8**). For example, students measure various objects in meters and in centimeters (using a meter stick). As they measure their items, they record the measurements in a table. They notice the object that measures about 300 centimeters also measures about 3 meters (**MP.8**). They explain why this pattern is true, arguing each of the meters has 100 centimeters, so 3 meters will have 300 centimeters and more generally explaining the smaller the unit the more of unit there will be when measuring the same object (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply the statistics process.	
Standards	**Clarifications**
KY.5.MD.2 Identify and gather data for statistical questions focused on both categorical and numerical data. Select an appropriate data display (bar graph, pictograph, dot plot). Make observations from the graph about the questions posed. **MP.4**, **MP.5**, **MP.6**	Generate questions for which data can be gathered and sort questions that are categorical (Possible question: What is your favorite after-school activity?) and questions that are numerical (Possible question: How many times can you say/write your name in one minute?). After gathering data on a question, students discuss which graphs are possible and which ones are not possible, and why. Students select one type of graph that fits the data gathered and create the graph, by hand or by using technology. KY.6.SP.2 Coherence KY.4.MD.4→ KY.5.MD.2→KY.6.SP.4
Attending to the Standards for Mathematical Practice	
After gathering data on a question of interest, students recognize they have many data points and therefore, decide they will do a scaled graph (**MP.4**). In creating the graph, they decide to do a picture graph and pick a scale of 1 picture = 4 data points (**MP.6**). In another situation, students recognize they have numerical data and create a dot plot and decide to use a spreadsheet on the computer to create the graph (**MP.5**). Students compare how dot plots and bar graphs are similar and different, recognizing when to use each (**MP.6**).	

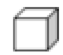
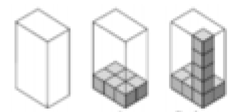
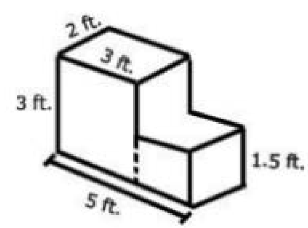
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Standards	Clarifications
KY.5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **MP.6**	a.   b. Coherence KY.3.MD.5→ KY.5.MD.3
KY.5.MD.4 Measure volumes by counting unit cubic cm, cubic in, cubic ft. and improvised units. **MP.5**, **MP.6**	Coherence KY.3.MD.6→ KY.5.MD.4
KY.5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes. b. Apply the formulas *V= l x w x h* and *V = B x h* for rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by	For example, students determine the volume of concrete needed to build the steps in the diagram below.  Coherence KY.4.MD.3→ KY.5.MD.5→KY.6.G.2

Standards	Clarifications
adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **MP.1**, **MP.4**, **MP.8**	

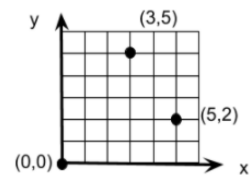
Attending to the Standards for Mathematical Practice
Students use cubes to cover a bottom layer of a rectangular prism, understanding cube as a unit cube (**MP.5**). As students place the cubes in layers to fill the rectangular solid, they notice the number of cubes in each layer can be found by multiplying [number of cubes in one row] x [number of rows] and this product (the base) can be multiplied by how many layers to determine how many unit cubes will fill the container (**MP.8**). Students connect this idea to the formulas for volume and use these formulas to solve problems (**MP.4**). When a three-dimensional shape is not a single rectangular solid, students analyze the shape and its measurements to determine how to decompose the shape and find the volume of each prism (**MP.1**).

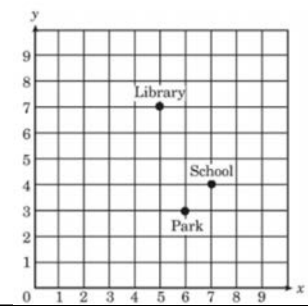
Geometry

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Graph points on the coordinate plane to solve real-world and mathematical problems.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.5.G.1 Use a pair perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second.
MP.4, MP.7</td><td>This standard pertains to the first quadrant only which limits to positive ordered pairs only.


Coherence KY.5.G.1→KY.6.NS.6</td></tr>
<tr><td>KY.5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
MP.1, MP.6</td><td>For example, students use the coordinate grid, which ordered pair represents locations of places or objects.


KY.6.NS.8
Coherence KY.5.G.2→KY.6.G.3</td></tr>
</table>

Attending to the Standards for Mathematical Practice

Students notice a coordinate axis, is in fact, coordinating a horizontal number line with a vertical number line (**MP.7**). These two lines need a title, scale and a label in order to be understood by a reader (**MP.6**). Students record data in their graph from exploring a pattern and gain insights about the pattern. For example, students graph data from a two-column table that shows the cost of buying pineapples (one pineapple costs $2, three pineapples costs $6) and use the coordinate axis to explain what they notice about the relationship between the number of pineapples and the cost of pineapples (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Classify two-dimensional figures into categories based on their properties.

Standards	Clarifications
KY.5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. **MP.3**, **MP.6**	For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. Coherence KY.4.G.2→ KY.5.G.3
KY.5.G.4 Classify two-dimensional figures in a hierarchy based on properties. **MP.1**, **MP.7**	Figures from previous grades: polygons, rhombus/rhombi, rectangle, square, triangle quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter, circle. For example: • Polygon - a closed plan figure formed from line segments that meet only at their endpoints. • Quadrilateral - a four-sided polygon • Rectangle - a quadrilateral with two pairs of congruent parallel sides and four right angles. • Rhombus - a parallelogram with all four sides equal in length • Square - a parallelogram with four congruent sides and four right angles. Coherence KY.4.G.2→ KY.5.G.4

Attending to the Standards for Mathematical Practice

As they have done in grade 3, students describe attributes they notice for a particular type of quadrilateral, focusing on side lengths and angles (**MP.6**). They compare the lists of defining attributes across shapes to notice what they have in common and what is different**.** (**MP.7**). They explain some types of quadrilaterals (parallelograms) are also rectangles because all the attributes of a parallelogram are also attributes of a rectangle (**MP.3**). They use this analysis to build an understanding of a rectangle as a special case of a parallelogram (a parallelogram with 90 degree angles) and use these understandings to create a hierarchy of quadrilaterals (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Grade 6 Overview

Ratios and Proportional Relationships (RP)	The Number System (NS)	Expressions and Equations (EE)	Geometry (G)	Statistics and Probability (SP)
• Understand ratio concepts and use ratio reasoning.	• Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Multiply and divide multi-digit numbers and find common factors and multiples. • Apply and extend previous understanding of numbers to the system of rational numbers.	• Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.	• Solve real-world and mathematical problems involving area, surface area and volume.	• Develop understanding of the process of statistical reasoning. • Develop understanding of statistical variability. • Summarize and describe distributions.

In grade 6, instructional time should focus on five critical areas:

1. **In the Ratios and Proportional Relationships domain, students will:**
   - use reasoning about multiplication and division to solve ratio and rate problems about quantities;
   - connect understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from and extending, pairs of rows (or columns) in the multiplication table and by analyzing simple drawings that indicate the relative size of quantities; and
   - expand the scope of problems for which they can use multiplication and division to solve problems and they connect ratios and rates.

2. **In the Number System domain, students will:**
   - use the meaning of fractions and relationships between multiplication and division to understand and explain why the procedures for dividing fractions make sense;
   - extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, particularly negative integers; and
   - reason about the order and absolute value of rational numbers and about the location of points on a coordinate plane.

3. **In the Expressions, Equations and Inequalities domain, students will:**
   - write expressions and equations that correspond to given situations, using variables to represent an unknown and describe relationships between quantities;
   - understand that expressions in different forms can be equivalent and use the properties of operations to rewrite and evaluate expressions in equivalent forms; and
   - use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations.

4. In the Geometry domain, students will:

- reason about relationships among shapes to determine area, surface area and volume. They find areas of right triangles, other triangles and special quadrilaterals by decomposing these shapes, rearranging or removing pieces and relating the shapes to rectangles.
- discuss, develop and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths

5. In the Statistics and Probability domain, students will:

- develop their ability to think statistically;
- recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally and also in the sense that it is a balance point.
- recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.
- learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps and symmetry, considering the context in which the data were collected.

Ratios and Proportional Relationships	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understanding ratio concepts and use ratio reasoning to solve problems.	
Standards	**Clarifications**
KY.6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **MP.2**, **MP.6**	Students use the concept of ratios as a comparison between related quantities; students also express these relationships in equivalent ratios in lowest terms, where appropriate. Coherence KY.5.NF.5→KY.6.RP.1
KY.6.RP.2 Understand the concept of a unit rate a/b associated with a ratio *a:b* with B ≠ 0 and use rate language in the context of a ratio relationship. **MP.2**, **MP.6**	Expectations for unit rates in grade 6 are limited to non-complex fractions; additionally, students reduce ratios of two whole numbers to a unit rate involving a fraction and a denominator of 1. Students describe real-life contexts using ratio language. Coherence KY.5.NF.3→KY.6.RP.2→KY.7.RP.1
KY.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve rate problems including those involving unit pricing and constant speed. c. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **MP.1**, **MP.4**, **MP.7**	a. Students find the missing values in a table, assuming the values in the table represent a proportional relationship; students plot the values from a table on a coordinate plane, with appropriate labels and scales; Students compare the ratios of tables, answering, which has a greater/less rate. b. Students find a unit rate from a given situation and reason to apply it to a future scenario. c. For example, convert miles per hour to feet per hour or meters per minute to meters per hour using appropriate conversion ratios. Coherence KY.6.RP.3→KY.7.RP.2

Attending to the Standards for Mathematical Practice
As students solve similar problems, they develop their skills in several mathematical practice standards, reasoning abstractly and quantitatively (**MP.2**), abstracting information from the problem, creating a mathematical representation of the problem and correctly working with both part-part and part-whole situations. Students attend to precision (**MP.6**) as they properly use ratio notation, symbolism and label quantities. Representing ratios in various ways help students see the additive and multiplicative structure of ratios (**MP.7**). Students model with mathematics (**MP.4**) when they solve real-world and mathematical problems using ratio and rate reasoning, especially when they make use of various representations in the modeling process.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
Standards	**Clarifications**
KY.6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. **MP.1**, **MP.2**, **MP.3**	For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient: How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mile? Coherence KY.5.NF.7→KY.6.NS.1→KY.7.NS.2
Attending to the Standards for Mathematical Practice	
Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" (**MP.1**). For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. Students confirm, for example, that 10 ÷ 2 can be found by determining how many groups of two are in ten. They apply that strategy to the division of fractions (**MP.3**). Students use pictorial representations such as area models, array models, number lines and drawings to conceptualize and solve problems.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.	
Standards	**Clarifications**
KY.6.NS.2 Fluently divide multi-digit numbers using an algorithm. a. Convert a rational number to a decimal using long division. b. Know that the decimal form of a rational number terminates in 0s or eventually repeats. **MP.7**, **MP.8**	a. Divide a rational number a/b using long division, making sure to include rational numbers equivalent to terminating decimals and rational numbers equivalent to repeating decimals. b. Students understand and explain when they have a 0 remainder in a long division problem, the quotient (answer) is a terminating decimal; students understand when they notice a pattern in the process of dividing, they conclude they will never reach a 0 remainder and they then notate the part of the quotient that is repeating by marking a bar over those values. Coherence KY.5.NBT.6→KY.6.NS.2
KY.6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. **MP.2**, **MP.6**	Emphasis is on the role of the decimal point in operations and how place value is critical to the overall fluency of the performed operations involving decimals. KY.5.NBT.5 Coherence KY.5.NBT.7→KY.6.NS.3→KY.7.NS.3
KY.6.NS.4 Use the distributive property to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **MP.8**	Express numerical expressions using the distributive property; understand there may be multiple equivalent expressions, but only one will have been completely factored (the greatest common factor removed using the distributive property) such as 6 + 21 = 3 (2 +7). Coherence KY.4.OA.4→KY.6.NS.4

Attending to the Standards for Mathematical Practice
Students understand and use connections between divisibility and the greatest common factor to apply the distributive property (**MP.2**). Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students use precise language and place value (**MP.6**) when adding, subtracting, multiplying and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, 326.31 is read as three hundred twenty-six and thirty-one hundredths (**MP.7**). Students calculate sums, differences, products and quotients of decimal numbers with a degree of precision appropriate to the problem context.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understanding of numbers to the system of rational numbers.	
Standards	**Clarifications**
KY.6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **MP.1. MP.2**, **MP.4**	For example, positive and negative temperatures or elevations, with the understanding that zero means the freezing point Celsius of water or sea level. Coherence KY.6.NS.5→KY.7.NS.1
KY.6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes, using appropriate range and intervals, to represent points on the line and in the plane, that include negative numbers and coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize 0 is its own opposite and the opposite of a positive number is a negative, and the opposite of a negative number is a positive, such as $-(-3) = 3$. b. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. c. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize the similarity between whole numbers, their negative opposites and their positions on a number line, ordered pairs differ only by signs and their locations on one or both axes. **MP.2**, **MP.4**	a. Emphasis is on student understanding that every positive location on a number line has an opposite the same distance from zero in the negative direction and vice versa. Logically following from this is the fact that zero, as it has no positive or negative sign, is its own opposite. b. Emphasis is on generalizing patterns about where coordinates are located on a coordinate plane. c. The intent is for students to see a coordinate axis is the combination of a vertical number line and a horizontal number line. KY.6.EE.6 Coherence KY.5.G.1→KY.6.NS.6→KY.7.NS.1

Standards	Clarifications
KY.6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order. **MP.1. MP.2**, **MP.4**	a. Interpret two numbers, including two negatives, as one is to the left or right (or above or below) the other on a number line diagram. b. Understand, as with 6.NS.7a, positive and negative rational numbers represent real-life situations and can be compared. c. Interpret a positive or negative direction from zero as an absolute value, or magnitude, to describe a real-life situation. d. Recognize a number's distance from zero can be compared to another number's distance from zero with a "less than" or "greater than" distinction. Coherence KY.5.NBT.3→KY.6.NS.7→KY.7.NS.1 KY.6.EE.8
KY.6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **MP.5**, **MP.7**	For example, represent the vertices of a rectangle in the coordinate plane and find distances between horizontal and vertical vertices accurately. Given a vertex of (-2, 3), a length of 5 and a width of 11, locate the other three vertices of the rectangle. Coherence KY.5.G.2→KY.6.NS.8

Attending to the Standards for Mathematical Practice
Students use vertical and horizontal number lines to visualize integers and better understand their connection to whole numbers. They divide number line intervals into sub-intervals of tenths to determine the correct placement of rational numbers (**MP.7**). Students may represent a decimal as a fraction or a fraction as a decimal to better understand its relationship to other rational numbers to which it is being compared (**MP.2**). To explain the meaning of a quantity in a real-life situation (involving elevation, temperature, or direction), students draw a diagram and/or number line to illustrate the location of the quantity in relation to zero or an established level that represents zero in that situation (**MP.4**). Students understand the placement of negative numbers on a number line by observing the patterns that exist between negative and positive numbers with respect to zero (**MP.7**). They recognize two numbers are opposites if they are the same distance from zero and zero is its own opposite. Students extend their understanding of the number line structure to the coordinate plane to determine a point's location. They recognize the relationship between the signs of a point's coordinates and the quadrant in which the point lies.

Expression and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.	
Standards	**Clarifications**
KY.6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. **MP.2**, **MP.6**	Interpret an exponent of size *n* as a repetitive multiplication expression of the base multiplied by itself *n* times; use the standard order of operations using exponents to evaluate numerical expressions. Coherence KY.5.NBT.2→KY.6.EE.1→KY.8.EE.1
KY.6.EE.2 Write, read and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sums, term, product, factor, quotient, coefficient); view one or more parts of an expression in a single entity. c. Evaluate expressions for specific values of their variables, including values that are non-negative rational numbers. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **MP.1**, **MP.3**, **MP.4**	For example, a. Express the calculation "y less than 5" as 5 – y. b. Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Use the formulas $V = s^3$ and $SA = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$ meter. KY.5.OA.1 Coherence KY.5.OA.2→KY.6.EE.2
KY.6.EE.3 Apply the properties of operations to generate equivalent expressions. **MP.7**, **MP.8**	Using Associative, Commutative and Distributive properties to generate equivalent expressions. Coherence KY.5.OA.2→KY.6.EE.3→KY.7.EE.1

Standards	Clarifications
KY.6.EE.4 Identify when two expressions are equivalent when the two expressions name the same number regardless of which value is substituted into them. **MP.2**, **MP.3**, **MP.7**	Students commonly think of variables as a missing number. The focus of this standard is recognizing the variable represents *any* number. In other words, they do not seek to find a single number to replace the letter, but they substitute any number and the expressions will be equivalent. When each expression (not just the variable) is altered by the same value, the expressions remain equivalent, no matter the value. Coherence KY.5.OA.2→KY.6.EE.4→KY.7.EE.1

Attending to the Standards for Mathematical Practice
Students connect symbols to their numerical referents. They understand exponential notation as repeated multiplication of the base number. Students realize 3^2 is represented as 3 × 3, with a product of 9 and explain how 3^2 differs from 3 × 2, where the product is 6. Students determine the meaning of a variable within a real-life context (**MP.2**). Students look for structure in expressions by deconstructing them into a sequence of operations. They make use of structure to interpret an expression's meaning in terms of the quantities represented by the variables. In addition, students make use of structure by creating equivalent expressions using properties. For example, students write $6x$ as $x + x + x + x + x + x$, $4x + 2x$, $3(2x)$, or other equivalent expressions (**MP.7**). Students look for regularity in a repeated calculation and express it with a general formula (**MP.8**). Students work with variable expressions while focusing more on the patterns that develop than the actual numbers that the variable represents. For example, students move from an expression such as $3 + 3 + 3 + 3 = 4 \cdot 3$ to the general form $m + m + m + m = 4 \cdot m$, or $4m$. Similarly, students move from expressions such as $5 \cdot 5 \cdot 5 \cdot 5 = 5^4$ to the general form $m \cdot m \cdot m \cdot m = m^4$. These are especially important when moving from the general form back to a specific value for the variable (**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Reason about and solve one-variable equation and inequalities.	
Standards	**Clarifications**
KY.6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **MP.1**, **MP.2**, **MP.7**	From a set of numbers, substitute values to choose which satisfy a given equation or inequality. An equation or inequality with no solutions from the list may be described as having no solutions or an empty set of solutions, given the set of possible values. Coherence KY.6.EE.5→KY.8.EE.8
KY.6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set. **MP.2**, **MP.6**	Represent an unknown quantity in real-world context appropriately with a variable and write an expression to show this. Coherence KY.6.EE.6→KY.7.EE.4
KY.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **MP.1**, **MP.2**, **MP.3**, **MP.4**	Emphasis is on understanding equations can be solved by using subtraction as an opposite operation of addition and division as an opposite operation of multiplication. Additionally, emphasis is on the importance of keeping the equations balanced when solving. Coherence KY.6.EE.7→KY.7.EE.4
KY.6.EE.8 Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of these forms have infinitely many solutions; represent solutions of such inequalities on vertical and horizontal number lines. **MP.3**, **MP.7**	Emphasis is on students understanding the phrases “more than”, “less than”, “at least” and “at most” represent constraints and conditions and are therefore associated with the operators listed in real-world problems. Students also understand an inequality does not yield a specific value, but rather an infinite range of values. Students also appropriately represent solutions to inequalities using both open and closed circles, along with direction, on vertical and horizontal number lines. Coherence KY.6.EE.8→KY.7.EE.4

Attending to the Standards for Mathematical Practice
Students have previously explored the concept of equality. In grade 6, students explore equations as one expression being set equal to a specific value. A solution is a value of the variable that makes the equation true and students may use various processes to identify such values that, when substituted for the variable, will make the equation true (**MP.2**). This reasoning is also applied when recognizing solutions for inequalities, such that students realize the value of a variable is one that would make the inequality statement true. Students use manipulatives and pictures (e.g., tape-like diagrams) to represent the equations and their solution strategies. When writing equations, students learn to be precise in their definition of a variable (**MP.6**), for example, writing “n equals John’s age in years” as opposed to writing “n is John”. Students use tables and graphs to compare different expressions or equations to make decisions in real-world scenarios. These models also create structure as students gain knowledge in writing expressions and equations (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and analyze quantitative relationships between dependent and independent variables.	
Standards	**Clarifications/Illustrations**
KY.6.EE.9 Use variables to represent two quantities in a real-world problem that changes in relationship to one another; a. Appropriately recognize one quantity as the dependent variable and the other as the independent variable. b. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. c. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the question. **MP.3**, **MP.4**, **MP.7**	Students understand in real-world problems, one quantity dependently changes relative to another independent quantity at a constant rate; understand, at times, the quantities given may not have a clear independent/dependent relationship. Coherence KY.5.OA.3→KY.6.EE.9→KY.8.EE.5
Attending to the Standards for Mathematical Practice	
Students show relationships between quantities with multiple representations, using language, a table, an equation, or a graph. Translating between multiple representations helps students understand each form represents the same relationship and provides a different perspective on the relationship. (**MP.3**) Students construct arguments supporting mathematical claims about the relationship between the dependent and independent variable using evidence from the different representations. Students are also equipped to examine the evidence and claims of other students while comparing the different representations. Students model with mathematics (**MP.4**) the relationship between dependent and independent variables. Students use many forms to represent the relationship between quantities. Students demonstrate a mathematical model by translating between multiple representations to provide different perspectives on the relationship at hand.	

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve real-world and mathematical problems involving area, surface area and volume.	
Standards	**Clarifications**
KY.6.G.1 Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and quadrilaterals; apply these techniques in the context of solving real-world and mathematical problems. **MP.1**, **MP.6**, **MP.8**	Area of the listed shapes may be thought of as a rectangle with larger area, subtracting the areas exterior to the actual shape to obtain the true area, or as a composite area of smaller triangles and rectangles which sum to the true area of the given shape. Students recognize given shapes can be combined to find area or decomposed to find area, such as surface area, and one method may be more efficient than the other. Coherence KY.5.NF.4→KY.6.G.1→KY.7.G.6
KY.6.G.2 Find the volume of a right rectangular prism with rational number edge lengths. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with rational number edge lengths in the context of solving real-world and mathematical problems. **MP.2**, **MP.5**, **MP.6**	Coherence KY.5.MD.5→KY.6.G.2→KY.7.G.6
KY.6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **MP.4**, **MP.5**, **MP.6**	For example, a gardener draws a map of his garden on a coordinate plane with vertices (-2, 7), (-2, -1), (4, 7). What is the base and height of this triangle? What is the area of his garden, assuming each unit on the coordinate plane is 1 meter? Coherence KY.5.G.2→KY.6.G.3
KY.6.G.4 Classify three-dimensional figures including cubes, prisms, pyramids, cones and spheres. **MP.2**, **MP.3**	Emphasis is on classifying three-dimensional shapes and specifically the attributes of each shape that make it unique to its classification.

	Coherence KY.6.G.4→KY.7.G.6

Attending to the Standards for Mathematical Practice
Students make sense of real-world problems involving area, volume and surface area. Students begin to understand any shape can be thought of as a series of simpler shapes, merely stitched together to form a composite shape (**MP.1**). They begin to visualize the volume of any given shape as a bounded region, filled with smaller cubes of equal size (**MP.2**) and understand, by doing so, they approximate the volume of a three-dimensional shape with integer edge lengths (**MP.5**) and then, continue this reasoning by precisely finding the volume of figures with rational edge lengths (**MP.1**, **MP.6**, **MP.8**). Generalizing the study of geometric shapes to the coordinate plane gives students a tool to precisely calculate side lengths and area of shapes. When two different units are given within a problem, students know to use previous knowledge of conversions to make the units match before solving the problem (**MP.4**, **MP.5**, **MP.6**).

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Develop understanding of the process of statistical reasoning.

Standards	Clarifications/Illustrations
KY.6.SP.0 Apply the four-step investigative process for statistical reasoning. a. Formulate Questions: Formulate a statistical question as one that anticipates variability and can be answered with data. b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual and comparing individual to group. d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. **MP.1**, **MP.4**	Emphasis is on understanding answering a statistical question is completed by an investigative process that encompasses questioning, collection, analysis and interpretation of the data gathered. Coherence KY.5.MD.2→KY.6.SP.0→KY.7.SP.1

Attending to the Standards for Mathematical Practice
The four-step investigative process provides a structure for students to follow that allows them to model many real-world situations with a model (**MP.4**). Students use the statistical process to seek to understand the world around them, taking time to pursue the entire process in order to gain insights, looping back to make revisions to the question or data gathering if the results they have do not adequately address their question (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Develop understanding of statistical variability.

Standards	Clarifications
KY.6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. **MP.1**, **MP.3**, **MP.6**	For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates a variety of values with associated variability in students' ages. Coherence KY.5.MD.2→KY.6.SP.1→KY.7.SP.1
KY.6.SP.2 Understand that a set of numerical data collected to answer a statistical question has a distribution which can be described by its center, spread and overall shape. **MP.2**, **MP.6**, **MP.7**	Students distinguish between graphical representations which are skewed or approximately symmetric; use a measure of center to describe a set of data. Coherence KY.5.MD.2→KY.6.SP.2→KY.7.SP.3
KY.6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number to describe a typical value, while a measure of variation describes how the values in the distribution vary. **MP.2**, **MP.5**, **MP.6**	Emphasis is on the sensitivity of measures of center to changes in the data, such as mean is generally much more likely to be pulled towards an extreme value than the median. Additionally, measures of variation (range, interquartile range) describe the data by giving a sense of the spread of data points. Coherence KY.6.SP.3→KY.7.SP.4

Attending to the Standards for Mathematical Practice

Students recognize a question such as "What did I eat for breakfast?" is not a statistical question, whereas "What is the most popular breakfast in my school?" will elicit data they can measure precisely (**MP.6**) and draw conclusions based on that data (**MP.3**). After collecting data, by creating a distribution of that data, students recognize data generally follows a structure and can be described in terms of that structure (**MP.7**). By accurately calculating the mean (or any other statistical measure), students are now more precise in describing data, going from, for example, describe the rainfall for the month as "about average" to "the rainfall this month is slightly higher than the mean of the last 10 years and within the interquartile range for that data." (**MP.6**)

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Summarize and describe distributions.	
Standards	**Clarifications**
KY.6.SP.4 Display the distribution of numerical data in plots on a number line, including dot plots, histograms and box plots. **MP.6**, **MP.7**	Students create the listed graphical representations in the appropriate context and describe the attributes of each. Coherence KY.5.MD.2→KY.6.SP.4→KY.7.SP.1
KY.6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Determining quantitative measures of center (median and/or mean) to describe distribution of numerical data. d. Describing distributions of numerical data qualitatively relating to shape (using terms such as cluster, mode(s), gap, symmetric, uniform, skewed-left, skewed-right and the presence of outliers) and quantitatively relating to spread/variability (using terms such as range and interquartile range). e. Relating the choice of measures of center and variability to the shape of the data distribution. **MP.3**, **MP.7**	a. Students understand larger numbers of observations create a more accurate statistical representation than smaller numbers of observations. b. Students describe how the data measured relates to answering a statistical question. c. Students know methods of finding measures of center, including finding median in non-ordered sets of data and a mean is a mathematical average. d. Students describe the shape of data by inspection using the terms listed and calculate the range and interquartile range of a set of data. e. Students recognize mean and range are appropriate measures for symmetrical data while the median and interquartile range may be better measures for skewed data. Coherence KY.6.SP.5→KY.7.SP.1
Attending to the Standards for Mathematical Practice	
Students make use of structure by aligning numerical data into plots and histograms. Students characterize their data in a distribution using mathematically precise terms, both quantitatively (mean, IQR, etc.) and qualitatively (skewed, clustered, etc.). (**MP.7**). Students summarize their data in a variety of ways, both numerically and graphically and use these summaries to draw conclusions about their results (**MP.3**). Additionally, because students are calculating precisely the measures of center and variability for their data, they accurately compare data sets in a variety of ways (**MP.6**).	

Kentucky Academic Standards for Mathematics: Grade 7 Overview

Ratio and Proportional Relationships (RP)	The Number System (NS)	Expressions and Equations (EE)	Geometry (G)	Statistics and Probability (SP)
• Analyze proportional relationships and use them to solve real-world and mathematical problems.	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.	• Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area and volume.	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use and evaluate probability models.

In grade 7, instructional time should focus on four critical areas:

1. **In the Ratios and Proportional Relationships domain, students will:**
    - extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems;
    - use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips and percent increase or decrease;
    - solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects;
    - graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope;
    - distinguish proportional relationships from other relationships.

2. **In the Number System and the Expressions, Equations and Inequalities domains, students will:**
    - develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation) and percents as different representations of rational numbers;
    - extend addition, subtraction, multiplication and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division--by applying these properties and by viewing negative numbers in terms of everyday contexts;
    - explain and interpret the rules for adding, subtracting, multiplying and dividing with negative numbers;
    - use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. **In the Geometry domain, students will:**
    - continue their work with area from grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects;

- reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions and they gain familiarity with the relationships between angles formed by intersecting lines;
- work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections;
- solve real-world and mathematical problems involving area, surface area and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. **In the Statistics and Probability domain, students will:**
    - build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations;
    - begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Ratios and Proportional Relationships	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Analyze proportional relationships and use them to solve real-world and mathematical problems.**	
Standards	**Clarifications**
KY.7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. **MP.2**, **MP.6**	For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour. KY.6.RP.2 Coherence KY.6.RP.3→ KY.7.RP.1
KY.7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities represent a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **MP.1**, **MP.2**, **MP.3**	a. Students test for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Students understand finding the unit rate in a table or graph is equivalent to the constant of proportionality in an equation or verbal description. KY.8.F.2 KY.8.F.4 Coherence KY.6.RP.3a →KY.7.RP.2b →KY.8.EE.6 c. If total cost *t* is proportional to the number n of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$. Coherence KY.7.RP.2c → KY.8.EE.5 d. Students describe points (*x, y*) in terms of the labels of the *x*- and *y*-axes; students understand in a proportional relationship (0, 0) is a valid point and (1, *r*) represents the unit rate and the constant of proportionality for the relationship between the quantities.

Standards	Clarifications
	Coherence KY.7.RP.2d → KY.8.F.5
KY.7.RP.3 Use percents to solve mathematical and real-world problems. a. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, a part and a percent, given two of these. b. Use proportional relationships to solve multistep ratio and percent problems. **MP.5**, **MP.6**	a. For example, 30% of a quantity means 30/100 times the quantity. b. Could include but not limited to simple interest, tax, markups and markdowns, gratuities and commissions, percent increase and decrease, percent error. Coherence KY.6.RP.3c → KY.7.RP.3

Attending to the Standards for Mathematical Practice

Translating a rate to a unit rate allows students to contextualize a complex ratio to something more likely for them to understand, for example, a rate of miles per ONE hour or gallons per ONE minute (**MP.2**). The use of unit rates allows students to be precise in their understanding, transferring "½ mile in ¼ hour" to something understandable, such as 2 miles per hour (**MP.1**). Students think about why some relationships are proportional where others are not. Students make sense of and solve multistep ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane and equations and relate these representations to each other and to the context of the problem. Students depict the meaning of the constant of proportionality in proportional relationships and the importance of (0, 0) and (1, *r*) on graphs (**MP.1**). Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Students use concrete numbers to create and implement equations, including $y = kx$, where *k* is the constant of proportionality. (**MP.2**) One special proportional relationship in common usage involves percents. Students may think about "percent" as "part of 100" and solve a proportional relationship for any missing part of the relationship between a number, a part of that number and the associated percentage (**MP.5**). Students reason about when their resulting solutions make sense, as when the resulting solution is greater than 100% or, when speaking about percent increase, decrease and error, when their resulting solution may be a negative value (**MP.6**).

The Number System

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.**

Standards	Clarifications
KY.7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. **MP.2**, **MP.4**, **MP.7**	a. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. The sum of numbers is a directional movement from one number to another for a specified amount of spaces on the number line. The sum of opposites is 0 due to the fact that opposites have equivalent absolute values. c. Subtracting a positive number is the same as adding the positive number's opposite. KY.6.NS.5 KY.6.NS.6 Coherence KY.6.NS.7→ KY.7.NS.1
KY.7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules	a. Emphasis is on exploring and understanding how the rules for multiplying and dividing with negative numbers are connected to properties for the operations, rather than to think of them as arbitrary rules. They explain 4 times (-3) could be four days of golfing 3 under par and therefore, having an overall score of -12. The remaining operations are based on applying properties.

Standards	Clarifications
for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. **MP.2**, **MP.7**, **MP.8**	b. Emphasis is on the equivalence relationship provided by the movement of one negative sign among the numerator, denominator, or in front of the entire fraction. Coherence KY.6.NS.1→ KY.7.NS.2 → KY.8.NS.1
KY.7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. **MP.1**, **MP.2**, **MP.5**	Emphasis is on applying mathematical operations to rational numbers that occur in real world context. Coherence KY.6.NS.3 → KY.7.NS.3

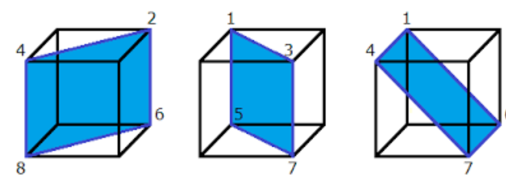
Attending to the Standards for Mathematical Practice
In grade 7, students build upon understanding by examining inverses and reason any number has an additive inverse, which is the mirror-image of the original number, albeit on the opposite side of zero, which brings the idea of absolute value to life (**MP.2**). The structure of working with the various properties of rational numbers cannot be ignored and students systematically apply these properties in a variety of scenarios (**MP.7**). Understanding these properties gives students a tool to model many real-world situations with simpler mathematical sentences. Through the use of number lines, tape diagrams, expressions and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples (**MP.4**). Students demonstrate fluency in applying the four operations to rational numbers in real life situations when they strategically apply the properties of operations to model real-world situations and truly making sense of the world around them with mathematics. Additionally, as students fluently solve word problems, they consider their steps and determine whether or not they make sense in relationship to the arithmetic understanding that served as their foundation in earlier grades (**MP.1**, **MP.2**, **MP.4**, **MP.5**). Students move from recall of applying rules of multiplying and dividing signed numbers to the ability to apply these rules strategically in a variety of situations. Students formulate rules for operations with signed numbers by observing patterns (**MP.2**, **MP.8**).

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Use properties of operations to generate equivalent expressions.**	
Standards	**Clarifications**
KY.7.EE.1 Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. **MP.2**, **MP.3**	Students demonstrate understanding of applying the order of operations to an expression involving multiple operations, including using the distributive property and variables in the expression. Students apply the properties of commutative, associative and distributive fluently. Coherence KY.6.EE.3 → KY.7.EE.1 → KY.8.EE.7
KY.7.EE.2 Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. **MP.7**, **MP.8**	Students apply mathematical properties in order to rewrite expressions and clarify the relationship of quantities in a problem. For Example: If Tom and Jim both get paid a wage of $11 per hour, but Tom was paid an additional $55 for overtime, the expression $11(T + J) + 55$ may be more clearly interpreted as $11T + 55 + 11J$ for purposes of understanding Tom's pay separated from Jim's pay. Coherence KY.6.EE.4 → KY.7.EE.2 → KY.8.EE.8c
Attending to the Standards for Mathematical Practice	
Students who fluently use the strategies of the properties of rational numbers to reason through the standard order of operations by applying these properties in a structured way. Students recognize the repeated use of the distributive property as they write equivalent expressions (**MP.7**). When given an example problem involving multiple operations containing a mistake, students answer the question "Where did the mistake occur and how do I know?" (**MP.3**). Students bring mathematical context to real-life situations by understanding multiple representations of quantities may exist. For example, adding 5% to quantity *a* leads to an expression of $a + .05a = 1.05a$, which clarifies the problem. Students access previous knowledge of working with percents to use the same structure to see equivalent expressions exist, even when taken out of the context of the real-world situation (**MP.7**). Students extend this reasoning to understand other situations (**MP.8**).	

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**	
Standards	**Clarifications**
KY.7.EE.3 Solve real-life and mathematical problems posed with positive and negative rational numbers in any form, using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. **MP.1**, **MP.4**, **MP.6**	Students solve multi-step real-world and mathematical problems containing integers, fractions and decimals, using previously acquired skills around converting fractions, decimals and percentages and use properties of operations to find equivalent forms of expressions when needed. Students solidify understanding by checking their solutions for reasonableness using estimation strategies such as rounding, compatible numbers and benchmark numbers. Coherence KY.7.EE.3 →KY.8.EE.4
KY.7.EE.4 Use variables to represent quantities in a real-world or mathematical problem and construct equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q* and *r* are specific rational numbers. Solve equations of these forms. Graph the solution set of the equality and interpret it in context of the problem. b. Solve word problems leading to inequalities of the form $px + q > r, px + q < r, px+q \geq r, px+q \leq r$; where *p*, *q* and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in context of the problem. **MP.2**, **MP.4**	a. Interpret word problems in the form of the initial value as a one-time occurrence within the problem and the coefficient as the recurring event within the problem. Coherence KY.6.EE.7 → KY.7.EE.4 → KY.8.EE.7 b. Interpret word problems having one or more solutions that satisfy the conditions of the problem. Graph on a number line the solution set that satisfies the conditions of the problems. Coherence KY.6.EE.8 → KY.7.EE.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
It is common for students to have difficulty in scaffolding from simple problems to more complex, multi-step problems; assistance in this regard is given by the use of estimation strategies to benchmark their work and lend confidence to more accurate solutions (**MP.1**, **MP.6**). Students apply the properties of rational numbers in order to solve equations and inequalities. Students must be precise when defining a variable (**MP.6**). Students reason a solution to a real-life situation but may struggle with modeling the problems with an equation or inequality involving a variable. For example, “I buy 6 pencils and a \$3 pen for a total of \$12. How much did each pencil cost?” Students with an understanding of numbers, but not the idea of a variable, may create an equation of $p = \frac{12-3}{6}$ = 1.50. Students who successfully model with mathematics understand the variable represents the cost of one pencil and use it appropriately, $6p + 3 = 12$, which more accurately represents the problem presented (**MP.4**).

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Draw, construct and describe geometrical figures and describe the relationships between them.**	
Standards	**Clarifications**
KY.7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **MP.1**, **MP.2**, **MP.5**	Emphasis is on being able to convert values from one given measurement to another based on a given scale factor. For example, 1 inch on the scale drawing equals how many feet in real life based on the scale factor given. Students reproduce a given drawing based on a scale factor. Coherence KY.6.G.1→KY.7.G.1→KY.8.EE.6
KY.7.G.2 Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **MP.6**, **MP.7**	Emphasis is on taking given conditions and converting them to geometric shapes, constructing triangles with given angle measures and side lengths and determining when the given conditions do not meet the conditions of a triangle. Coherence KY.7.G.2→KY.8.G.1
KY.7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **MP.5**, **MP.6**	Cross sections may be taken from horizontal, vertical and oblique angles, such as: 

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice
Students extend their knowledge of proportional reasoning to solve problems involving dimensions and area. Proper use of tools help them understand the conditions by which three side lengths will determine one triangle or no triangle. Students have opportunities to reflect on the appropriateness of a tool for a particular task (**MP.5**). Initially, students may struggle with moving from a concrete understanding of a real-world situation to a miniature version, or vice versa; hands-on measurements and the use of technology can assist students with this abstract idea. In many cases, students make sense of new and different contexts and engage in significant struggle to solve problems (**MP.1**, **MP.2**). Students begin to understand it may not be possible to draw a certain shape with given measurements, or, if possible, may not yield a unique shape and reason why this may be the case (**MP.7**). By finding the constraints that exist in the Triangle Inequality Theorem, for example, a student determines precisely when a triangle may or may not exist (**MP.6**). By emphasizing the differences in various slicing planes, students accurately represent the resulting sections (**MP.6**).

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-life and mathematical problems involving angle measure, area, surface area and volume.**	
Standards	**Clarifications**
KY.7.G.4 Use formulas for area and circumference of circles and their relationships. a. Apply the formulas for the area and circumference of a circle to solve real-world and mathematical problems. b. Explore and understand the relationship between the radius, diameter, circumference and area of a circle. **MP.1**, **MP.2**, **MP.8**	Circle Formulas: C=dπ $C = 2r\pi$ A=πr^2 Note: Calculating the radius or diameter of a circle given its area is not expected, as finding the square root of a number is reserved for 8th grade. a. Both area and circumference are represented; students recognize when circumference is needed and when area is needed. b. Emphasis is on calculating area given diameter; finding circumference given radius or diameter; and finding radius or diameter given circumference. Special attention given to the relationship between diameter and circumference as a ratio that leads to pi. Coherence KY.7.G.4 → KY.8.G.9
KY.7.G.5 Apply properties of supplementary, complementary, vertical and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **MP.3**, **MP.6**, **MP.7**	Emphasis is on the relationships between the various angles listed to find missing angles based on the relationships and to write and solve equations to find unknown angles. KY.8.G.1 Coherence KY.4.MD.7 →KY.7.G.5 →KY.8.G.5
KY.7.G.6 Solve problems involving area of two-dimensional objects and surface area and volume of three-dimensional objects. a. Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles, quadrilaterals and other polygons.	a. Emphasis is on finding the area of composite figures composed of convex polygons. b. Students understand volume and surface area are two different quantities used to describe the same three-dimensional figure. Building upon their understanding of area, students use nets of three dimensional objects to conceptualize surface area.

Standards	Clarifications
b. Solve real-world and mathematical problems involving volume and surface area, using nets as needed, of three-dimensional objects including cubes, pyramids and right prisms. **MP.3**, **MP.4**, **MP.5**	Students calculate with appropriate units, using nets as a possible strategy for calculation as well as formulas for volume and surface area, where appropriate. KY.6.G.1 KY.6.G.2 Coherence KY.6.G.4→ KY.7.G.6 → KY.8.G.6

Attending to the Standards for Mathematical Practice
A student who merely memorizes the area and circumference formulas for a circle or the area, volume and surface area formulas of other shapes does not have a deep, conceptual understanding of the basis for these equations. Exploring the relationships between radius, diameter, area and circumference limits the confusion inherent in rote memorization, because students are given a context to the concepts (**MP.2**, **MP.8**). Solving real-world situations involving these quantities gives the student context for their understanding of the mathematics (**MP.1**). In addition, precise drawing or manipulation of technology lends itself to generate definitions (**MP.6**). Students continue their work from grade 6 from solving area problems involving triangles and rectangles to those involving more complex shapes, such as rhombi or trapezoids (**MP.4**). Students may mischaracterize volume and surface area of three dimensional shapes, leading them to develop ways to decide upon whether a situation calls for the volume of a figure, or the surface area of a figure (**MP.3**). The use of nets and other appropriate tools gives students a structure to foster greater understanding of the concept of surface area (**MP.5**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use random sampling to draw inferences about a population.	
Standards	**Clarifications**
KY.7.SP.0 Create displays, including circle graphs (pie charts), scaled pictographs and bar graphs, to compare and analyze distributions of categorical data from both matching and different-sized samples. **MP.2**, **MP.3**, **MP.6**	Students have been introduced to pictographs and bar graphs in grades 2 and 3; Circle graphs are new and connect to the grade 7 focus on percents. Also, students' knowledge of rates mean they can approach scaled pictographs in a more sophisticated manner. An important aspect of doing statistics is *selecting* an appropriate data display for the question under investigation. Students need to be asked, "Which data display fits this data set and why?" The circle graph focuses more on the relative values of the clustering of data, whereas the bar and pictographs add a dimension of quantity. The choice of which data display (and how categories are set up within each display) will result in different pictures of the shape of data. Finally students are comparing two distributions. When comparing two different distributions, circle graphs lend to comparing different sized samples, because circle graphs are based on percentages. KY.7.SP.0 KY.7.SP.2 Coherence KY.6.SP.O→KY.7.SP.4
KY.7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random	Recognize what makes a valid and non-valid sample of a population. Recognize the size of the sample holds importance to the accuracy of the sample. KY.6.SP.0

Standards	Clarifications
sampling tends to produce representative samples and support valid inferences. **MP.3**, **MP.6**	KY.6.SP.1 Coherence KY.6.SP.2→KY.7.SP.1→KY.HS.SP.9
KY.7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. a. Generate multiple samples of categorical data of the same size to gauge the variation in estimates or predictions. b. Generate multiple samples (or simulated samples) of numerical data to gauge the variation in estimates or predictions. c. Gauge how far off an estimate or prediction might be related to a population character of interest. **MP.2**, **MP.3**, **MP.7**	Emphasis is on the sample size and how this affects the validity of the estimate or prediction. Examples: a. Randomly sample 6th, 7th and 8th graders about who their favorite superhero is to generate samples of data that are roughly the same size, looking specifically at patterns, if any. b. Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Coherence KY.6.SP.0→ 7.SP.2→ KY.HS.SP.12

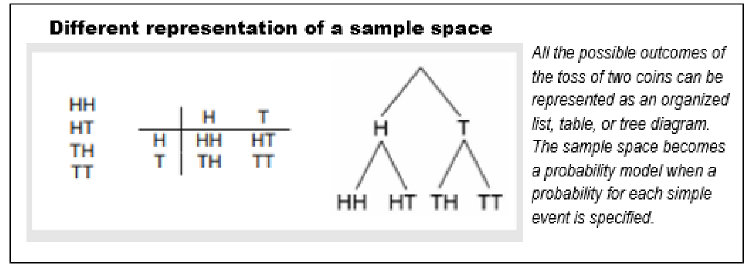
Attending to the Standards for Mathematical Practice
Students understand the method of sampling a population affects the reliability and validity of the data gleaned, so they justify their conclusions and inferences in a valid way (**MP.3**). In doing so, they create an accurate picture of the question posed (**MP.6**). In drawing inferences and reasoning about the variation of their estimates, students construct arguments based on data (**MP.2**, **MP.3**). When students, for example, examine a sample of 10 data points, versus a sample of 100 data points, they generalize why the samples may have two different sample errors (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Draw informal comparative inferences about two populations.**	
Standards	**Clarifications**
KY.7.SP.3 Describe the degree of visual overlap (and separation) from the graphical representations of two numerical data distributions (box plots, dot plots) with similar variabilities with similar contexts (same variable), measuring the difference between the centers (medians or means) by expressing this difference as a multiple of a measure of variability (interquartile range when comparing medians or the mean absolute deviation when comparing means). **MP.1**, **MP.5**, **MP.7**	For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. KY.6.SP.2 Coherence KY.6.NS.1→KY.7.SP.3→KY.HS.SP.13 KY.HS.SP.10
KY.7.SP.4 Calculate and use measures of center (mean and median) and measures of variability (interquartile range when comparing medians and mean absolute deviation when comparing means) for numerical data from random samples to draw informal comparative inferences about two populations. **MP.2**, **MP.5**, **MP.7**	For example, decide whether the words in a chapter of a grade seven science book are generally longer than the words in a chapter of a grade four science book. KY.HS.SP.10 Coherence KY.6.SP.2→KY.7.SP.4→KY.HS.SP.13
Attending to the Standards for Mathematical Practice	
When comparing two data distributions, students visually note differences, for example, of two dot plots. What is more difficult at times is to conceptualize this in mathematical terms, such that one distribution may have twice the variability of the other (**MP.2**). In moving from visual representation to measures of center and variability, students using these measures mathematically describe a situation that may be difficult to otherwise describe (**MP.5**, **MP.7**). Categorically summarizing data in circle graphs, gives students a basis for bringing their number sense from percents to statistics, allowing them to be precise when describing data (57% of students have brown shoes) (**MP.6**), while reasoning and drawing conclusions from data presented (**MP.2**, **MP.3**). Now, students drawing inferences from their calculations they have learned in grade 6 and earlier in grade 7 allows them to use these tools (**MP.5**) and allows them to mathematically compare (**MP.7**) in such a way that their inferences and conclusions make sense in context (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Investigate chance processes and develop**, **use and evaluate probability models.**	
Standards	**Clarifications**
KY.7.SP.5 Describe the probability of a chance event is a number between 0 and 1, which tells how likely the event is, from impossible (0) to certain (1). A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event. **MP.5**, **MP.6**, **MP.7**	Emphasis is on descriptive language used to describe numerical probabilities; impossible event, unlikely event, equally likely event, likely event, certain event. Students understand all probabilities must fall between 0 and 1.
KY.7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability. **MP.1**, **MP.2**	Estimate the likelihood of an event, test the estimate by trial and collect data. Students observe accuracy of the estimate will increase with the frequency of repeated trials. Coherence KY.7.SP.6→ KY.HS.SP.10
KY.7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. **MP.4**, **MP.7**, **MP.8**	For example: a. If a student is selected at random from a class, find the probability Jane will be selected and the probability a girl will be selected. b. Find the approximate probability a spinning penny will land heads up or a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? KY.7.RP.3 Coherence KY.7.SP.7→ KY.HS.SP.14
KY.7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.	Example: a. If the probability of heads occurring on a coin is ½, then the probability of three heads in a row is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

Standards	Clarifications
a. Explain just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events described in everyday language using methods such as organized lists, tables and tree diagrams. c. Design and use a simulation to generate frequencies for compound events. **MP.2**, **MP.4**, **MP.7**	b. For a simulation of tossing two fair coins:  c. Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability it will take at least 4 donors to find one with type A blood? Coherence KY.7.SP.8→ KY.HS.SP.14

Attending to the Standards for Mathematical Practice

Thinking of probability as being on a continuum ranging from a probability of 0 to a probability of 1 allows students to visualize the structure of ranking the chances of an event occurring (**MP.7**). When they relate these broader terms to actual calculated probability, this lends precision to otherwise vague concepts (**MP.6**). In addition, students note the opposite is also true; a calculated probability close to ½ means the event is neither unlikely nor likely, or equally likely (**MP.5**). Looking at the process that generates a set of probabilities (experimental probability) in a specific scenario gives students the opportunity to examine a situation in depth (**MP.1**) and reason about why the conclusion they draw may or may not be accurate (**MP.2**). Student thinking about theoretical probability is extended to developing a model (**MP.4**) that lends structure (**MP.7**) to an otherwise abstract idea. Students may use this model to explain why a penny comes up heads half the time and tails the other half, but in an experiment where this event is repeated multiple times, the experimental probability may not be exactly ½ and ½. (**MP.8**). Compound probability may be more difficult for students to understand; tree diagrams, lists, etc. may help students understand the concept (**MP.7**). Difficult to understand compound events may necessitate a simulation tool, for example a random digit generator (**MP.4**).

Kentucky Academic Standards for Mathematics: Grade 8 Overview

The Number System (NS)	Expressions and Equations (EE)	Functions (F)	Geometry (G)	Statistics and Probability (SP)
• Know that there are numbers that are not rational and approximate them by rational numbers.	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.	• Define, evaluate and compare functions. • Use functions to model relationships between quantities.	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.	• Investigate patterns of association in bivariate data.

In grade 8, instructional time should focus on three critical areas:

1. **In the Number System, the Expressions, Equations and Inequalities, and the Probability and Statistics domains, students will:**
   - recognize equations for proportions (y/x = m or y=mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope and the graphs are lines throughout the origin;
   - understand that the slope (m) of a line is a constant rate of change, as well as how the input and output changes as a result of the constant rate of change;
   - interpret a model in the context of the data by expressing a linear relationship between the two quantities in question and interpret components of the relationship (such as slope and y-intercept) in terms of the situation;
   - solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line;
   - use linear equations, systems of linear equations, linear functions and their understanding of slope of a line to represent, analyze and solve a variety of problems.
2. **In the Functions and the Expressions, Equations and Inequalities domains, students will:**
   - grasp the concept of a function as a rule that assigns to each input exactly one output;
   - understand that functions describe situations where one quantity determines another;
   - translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations of the function) and describe how aspects of the function are reflected in the different representations.
3. **In the Geometry domain, students will:**
   - use ideas about distance and angles, how they behave under translations, rotations, reflections and dilations and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems;
   - show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines;
   - understand the statement of the Pythagorean Theorem and its converse, and why the Pythagorean Theorem holds;
   - apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths and to analyze polygons.

The Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Know that there are numbers that are not rational and approximate them by rational numbers.**

Standards	Clarifications
KY.8.NS.1 Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational. **MP.2**, **MP.6**, **MP.7**	Emphasis is placed on how all rational numbers can be written as an equivalent decimal. The end behavior of the decimal determines the classification of the number. Coherence KY.7.NS.2→ KY.8.NS.1→ KY.HS.N.3
KY.8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions. **MP.2**, **MP.7**, **MP.8**	For example, by shortening the decimal expansion of $\sqrt{2}$ by dropping all decimals past a certain point and showing $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5 and so on. Students recognize this process could be repeated an infinite number of times. Coherence KY.8.NS.2→ KY.HS.N.3

Attending to the Standards for Mathematical Practice

Students attend to precision (**MP.6**) by recognizing and identifying numbers as rational or irrational. Students know the definition of an irrational number and represent the number in different ways, as a root, non-repeating decimal block, or symbol. Students attend to precision when clarifying the difference between an exact value of an irrational number compared to the decimal approximation of the irrational number. Ultimately, students come to an informal understanding (**MP.2**) the set of real numbers consists of rational numbers and irrational numbers. They continue to work with irrational numbers and rational approximations when solving equations such as $x^2 = 18$. While using the long division algorithm to convert fractions to decimals, students recognize when a sequence of remainders repeats that the decimal form of the number will contain a repeat block (**MP.8**). Students recognize when the decimal expansion of a number does not repeat or terminate, the number is irrational and can be represented with a method of rational approximation using a sequence of rational numbers to get closer and closer to the given number (**MP.7**). Students look for structure in repeating decimals, recognize repeating blocks and know every fraction is equal to a repeating decimal.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Work with radicals and integer exponents.**

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
MP.3, MP.7, MP.8</td><td><table>
<tr><th>Name</th><th>Product of Powers</th><th>Quotient of Powers</th><th>Power of a Product</th><th>Power of a Quotient</th><th>Power of a Power</th><th>Negative Exponent</th></tr>
<tr><td>Property</td><td>$a^m \cdot a^n = a^{m+n}$</td><td>$\frac{a^m}{a^n} = a^{m-n}$</td><td>$(a \cdot b)^n = a^n \cdot b^n$</td><td>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$</td><td>$(a^m)^n = a^{mn}$</td><td>$a^{-n} = \frac{1}{a^n}$</td></tr>
</table>
Coherence KY.8.EE.1→ KY.HS.N.1</td></tr>
<tr><td>KY.8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that perfect squares and perfect cubes are rational.
MP.5, MP.6</td><td>Students do not prove these are the only solutions, but rather use informal methods, such as guess and check. For example, $\sqrt{64} = \sqrt{8^2} = 8$ and $\sqrt[3]{5^3 = 5}$. Since $\sqrt{p}$ is defined to mean the positive solution to the equation $x^2 = p$ (when it exists), it is not correct to say (as is common) $\sqrt{64} = \pm 8$. In describing the solutions to $x^2 = 64$, students write $x = \pm\sqrt{64} = \pm 8$.
Coherence KY.8.EE.2→ KY.HS.A.12</td></tr>
<tr><td>KY.8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 (Scientific Notation) to estimate very large or very small quantities and express how many times larger or smaller one is than the other.
MP.3, MP.5, MP.6</td><td>Students conceptualize why a number could be written in scientific notation and the benefits of doing so and connect exponent rules learned earlier to the methods of writing a quantity in scientific notation.
Coherence KY.8.EE.3→ KY.HS.N.6</td></tr>
<tr><td>KY.8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.
MP.2, MP.5, MP.6</td><td>Choose appropriate units for real-life situations. When solving problems and using technology, it is possible solutions are given that take the form of $1.2\ x\ 10^{00}$ or $3.4\ x\ 10^{-07}$. Some technologies also use a capital E when denoting numbers such a $1.45E07$ or $4.665E$-11.
Coherence KY.8.EE.4→ KY.HS.N.4</td></tr>
</table>

Attending to the Standards for Mathematical Practice
Students construct mathematical arguments and reasoning emphasized as students learn the properties of exponents (**MP.3**). Students reason $5^3 \cdot 5^2 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5) = 5^5$ through numerous experiences of working with exponents, students generalize the properties of exponents (**MP.7**) before using them fluently. Students notice if calculations are repeated (**MP.8**) and look both for general methods and for shortcuts. Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used (**MP.2**, **MP.7**, **MP.8**). Students compare and interpret scientific notation quantities in the context of the situation, recognizing the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand the connections between proportional relationships, lines and linear equations.**	
Standards	**Clarifications**
KY.8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **MP.2**, **MP.3**, **MP.4**	Emphasis is on relating previous knowledge of unit rate to slope in tables, graphs, equations and sets of ordered pairs and comparing the slopes of two different proportional relationships. Different ways the proportional relationships can be represented include tables, graphs, equations, or sets of ordered pairs. KY.8.F.2 Coherence KY.7.RP.2→ KY.8.EE.5→ KY.HS.A.23
KY.8.EE.6 Use similar triangles to explain why the slope, *m*, is the same between any two distinct points on a non-vertical line in the coordinate plane; know the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. **MP.3**, **MP.4**, **MP.7**	Using the properties of similar triangles, demonstrate the slope between any two pairs of points on a non-vertical line create the same rise-run ratio when simplified. Understand $y = mx$ and $y = mx + b$ differ in that $y = mx$ only has the possibility of 0 being the *y*-intercept and that $y = mx + b$ has infinite possibilities, including 0, for the *y*-intercept depending on the value of *b*. KY.HS.G.22 Coherence KY.7.RP.2→ KY.8.EE.6→ KY.HS.A.23
Attending to the Standards for Mathematical Practice	
Students represent real-world situations symbolically (**MP.4**). Students identify important quantities from a context and represent the relationship in the form of an equation, a table and a graph. Students analyze the various representations and draw conclusions and/or make predictions (**MP.3**). Once a solution or prediction has been made, students reflect on whether the solution makes sense in the context presented (**MP.4**). One example of this is when students determine how many buses are needed for a field trip. As this is most probably not an exact solution, students must interpret their fractional solution and make sense of it as it applies to the real world. Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. Students use the structure of an equation to make sense of the information in the equation (**MP.7**). For example, students write equations that represent the constant rate of motion for a person walking. In doing so, they interpret an equation such as $y = \frac{3}{5}x$ as the total distance a person walks, y, in x	

amount of time, at a rate of $\frac{3}{5}$. Students look for patterns or structure in tables and show a rate is constant; students also understand a lack of a pattern represents a non-constant (non-linear) rate.

Expressions and Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Analyze and solve linear equations and pairs of simultaneous linear equations.**	
Standards	**Clarifications**
KY.8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms. **MP.2**, **MP.3**, **MP.7**	Building upon skills from grade 7, students combine like terms on the same side of the equality and use the distributive property to simplify the equation when solving. Emphasis in this standard is also on using rational number coefficients. Solutions of certain equations may elicit infinitely many or no solutions. Coherence KY.7.EE.1→ KY.8.EE.7→ KY.HS.A.18
KY.8.EE.8 Analyze and solve a system of two linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously; understand that a system of two linear equations may have one solution, no solution, or infinitely many solutions. b. Solve systems of two linear equations in two variables algebraically by using substitution where at least one equation contains at least one variable whose coefficient is 1 and by inspection for simple cases c. Solve real-world and mathematical problems leading to two linear equations in two variables.	a. Examples are both mathematical and real-life contexts. Emphasis is on determining what types of contexts lead to having no solutions or infinitely many solutions. Students use tables, graphs and equations to explain why a graphed system has infinitely many or no solutions. b. Elimination and/or matrices are not required for grade 8. Emphasis is on *choosing* a method. Students solve simple cases by inspection, for example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6 and select from the other approaches, based on the numbers in the problem. Solving systems algebraically will be

Standards	Clarifications
MP.1, **MP.3**, **MP.4**	limited to at least one equation containing at least one variable with a coefficient of 1; for example, $y = 3x$, $y = -12x + 6, x = 2, x = 2y + 1$. Coherence KY.7.EE.2→ KY.8.EE.8→ KY.HS.A.20

Attending to the Standards for Mathematical Practice
Students solve linear equations in one variable, including cases with one solution, an infinite number of solutions and no solutions. Students show examples of each of these cases by successively transforming an equation into simpler forms. Some linear equations require students to expand expressions by using the distributive property and to collect like terms (**MP.2**, **MP.7**). Solving pairs of simultaneous linear equations builds on the skills and understandings students used to solve linear equations with one variable and systems of linear equations may also have one solution, an infinite number of solutions, or no solutions (**MP.2**, **MP.3**). Students discover these cases as they graph systems of linear equations and solve algebraically.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Define**, **evaluate and compare functions.**

Standards	Clarifications
KY.8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **MP.7**, **MP.8**	Students understand the reasoning that not all relations are functions. Note: Function notation is not required in grade 8. Coherence KY.8.F.1→ KY.HS.F.1
KY.8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **MP.1**, **MP.2**, **MP.4**	Given a linear function represented using one method listed and another linear function represented by different method listed, determine which function has the greater or lesser rate of change or greater or lesser initial value. Coherence KY.7.RP.2→ KY.8.F.2→ KY.HS.F.1
KY.8.F.3 Understand properties of linear functions. a. Interpret the equation *y* = *mx* + *b* as defining a linear function, whose graph is a straight line. b. Identify and give examples of functions that are not linear. **MP.7**	a. For example, the equation $c = 3g + 5$ models the linear function for the total cost, *c*, of bowling, where *g* represents the number of games played and shoe rental is $5. b. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. Coherence KY.7.EE.4→ KY.8.F.3→ KY.HS.F.11

Attending to the Standards for Mathematical Practice

Students examine, interpret and represent functions symbolically (**MP.2**, **MP.4**). They make sense of quantities and their relationships in problem situations (**MP.2**). For example, students make sense of values as they relate to the total cost of items purchased or a phone bill based on usage in a particular time interval. Students use what they know about rate of change to distinguish between linear and nonlinear functions (**MP.8**). Further, students contextualize information gained from the comparison of two functions (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use functions to model relationships between quantities.

Standards	Clarifications
KY.8.F.4 Construct a function to model a linear relationship between two quantities. a. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. b. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. **MP.4**, **MP.5**, **MP.8**	Examining a relationship between two quantities yields a function rule. This function rule can be described using its initial value and rate of change, from a variety of representations, including tables, graphs, equations and verbal descriptions. Understand the rate of change and initial value in terms of the situation it models. KY.HS.F.6 Coherence KY.7.RP.2→ KY.8.F.4→ KY.HS.F.3
KY.8.F.5 Use graphs to represent functions. a. Describe qualitatively the functional relationship between two quantities by analyzing a graph. b. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **MP.3**, **MP.7**	Students describe whether a function is increasing or decreasing and linear or nonlinear. Function examples are described in contexts as well as in symbols. Coherence KY.7.RP.2→ KY.8.F.5→ KY.HS.F.4

Attending to the Standards for Mathematical Practice

Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data (**MP.4**). There are many real-world problems that can be modeled with linear functions, including instances of constant payment plans (phone plans), costs associated with running a business and relationships between associated bivariate data. When students are analyzing graphs, they focus on how the function is changing. Students take verbal descriptions and create graphs, while also being able to take a graph and create a verbal description (**MP.2**, **MP.5**). Students look for patterns within the graphs to provide justification of the verbal description being represented by the graph (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand congruence and similarity using physical models, transparencies, or geometry software.**	
Standards	**Clarifications**
KY.8.G.1 Verify experimentally the properties of rotations, reflections and translations: ● Lines are congruent to lines. ● Line segments are congruent to line segments of the same length. ● Angles are congruent to angles of the same measure. ● Parallel lines are congruent to parallel lines. **MP.5**, **MP.6**	Emphasis is congruence transformations preserve corresponding congruent lines, segments and angles. KY.HS.G.2 Coherence KY.8.G.1→ KY.HS.G.3(+)
KY.8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them. **MP.2**, **MP.7**	Students understand a figure, called a pre-image, is congruent to another figure, called the image, if the second figure can be obtained by a sequence of congruence transformations performed on the first figure. Students describe the sequence of congruence transformations necessary to transform one figure to a congruent second figure. KY.HS.G.4 Coherence KY.8.G.2→ KY.HS.G.5
KY.8.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates. **MP.3**, **MP.5**, **MP.6**	Emphasis is on noticing patterns across examples, noting how the *x* and *y* values change for different kinds of transformations. Coherence KY.8.G.3→ KY.HS.G.9
KY.8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **MP.2**, **MP.5**, **MP.7**	If similar, non-congruent figures are given, students understand a dilation must have taken place in the sequence of transformations to obtain the image from the pre-image. KY.HS.G.2 Coherence KY.8.G.4→ KY.HS.G.10

Standards	Clarifications
KY.8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. **MP.3**	Students use technology or physical tools to explore triangles. They arrange three copies of the same triangle so that the sum of the three angles appears to form a line and give an argument in terms of transversals of why this is so. KY.HS.G.7 Coherence KY.7.G.5→ KY.8.G.5→ KY.HS.G.10

Attending to the Standards for Mathematical Practice
Students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles and properties of polygons like rectangles and parallelograms (**MP.3**). With the aid of physical models, transparencies and geometry software, students in grade eight gain an understanding of transformations and their relationship to congruence of shapes (**MP.5**, **MP.6**). Through experimentation, students verify the properties of rotations, reflections and translations, including discovering these transformations change the position of a geometric figure but not its shape or size (**MP.7**). Finally, students understand congruent shapes are precisely those that can be "mapped" one onto the other by using rotations, reflections, or translations (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Understand and apply the Pythagorean Theorem.**	
Standards	**Clarifications**
KY.8.G.6 Explain a proof of the Pythagorean Theorem and its converse. **MP.3**, **MP.7**	Students verify, using a model, the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students understand if the sum of the squares of the two smaller legs is equal to the square of the third leg, then the triangle is a right triangle. Coherence KY.7.G.6→ KY.8.G.6→KY.HS.G.11
KY.8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **MP.1**, **MP.2**, **MP.4**	Students apply the Pythagorean Theorem to mathematical real-world problems. For example, finding the width of a television given the length and diagonal distance (two-dimensional) and the distance from the top left rear corner of a prism to the bottom right front corner of the prism (three-dimensional). Coherence KY.8.G.7→KY.HS.G.12
KY.8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **MP.5**, **MP.6**	Students calculate distances on the coordinate plane between two non-vertical or non-horizontal points by applying the Pythagorean Theorem. Students calculate distances between two non-vertical or non-horizontal points not given on a coordinate plane by applying the Pythagorean Theorem to absolute horizontal and vertical distances the student calculates. KY.HS.G.19 Coherence KY.8.G.8→ KY.HS.G.21
Attending to the Standards for Mathematical Practice	
By explaining a proof of the Pythagorean Theorem and its converse, students are constructing and defending arguments as to why the relationship is true (**MP.3**). The structure inherent in the use of the Theorem is a set of guidelines students seek to apply when applying the Theorem to right triangle relationships (**MP.7**). Students make sense of the world around them by applying the Pythagorean Theorem in a variety of ways (**MP.1**). Investigation into Pythagorean Triples and the relationships among similar triangles with the same ratio of Pythagorean Triples	

allows students to reason about the relationships (**MP.2**). Extending knowledge of the Pythagorean Theorem to the coordinate plane gives students another tool to prove the relationship exists and to apply the relationship to quantitative tasks (**MP.5**). Attending to precision is inherent in the study of this cluster, as a discussion will inevitably occur involving leaving a solution in terms of a radical, or a rational approximation ($\sqrt{50}\ vs.\ 7.07106\ldots$)(**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

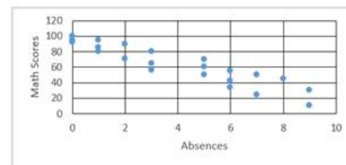
Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: **Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.**	
Standards	**Clarifications**
KY.8.G.9 Apply the formulas for the volumes and surface areas of cones, cylinders and spheres and use them to solve real-world and mathematical problems. **MP.1**, **MP.7**, **MP.8**	Cones: $V = \frac{1}{3}\pi r^2 h \quad SA = \pi r\,(r + \sqrt{(r^2 + h^2)})$ Cylinders: $V = \pi r^2 h \quad SA = 2\pi rh + 2\pi r^2$ Spheres: $V = \frac{4}{3}\pi r^3 \quad SA = 4\pi r^2$ KY.HS.G.29 Coherence KY.7.G.4→ KY.8.G.9→ KY.HS.G.25
Attending to the Standards for Mathematical Practice	
Students may confuse the three formulas given if they try to apply a formula to a specific shape. Student understanding of the volume formulas is enhanced by investigations into the derivations of the volume formulas (**MP.1**). Students examining structure in real-world problems in order to apply the correct volume formula (if needed) begin to see where these are useful in real life (**MP.7**). If students can successfully compare volumes of similar shapes, for example, which of two storage tank can hold the most fuel, they begin to use repeated reasoning in the real-world (**MP.8**).	

Statistics and Probability

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Investigate patterns of association in bivariate data.**

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.8.SP.1 Construct and interpret scatter plots for bivariate numerical data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
MP.2, MP.7</td><td>
<table>
<tr><th>Absences</th><th>Math Scores</th></tr>
<tr><td>3</td><td>65</td></tr>
<tr><td>5</td><td>50</td></tr>
<tr><td>1</td><td>95</td></tr>
<tr><td>1</td><td>85</td></tr>
<tr><td>3</td><td>80</td></tr>
<tr><td>6</td><td>34</td></tr>
<tr><td>5</td><td>70</td></tr>
<tr><td>3</td><td>56</td></tr>
<tr><td>0</td><td>100</td></tr>
<tr><td>7</td><td>24</td></tr>
<tr><td>8</td><td>45</td></tr>
<tr><td>2</td><td>71</td></tr>
<tr><td>9</td><td>30</td></tr>
<tr><td>0</td><td>95</td></tr>
<tr><td>6</td><td>55</td></tr>
<tr><td>6</td><td>42</td></tr>
<tr><td>2</td><td>90</td></tr>
<tr><td>0</td><td>92</td></tr>
<tr><td>5</td><td>60</td></tr>
<tr><td>7</td><td>50</td></tr>
<tr><td>9</td><td>10</td></tr>
<tr><td>1</td><td>80</td></tr>
</table>
Given data from students' math scores and absences, make a scatterplot.


For example, given the data and scatter plot to the left, students explain the relationship between students' absences and math scores shows a negative, linear association and has no obvious outliers.
KY.HS.SP.6
Coherence KY.8.SP.1→ KY.HS.SP.8</td></tr>
<tr><td>KY.8.SP.2 Know that lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a line and informally assess the model fit by judging the closeness of the data points to the line.
MP.2</td><td>Students are informally fitting a line to data; they judge whether or not a given line is a good fit for the data and describe needed adjustments. Students recognize some scatter plots cannot be described by a line.
KY.HS.SP.6
Coherence KY.8.SP.2→ KY.HS.SP.8</td></tr>
<tr><td>KY.8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate numerical data, interpreting the slope and intercept.
MP.2, MP.4</td><td>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height and an initial value of 4 cm means the plant was 4 cm tall when measuring began.
KY.HS.SP.6
Coherence KY.8.SP.3→ KY.HS.SP.7</td></tr>
</table>

Attending to the Standards for Mathematical Practice
Students reason quantitatively by symbolically representing the verbal description of a relationship between two bivariate variables. They attend to the meaning of data based on the context of problems and the possible linear or nonlinear functions that explain the relationships of the variables. When classifying characteristics of sets of data, students reason about the descriptions that apply based on definition (**MP.2**). Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data. This requires a deep understanding of the parts of the model used and their interpretations (**MP.4**). Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. Students identify patterns or structures in scatter plots. They fit lines to data displayed in a scatter plot and determine the equations of lines based on points or the slope and initial value (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Conceptual Category Number and Quantity

Number and Quantity Overview

The Real Number System	Quantities	The Complex Number System	Vector and Matrix Quantities
• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.	• Reason quantitatively and use units to solve problems.	• Perform arithmetic operations with complex numbers. • Represent complex numbers and their operations on the complex plane. • Use complex numbers in polynomial identities and equations.	• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Number and Quantity-The Real Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Extend the properties of exponents to rational exponents.	
Standards	**Clarifications**
KY.HS.N.1 Extend the properties of integer exponents to rational exponents, allowing for the expression of radicals in terms of rational exponents. **MP.2**, **MP.7**	Students understand that a single root can be expressed as a rational exponent with a numerator of one and a base that is equal to the root index. Students understand that powers and roots can be concisely expressed as a single rational exponent where the numerator is the power and the denominator is the root index. For example, students understand that defining $4^{1/3}$ is the same as the cube root of 4 because $4^{(1/3)^3} = (4^{1/3})^3$ so $4^{(1/3)^3}$ must equal 4.
KY.HS.N.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. **MP.7**	Standards KY.HS.N.2 builds on standard KY.HS.N.1 by extending student understanding to situations where the numerator is not one. For example, students understand that defining $4^{m/n}$ is the same as $\sqrt[n]{4}^{m}$ and $(\sqrt[n]{4})^{m}$. Include contextual examples, such as rewriting the volume of a sphere to identify the radius as a function of volume.
Attending to the Standards for Mathematical Practice	
Students flexibly move between notating expressions as roots/powers or as integers with rational coefficients (**MP.2**). They explain why rational expressions can be more desirable and what the notation means (**MP.7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Real Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use properties of rational and irrational numbers.

Standards	Clarifications
KY.HS.N.3 (+) Justify why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **MP.3**, **MP.6**	An important difference between rational and irrational numbers is that rational numbers form a number system. Students understand that if you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Students also understand that multiplying the irrational number 2 by itself, yields a rational number, 2. Irrational numbers are defined by not being rational and this definition can be exploited to generate many examples of irrational numbers from just a few.

Attending to the Standards for Mathematical Practice
Students say or write what makes a number rational or irrational and use these definitions precisely to explain the properties of rational and irrational numbers **(MP.6)**. As students listen to the rationales or proofs of their peers, they determine whether the arguments make sense and prove the properties for all rational and irrational numbers **(MP.3)**.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Reason quantitatively and use units to solve problems.

Standards	Clarifications
KY.HS.N.4 Use units in context as a way to understand problems and to guide the solution of multi-step problems; ★ a. Choose and interpret units consistently in formulas; b. Choose and interpret the scale and the origin in graphs and data displays. **MP.5**, **MP.6**	Graphical representations and data displays include but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots and multi-bar graphs.
KY.HS.N.5 Define appropriate units in context for the purpose of descriptive modeling. ★ **MP.1**, **MP.6**	In real-world situations, answers are usually represented by numbers with units. Units involve measurement, which requires precision and accuracy. For example, students should recognize that units measuring speed would not be appropriate for situations involving volume. Additionally students should understand when one dimensional, two dimensional, or three dimensional units are most applicable.
KY.HS.N.6 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★ **MP.2**, **MP.6**	While KY.HS.N.6 does not require a formal discussion or use of significant digits in the scientific sense, students understand a level of precision. For example, when using the Pythagorean Theorem with measurements given in tenths of an inch, it is appropriate for students to express answers to the nearest tenth, but not to the nearest hundredth because that level of precision was not used in the original measures.

Attending to the Standards for Mathematical Practice

Students attend to units in real-world problems, reasoning about the level of precision needed and the related error that may be introduced to the problem **(MP.2)**. Students describe what is (and is not) an appropriate level of precision for their answers, describing the relationship between the precision that was used in the original measures and the precision that can be used in an answer (**MP.6**).

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform arithmetic operations with complex numbers.	
Standards	**Clarifications**
KY.HS.N.7 Understanding properties of complex numbers. a. Know there is a complex number i such that $i^2 = -1$ and every complex number has the form $a + bi$ with a and b real. b. Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. **MP.7**, **MP.8**	a. Students understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations. c. Students understand the complex conjugate as the pair of binomial complex factors, $(a + bi)(a - bi)$, whose product is a difference of squares: a^2+b^2, which is a real number. Students understand that the denominator of a fraction can be resolved of an imaginary number by multiplying by both the numerator and the denominator by the conjugate of the denominator.
Attending to the Standards for Mathematical Practice	
Students use the relation $i^2 = -1$ as a basis for describing properties and then apply those properties to solving problems (**MP.7**). As they solve sets of problems with complex numbers, they notice patterns. For example, students explain how multiplying complex numbers is both alike and different from multiplying binomial expressions (**MP.8**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent complex numbers and their operations on the complex plane.	
Standards	**Clarifications**
KY.HS.N.8 (+) Understanding representations of complex numbers using the complex plane. a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. b. Represent addition, subtraction, multiplication, modulus and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. **MP.2**, **MP.5**	a. Students graph in both rectangular and polar form and convert rectangular coordinates to polar coordinates and vice versa. Students understand this conversion preserves the equality of the two forms. c. Students understand that calculating the distance between numbers in the complex plane is fundamentally the same as calculating distances in the standard coordinate plane using the distance formula from grade 8. Students understand calculating the midpoint of a segment in the complex plane as the average of the *a* values and average of the *b* values in any two endpoints expressed as *a +bi.*
Attending to the Standards for Mathematical Practice	
Students use technology to graph complex numbers in rectangular and polar form (**MP.5**) and explain how these representations are equivalent (**MP.2**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Complex Number System	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use complex numbers in polynomial identities and equations.	
Standards	**Clarifications**
KY.HS.N.9 Solve quadratic equations with real coefficients that have complex solutions. **MP.1**, **MP.2**	Students use the Quadratic Formula to solve for complex solutions. Students recognize that when a quadratic equation yields complex solutions its graph does not cross the x-axis.
KY.HS.N.10 (+) Extend polynomial identities to the complex numbers. **MP.7**, **MP.8**	When multiplying complex binomials, students recognize and understand the value of i^2 as -1 and fluently simplify each polynomial appropriately navigating between the real number system and complex numbers. One example of this might be that students should understand that it would be appropriate to rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
KY.HS.N.11 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **MP.1**, **MP.3**	
Attending to the Standards for Mathematical Practice	
Students make sense of quadratic equations, looking to see if there are rational roots that can be found by factoring, or if other methods such as completing the square or the quadratic formula are needed (**MP.1**). They justify that their answer is reasonable and describe why there are no real roots, if that is the case (**MP.2**).	

Number and Quantity-Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Represent and model with vector quantities.	
Standards	**Clarifications**
KY.HS.N.12 (+) Understand and apply properties of vectors. a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. c. Solve problems involving velocity and other quantities that can be represented by vectors. **MP.1**, **MP.6**	a. Vectors are directed by an angle and continue in that direction for a set length. b. Students connect 1) finding vertical and horizontal components and the magnitude of a vector with 2) using the Pythagorean Theorem in the coordinate plane. Limit to two-dimensional vectors.

Number and Quantity-Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Perform operations on vectors.	
Standards	**Clarifications**
KY.HS.N.13 (+) Perform operations with vectors (addition, subtraction and multiplication by a scalar). a. Add vectors end-to-end, component-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise. d. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. e. b. Compute the magnitude of a scalar multiple cv using $\|\|cv\|\|$ = $\|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). **MP.3**, **MP.7**	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Vector and Matrix Quantities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Perform operations on matrices and use matrices in applications.

Standards	Clarifications
KY.HS.N.14 Use matrices to represent and manipulate data. **MP.4**, **MP.5**	Students understand matrices are rectangular arrays comprised of elements that are useful for solving problems in context.
KY.HS.N.15 Perform operations with matrices. a. Add, subtract and multiply matrices of appropriate dimensions. b. Multiply matrices by scalars to produce new matrices. **MP.7**, **MP.8**	
KY.HS.N.16 (+) Understand properties of square and identity matrices. a. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. b. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. c. Work with 2×2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. **MP.3**, **MP.7**	
KY.HS.N.17 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **MP.2**, **MP.7**	

Attending to the Standards for Mathematical Practice
Students create numerical arrays of data (matrices), taken from a variety of sources (e.g., tables, systems of equations, or coordinate points from a series of transformations) (**MP.2**) **and** they use technology to manipulate data when appropriate (**MP.5**). When performing matrix operations by hand, students look for patterns and make generalizations (**MP.8**).

Kentucky Academic Standards for Mathematics: Conceptual Category Algebra

Algebra Overview

Seeing Structure in Expressions	Arithmetic with Polynomials and Rational Expressions	Creating Equations ★	Reasoning with Equations and Inequalities
• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems.	• Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions.	• Create equations that describe numbers or relationships.	• Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations. • Represent and solve equations and inequalities graphically.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Algebra-Seeing Structure in Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret the structure of expressions.	
Standards	**Clarifications**
KY.HS.A.1 Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors and coefficients. b. Interpret complicated expressions, given a context, by viewing one or more of their parts as a single entity. **MP.2**, **MP.6**	Students encounter simpler scenarios where they interpret r·t as the product of a given rate and time or interpret the perimeter expression ($2l+2w$) contextually as the sum of twice the length and twice the width of a rectangle. Students encounter more complicated scenarios where they interpret $P(1+r)^n$ contextually as the product of a principal investment, P and $(1+r)^n$ which represents an investment rate, compounding factor and time.
KY.HS.A.2 Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. **MP.7**, **MP.8**	Students see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares factored as $(x^2 – y^2)(x^2 + y^2)$. Additionally, students see there are three commonly used forms for a quadratic expression: • Standard form • Factored form • Vertex form and can identify when one form might be more useful than another.

Attending to the Standards for Mathematical Practice

Students not only simplify problems, they use vocabulary, such as terms, coefficients and degrees, appropriately as they describe their process (**MP.6**). Students describe the meaning of parts of an expression, such as a particular term or coefficient and also explain the meaning of the full expression (**MP.7**). Students fluently manipulate expressions into equivalent forms, based on patterns they have noticed across problems (**MP.8**).

Algebra-Seeing Structure in Expressions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Write expressions in equivalent forms to solve problems.

Standards

KY.HS.A.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient and constant term.
b. Factor a quadratic expression to reveal the zeros of the function it defines.
c. Use the properties of exponents to rewrite exponential expressions.
d. (+) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

MP.5, **MP.7**

Clarifications

KY.HS.A.3b Students recognize the connection between the zero product property and solving a quadratic in one variable by setting factored expressions equal to zero.

KY.HS.A.3c

Name	Product of Powers	Quotient of Powers	Power of a Product	Power of a Quotient	Power of a Power	Negative Exponent
Property	$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a \cdot b)^n = a^n \cdot b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(a^m)^n = a^{mn}$	$a^{-n} = \frac{1}{a^n}$

KY.HS.A.3d (+) Students recognize being able to complete the square allows them to identify the coordinates of the maximum or minimum value more easily than when the quadratic is in standard form and there are pros and cons of each equivalent form.

Standards

KY.HS.A.4 (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. ★

MP.1, **MP. 4**

Clarifications

$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

Attending to the Standards for Mathematical Practice

Students explain that they need to rewrite quadratic expressions into equivalent factored forms in order to find the zeros of the function it defines (**MP.7**). Using technology, students change the exponents to reinforce their understanding of exponent properties (**MP.5**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Perform arithmetic operations on polynomials.	
Standards	**Clarifications**
KY.HS.A.5 Add, subtract and multiply polynomials. **MP.7**, **MP.8**	Students combine like terms and make use of the distributive property when adding, subtracting and multiplying polynomials.
Attending to the Standards for Mathematical Practice	
Students flexibly rewrite expressions in equivalent forms using algebraic properties, including properties of addition, subtraction and multiplication (**MP.7**). When multiplying binomials, students identify and describe shortcuts after noticing that calculations are repeated (**MP.8**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the relationship between zeros and factors of polynomials.	
Standards	**Clarifications**
KY.HS.A.6 (+) Know and apply the Remainder Theorem. **MP.1**, **MP.8**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
KY.HS.A.7 Identify roots of polynomials when suitable factorizations are available. Know these roots become the zeros (x-intercepts) for the corresponding polynomial function. **MP.2**, **MP.5, MP.7**	Methods of finding roots could include, but are not limited to: • factoring • synthetic division • long division • an analysis of the graph (created by hand or through use of technology).
Attending to the Standards for Mathematical Practice	
Students reason quantitatively as they select a method for finding roots and justify why they selected and applied a particular method (**MP. 2**). Students use technology to identify the x-intercepts from a polynomial graph and explain that the x-intercepts are zeros and therefore roots of the polynomials (**MP.5**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use polynomial identities to solve problems.	
Standards	**Clarifications**
KY.HS.A.8 (+) Prove polynomial identities and use them to describe numerical relationships. **MP.2**, **MP.3**, **MP.6**	Students observe the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
KY.HS.A.9 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. **MP.7**, **MP.8**	Students understand the Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Arithmetic with Polynomials and Rational Expressions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Rewrite rational expressions.

Standards	Clarifications
KY.HS.A.10 (+) Rewrite simple rational expressions in different forms. **MP.7**, **MP.8**	Students observe how to write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$. Methods of rewriting rational expressions could include, but are not limited to: • Inspection • Synthetic division • Long division • Use of technology
KY.HS.A.11 (+) Add, subtract, multiply and divide rational algebraic expressions. **MP.2**, **MP.3**	Students go beyond demonstrating procedural fluency and apply this standard in a variety of contextual situations.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Creating Equations ★

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Create equations that describe numbers or relationships.

Standards	Clarifications
KY.HS.A.12 Create equations and inequalities in one variable and use them to solve problems. **MP.1**, **MP.4**	Students use the addition, subtraction, multiplication and division properties for both equations and inequalities to solve problems. These equations may arise from linear and quadratic functions and simple rational and exponential functions.
KY.HS.A.13 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **MP.2**, **MP.5**	Students solve systems of equations with two or more variables to solve problems in the real world setting.
KY.HS.A.14 Create a system of equations or inequalities to represent constraints within a modeling context. Interpret the solution(s) to the corresponding system as viable or nonviable options within the context. **MP.4**, **MP.5**	Students may be asked to find an optimal solution and the conditions under which the optimal solution would occur for a given real world situation.
KY.HS.A.15 Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations. **MP.2**, **MP.7**	Students encounter scenarios where they rewrite formulas/equations for variables different from the commonly used formulas. An example may include, but not being limited to, students rearranging Ohm's law (V = IR) to highlight resistance R, rather than the variable for voltage V.

Attending to the Standards for Mathematical Practice

Students interpret a story or situation into an equation or inequality, connecting the terms and symbols within the equation or inequality to the context (**MP.1**) and relate how the solution to the equation or inequality connects back to the original problem (**MP.4**). Students utilize technology to graph equations and use the graph to describe qualitatively and quantitatively the relationship between variables (**MP.5**). Students explain when they would opt for different equivalent forms an equation (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand solving equations as a process of reasoning and explain the reasoning.	
Standards	**Clarifications**
KY.HS.A.16 Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **MP.1**, **MP.3**	Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x+4=10$, $2(x-3)+4=10$, $2(3x-4)+4=10$, etc. After solving many linear equations in one variable, students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b= cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.
KY.HS.A.17 Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. a. Solve rational equations written as proportions in one variable. b. Solve radical equations in one variable. **MP.3**, **MP.5, MP.7**	Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation.
Attending to the Standards for Mathematical Practice	
Students use properties, such as the distributive property of multiplication over addition, to describe why two expressions are equivalent. They explain their approach to a problem, as well as critique the solutions of others, comparing the different approaches in terms of whether they are	

accurate and efficient (**MP.3**). Students approximate solutions with technology (**MP.5**). Students use structure of an equation (rational, radical), to determine an efficient strategy for finding a solution, if one exists (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Solve equations and inequalities in one variable.	
Standards	**Clarifications**
KY.HS.A.18 Solve linear equations and inequalities in one variable, including literal equations with coefficients represented by letters. **MP.2**, **MP.7**	Students use all properties of both equations and inequalities to solve for one variable.
KY.HS.A.19 Solve quadratic equations in one variable. a. Solve quadratic equations by taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. b. (+) Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. c. (+) Solve quadratic equations by completing the square. **MP.1**, **MP.8**	Students observe that methods for solving quadratic equations are interrelated and certain situations may more appropriately call upon one method as opposed to the other methods. b & c. (+) Students understand completing the square involves factoring and the quadratic formula is nothing more than an encapsulation of the method of completing the square. While all students are not required to be able to use completing the square as a method for solving quadratic equations, exposure to this method is needed to explain how the quadratic formula is derived.

Attending to the Standards for Mathematical Practice
Students reason about which symbolic representation is needed in order to focus on a particular feature and then efficiently rewrite literal equations to feature that characteristic (**MP.2**). Students analyze the structure of a quadratic equation to determine an efficient strategy to find a solution (**MP.7**).

Algebra-Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Solve systems of equations.

Standards	Clarifications
KY.HS.A.20 Solve systems of linear equations in two variables. a. Understand a system of two equations in two variables has the same solution as a new system formed by replacing one of the original equations with an equivalent equation. b. Solve systems of linear equations with graphs, substitution and elimination, focusing on pairs of linear equations in two variables. **MP.3**, **MP.6**	a. This part of the standard is not focused on the actual process of solving a system of equations, but rather the proof of the method (specifically the elimination method). b. Students utilize a variety of methods to solve system of equations including graphing the system, solving using the substitution method, solving the system with elimination both with and without involving multiplication. Students recognize the conclusion of these processes may result in obtaining one solution (expressed as an ordered pair), no solution or infinitely many solutions.
KY.HS.A.21 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **MP.3**, **MP.6**	Students utilize algebra techniques and graphical representations to determine points of intersection between lines and parabolas that indicate solution sets for a system of linear and quadratic equations.
KY.HS.A.22 (+) Use matrices to solve a system of equations. a. Represent a system of linear equations as a single matrix equation in a vector variable. b. Find the inverse of a matrix if it exists. c. Use matrices to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **MP.4**, **MP.7**	a. Students do not focus on the solving of the system, but rather translating between the two different representations for this part of the standard. b. Methods of solving systems with matrices could include, but are not limited to: • utilizing inverse matrices • row reduction • Cramer's rule

Attending to the Standards for Mathematical Practice

Students use a variety of methods to solve systems of equations, understanding that tables and graphs may produce estimates rather than exact solutions (**MP.6**)**.** Students construct a viable argument to justify their solution(s) in a system of equations. (**MP.3**)

Algebra- Reasoning with Equations and Inequalities	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Represent and solve equations and inequalities graphically.

Standards	Clarifications
KY.HS.A.23 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. **MP.1**, **MP.4**	Students make connections between algebra and geometry within this standard. Students acquire the basic understanding that the coordinates of the points of intersection of the graphs are the pairs of values of the variables that solve the system.
KY.HS.A.24 Justify that the solutions of the equations $f(x) = g(x)$ are the x-coordinates of the points where the graphs of $y = f(x)$ and $y = g(x)$ intersect. Find the approximate solutions graphically, using technology or tables. ★ **MP.3**, **MP.5**	Students justify solutions for equations which Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. ★
KY.HS.A.25 Graph linear inequalities in two variables. a. Graph the solutions to a linear inequality as a half-plane (excluding the boundary in the case of a strict inequality). b. Graph the solution set to a system of linear inequalities as the intersection of the corresponding half-planes. **MP.5**, **MP.6**	Students recall skills regarding graphing the solutions of a linear inequality in the coordinate plane in order to graph the solution set for a system of linear inequalities. Students utilize these skills in other standards via linear programming.

Attending to the Standards for Mathematical Practice
Students explain that the solutions of a system of equations or inequalities are all the points represented on the graph and therefore, where two functions overlap illustrates solutions to two functions (**MP.1, MP.3**). Students use technology to determine solutions to a system of linear inequalities (e.g., using DESMOS or graphing calculators) (**MP.5**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Conceptual Category Functions

Functions Overview

Interpreting Functions	Building Functions	Linear, Quadratic and Exponential Models	Trigonometric Functions
• Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of the context. • Analyze functions using different representations.	• Build a function that models a relationship between two quantities. • Build new functions from existing functions.	• Construct and compare linear, quadratic and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the concept of a function and use function notation.	

Standards	Clarifications
KY.HS.F.1 Understand properties and key features of functions and the different ways functions can be represented. a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. b. Using appropriate function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. c. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. d. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. e. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **MP.2**, **MP.4**, **MP.7**	a. When describing relationships between quantities, the defining characteristic of a function is the input value determines the output value or, equivalently, the output value depends upon the input value. In some situations where two quantities are related, each can be viewed as a function of the other. c. A function is often described and understood in terms of the output behavior, or over what input values is it increasing, decreasing, or constant. Important questions include, “For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?” Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions. Key features include, but are not limited to: intercepts; intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior; periodicity. e. Students compare characteristics from various representations for one type of family of function at a time. For quadratics, students might determine which function has the larger maximum when given two different representations of quadratic functions.

Standards	Clarifications
KY.HS.F.2 Recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **MP.7**, **MP.8**	Sequences are functions with a domain consisting of whole numbers.

Attending to the Standards for Mathematical Practice
Students reason quantitatively about the relationship between domain and range of functions across abstract and concrete representations (**MP.2**). Students look closely to discern arithmetic and geometric relationships as patterns with additive and multiplicative changes, respectively **(MP.7)**. Students notice the regularity in the pattern to write a general formula for arithmetic or geometric sequence (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret functions that arise in applications in terms of the context.	
Standards	**Clarifications**
KY.HS.F.3 Understand average rate of change of a function over an interval. a. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. b. Estimate the rate of change from a graph. ★ **MP.2**, **MP.4**	The rate of change over an interval is equivalent to the slope between the endpoints of the interval. For linear functions, the rate of change is constant, over all intervals. However, for nonlinear functions, the average rate of change may vary depending on the interval.
Attending to the Standards for Mathematical Practice	
Students make sense of the rate of change, recognizing it captures how the input and the output of a function vary simultaneously (**MP. 2**). For example, students explain that the rate of change for nonlinear functions is not constant. Students use equations, tables and graphs to analyze rate of change in applied and mathematical contexts (**MP.4**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Analyze functions using different representations.	
Standards	**Clarifications**
KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★ a. Graph linear and quadratic functions and show intercepts, maxima and minima. b. Graph square root, cube root and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior. e. (+) Graph trigonometric functions, showing period, midline and amplitude. f. (+) Graph piecewise functions, including step functions. g. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. **MP.4**, **MP.5**	Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features important for identifying and describing functions. This standard indicates the function families in students' repertoires, detailing which features are required for several key families. Students demonstrate fluency with linear, quadratic and exponential functions, including the ability to graph without using technology. In other function families, students graph simple cases without technology and more complex ones with technology.
KY.HS.F.5 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Identify zeros, extreme values and symmetry of the graph within the context of a quadratic function.	a. Quadratic functions provide a rich playground for developing this ability, since the three principal forms for a quadratic expression (expanded, factored and completed square) each give insight into different aspects of the function. b. Students examine real-world situations with constant multiplicative change, represented as expressions, such as growth or decay.

Standards	Clarifications
b. Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3**, **MP.6**	

Attending to the Standards for Mathematical Practice
Students use graphs to answer questions and/or make predictions for a given context (**MP. 4**). Students use technology to explore concepts of function families and show key features of the graph (**MP. 5**). Students compare and contrast different characteristics of functions to connect features of the graph with different real-world contexts (**MP.6**). Students manipulate expressions, being careful to preserve equivalence and describe why a particular expression provides insights into the function (**MP.3, MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Building Functions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Build a function that models a relationship between two quantities.

Standards	Clarifications
KY.HS.F.6 Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions. **MP.4**, **MP.7**	b. Use real-world examples when appropriate. c. Consider contextual examples for composition functions, such as, if $T(y)$ is the temperature in the atmosphere as a function of height and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
KY.HS.F.7 Use arithmetic and geometric sequences to model situations and scenarios. a. Use formulas (explicit and recursive) to generate terms for arithmetic and geometric sequences. b. Write formulas to model arithmetic and geometric sequences and apply those formulas in realistic situations. ★ c. (+) Translate between recursive and explicit formulas. **MP.4**, **MP.8**	Examples include, but are not limited to: • calculating mortgages • drug dosages • simple interest

Attending to the Standards for Mathematical Practice

For real-world problems, students formulate the problem, make assumptions, define variables and create functions to model the situation (**MP.4**). Students notice the regularity in real-world growing patterns and use these insights to write a general formula to describe arithmetic or geometric sequences (**MP.8**).

Functions- Building Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Build new functions from existing functions.	
Standards	**Clarifications**
KY.HS.F.8 Understand the effects of transformations on the graph of a function. a. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. b. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **MP.3**, **MP.5**	a. Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.
KY.HS.F.9 Find inverse functions. a. Given the equation of an invertible function, find the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2**, **MP.6**	a. Students can complete the process of finding the inverse when given an equation of a function that is invertible. b-d. Students need a formal sense of inverse functions. Students understand a function and its inverse describe the exact same relationship but in different ways.
KY.HS.F.10 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents with the use of technology. **MP.1**, **MP.7**	Students can use inverses of simple logarithmic and exponential equations in order to solve those equations. The inverse relationship between logarithmic and exponential functions is special in that each function's inverse is also a function
Attending to the Standards for Mathematical Practice	
Students use technology to explore how changing the value of *k* impacts the graph of the function (**MP.5**). Students use the graphical representation to create plausible arguments about the effects of transformations, instead of relying on computational rules (**MP.3**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Construct and compare linear, quadratic and exponential models and solve problems.	
Standards	**Clarifications**
KY.HS.F.11 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Recognize and justify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **MP.3**, **MP.8**	Linear functions have the same average rate of change over same-sized intervals; the same value is added to the output over each interval. In contrast, the outputs of exponential functions grow or decay by the same percent over same-sized intervals; the same value is multiplied by the output over each interval.
KY.HS.F.12 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **MP.7**, **MP.8**	Students construct functions with and without technology.
KY.HS.F.13 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7**, **MP.8**	Students compare functions by focusing on how the output values change over intervals of equal length. Even though a linear function may initially be increasing faster than an exponential function, an increasing exponential function always eventually exceeds an increasing linear function.
Attending to the Standards for Mathematical Practice	
Students reason about particular characteristics of linear, quadratic and exponential functions, for example comparing how rates of change across different types of functions (**MP.3**). Students recognize families of functions in a more general sense to discern that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically (**MP.8**).	

Functions- Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret expressions for functions in terms of the situation they model.	
Standards	**Clarifications**
KY.HS.F.14 Interpret the parameters in a linear or exponential function in terms of a context. **MP.1**, **MP.2**	More than just substituting values into a given formula, this requires students to understand how changing specific parameters will change the function output. An example of this with an exponential function ($f(x) = a \bullet b^x$) might be changing the "b" from a number greater than 1 to a number between 0 and 1. Students should recognize this creates a decay problem instead of a growth problem. Similarly, changing the "a" parameter creates corresponding changes to the graph and has different implications within the realistic context.
Attending to the Standards for Mathematical Practice	
Students quantitatively reason to consider the units, limitations and parameters in linear and exponential functions in terms of a context (**MP.2**). When solving problems, students ask themselves, "Does this make sense?" (**MP.1**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Extend the domain of trigonometric functions using the unit circle.	
Standards	**Clarifications**
KY.HS.F.15 (+) Understand the relationship of radian measure of an angle to its arc length. **MP.1**, **MP.6**	Understanding radian measure of an angle as arc length on the unit circle enables students to build on their understanding of trigonometric ratios associated with acute angles and to explain how these ratios extend to trigonometric functions whose domains are included in the real numbers.
KY.HS.F.16 (+) Understand and use the unit circle. a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. b. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π /4 and π /6 and use the unit circle to express the values of sine, cosine and tangent for π –*x*, π +*x* and 2 π –*x* in terms of their values for *x*, where *x* is any real number. c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MP.7**, **MP.8**	This standard is sometimes called “unwrapping the unit circle.” For each function, the angle θ is represented by values on the horizontal axis and the resulting outputs are graphed on the vertical axis. c. Students understand symmetry exists within the unit circle for paired reference angles: sin (-θ) = -sin(θ), so sine is an odd function; and cos (-θ) = cos(θ), so cosine is an even function.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Model periodic phenomena with trigonometric functions.	
Standards	**Clarifications**
KY.HS.F.17 (+) Choose trigonometric functions to model periodic phenomena with specified period, midline and amplitude. ★ **MP.4**, **MP.5, MP.6**	A function is described as sinusoidal or is called a sinusoid if it has the same shape as the sine graph, for example, has the form $f(t) = A + B(\sin Ct + D)$. Many real-world phenomena can be approximated by sinusoids, including sound waves, oscillation on a spring, the motion of a pendulum, tides and phases of the moon. Because $\sin(t)$oscillates between 1 and 1, $A + B(\sin Ct + D)$will oscillate between $A - B$ and $A + B$. Thus, $y = A$ is the midline and B is the amplitude of the sinusoid. Students can obtain the frequency of f: the period of $\sin(t)$ is 2π, so (knowing the effect of multiplying t by C) the period of $\sin(Ct)$ is $2\pi/C$ and the frequency is its reciprocal. When modeling, students have the sense that C affects the frequency and that C and D together produce a phase shift, but finding a correct solution might involve technological support, except in simple cases.
KY.HS.F.18 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MP.2**, **MP.3**	Students experience restricting the domain of a function so it has an inverse. For trigonometric functions, a common approach to restricting the domain is to choose an interval on which the function is always increasing or always decreasing.

Standards	Clarifications
KY.HS.F.19 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. ★ **MP.4**, **MP.5**	Include $\sin^{-1} x, \cos^{-1} x \ and \ \tan^{-1} x$.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Prove and apply trigonometric identities.	
Standards	**Clarifications**
KY.HS.F.20 (+) Proving identities and formulas within the context of trigonometry. a. Prove the Pythagorean identity and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. b. Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems. **MP.3**, **MP.7**	In the unit circle, the x-value is the cosine and the y-value represents the sine. Since the hypotenuse of any right triangle on the unit circle is 1, the Pythagorean relationship of $x^2 + y^2 = 1$ holds. Students connect the Pythagorean Theorem in geometry and the study of trigonometry to understand this relationship.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Conceptual Category Geometry

Geometry Overview

Congruence	Similarity, Right Triangles and Trigonometry	Circles	Expressing Geometric Properties with Equations	Geometric Measurement and Dimensions	Modeling with Geometry
• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Prove geometric theorems. • Make geometric constructions.	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. • Define trigonometric ratios and solve problems involving right triangles. • Apply trigonometry to general triangles.	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles.	• Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically.	• Explain volume formulas and use them to solve problems. • Visualize relationships between two-dimensional and three-dimensional objects.	• Apply geometric concepts in modeling situations.

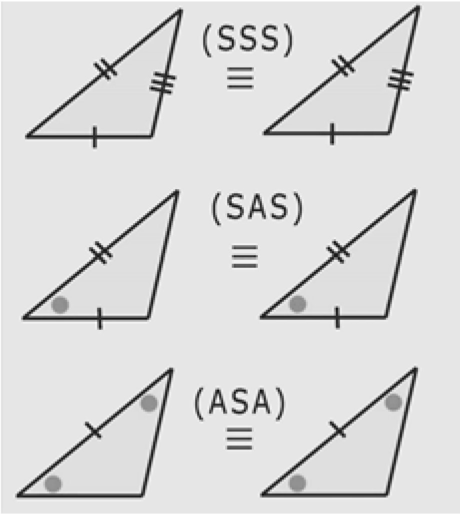
Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Experiment with transformations in the plane.	
Standards	**Clarifications**
KY.HS.G.1 Know and apply precise definitions of the language of Geometry: a. Understand properties of line segments, angles and circle. b. Understand properties of and differences between perpendicular and parallel lines. **MP.3**, **MP.6**	Students in high school start to formalize the intuitive geometric notions they developed in grades 6–8 and give specificity to geometric concepts that can serve as a good basis for developing precise definitions and arguments. a. Students understand a more formal knowledge of postulates, theorems and various properties relating to line segments, angles and circles. This knowledge is based on the undefined notions of point, line, distance along a line and distance around a circular arc. b. Students understand important properties of both parallel and perpendicular lines, prior to making the connections between these types of lines and how they relate to their calculated or given slope.
KY.HS.G.2 Representing transformations in the plane. a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs b. Compare transformations that preserve distance and angle measures to those that do not. c. Given a rectangle, parallelogram, trapezoid, or regular polygon, formally describe the rotations and reflections that carry it onto itself, using properties of these figures. **MP.5**, **MP.7**	Software, transparencies, etc. may be used to accurately represent congruence transformations in the plane. a. Students understand any point (a,b) can be thought of as an input and any image of point (a,b) can be thought of as the output of a specific transformation function. b. Students make connections between which transformations are a rigid motion (isometry) and which transformations do not have that characteristic. c. Students practice and understand the procedures needed to carry out multiple transformations that carry the figure onto itself, recognizing the important properties of these figures.

Standards	Clarifications
KY.HS.G.3 (+) Develop formal definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. **MP.6**, **MP.7**	Students understand and recognize characteristics of various transformations of multiple different geometric figures. Students develop formal definitions that reflect those transformations.
KY.HS.G.4 Understand the effects of transformations of geometric figures. a. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. b. Specify a sequence of transformations that will carry a given figure onto another. c. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **MP.2**, **MP.8**	Students understand a figure, called a pre-image, is congruent to another figure, called the image, if that second figure can be obtained by a sequence of congruence transformations performed on the first figure. Students can draw the image of a transformed pre-image using a variety of tools, including but not limited to: • graph paper • manipulatives • tracing paper • computer programs Students perform such sequences and describe the sequence of congruence transformations necessary to transform one figure to an congruent second figure.

Attending to the Standards for Mathematical Practice
Students make careful calculations when transforming figures by hand (**MP.6**) and use technology (**MP.5**) to analyze more complicated cases and to make generalizations (**MP.7**). Students use correct terminology when discussing figures and the effects of their transformed figure (**MP.3, MP.6**), identifying congruent, distance-preserving, figures when possible. For example, students connect geometric transformations with algebra when comparing a figure *F* and the transformed figure *T(F)* or a figure that has undergone multiple transformations *T(R(F))* (**MP.2**).

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand congruence in terms of rigid motions.	
Standards	**Clarifications**
KY.HS.G.5 Know and apply the concepts of triangle congruence: a. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Explain how the criteria for triangle congruence (ASA, SAS and SSS) follow from the definition of congruence in terms of rigid motions. **MP.3**, **MP.6**	
Attending to the Standards for Mathematical Practice	
Students fluently and intentionally select and/or calculate measures (**MP.6**) when deliberating criteria for triangle congruence (**MP.3**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Prove geometric theorems.

Standards	Clarifications
KY.HS.G.6 Apply theorems for lines, angles, triangles, parallelograms. **MP.2**, **MP.3**	Students use previously learned definitions, theorems, postulates and properties of lines, angles, triangles and parallelograms to draw conclusions and to make inferences. Theorems for lines and angles include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Theorems for triangles include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Theorems for parallelograms include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.
KY.HS.G.7 Prove theorems about geometric figures. a. Construct formal proofs to justify theorems for lines, angles and triangles. b. (+) Construct formal proofs to justify theorems for parallelograms. **MP.6**, **MP.7**	Students recall definitions, theorems, postulates and properties to construct formal proofs based on theorems established in other standards. (+)Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.

Attending to the Standards for Mathematical Practice
Students experiment with lines, angles, triangles and parallelograms to make connections and conjectures about their properties (**MP.7**), using dynamic software when appropriate (**MP.5**). Students routinely use various forms of proof (formal, informal, direct and indirect) to outline their logic and defend their conjectures (**MP.3**). Students consider alternate approaches to a proof or a conjecture and debate the alternatives for effectiveness and accuracy (**MP.2**, **MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Congruence	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Make geometric constructions.

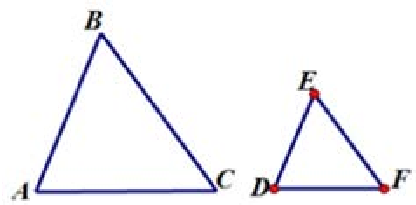
Standards	Clarifications
KY.HS.G.8 Create and apply geometric constructions. a. Make formal geometric constructions with a variety of tools and methods. b. Apply basic construction procedures to construct more complex figures. **MP.5**, **MP.6**	Methods for formal constructions may include but are not limited to: • compass and straightedge • string • reflective devices • paper folding • technology Students demonstrate the ability to copy a segment, copy an angle, bisect a segment, bisect an angle, construct perpendicular lines which includes the perpendicular bisector of a line segment and construct a line parallel to a given line through a point not on the line.

Attending to the Standards for Mathematical Practice
Students select and use a variety of tools to generate geometric constructions (**MP.5**). Students use precision when constructing shapes and figures by hand and select and use appropriate technology for complicated constructions (**MP.5, MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand similarity in terms of similarity transformations.

Standards	Clarifications
KY.HS.G.9 Understand properties of dilations. a. Verify the properties that result from that dilations given by a center and a scale factor. b. Verify that a dilation produces an image that is similar to the pre-image. **MP.5**, **MP.7**	• Methods to verify properties could include, but not limited to: scale models, moving an object closer to a light source and examining changes, changing the scale factor on a copier. • Students explain the effect of dilations on objects that pass through the center verses those that do not pass through the center of a figure. • Students understand within this standard, the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides is a result that occurs because two objects are similar.
KY.HS.G.10 Apply the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **MP.3**, **MP.6**	The AA Similarity Theorem  If $\angle A \cong \angle D, and\ \angle B \cong \angle E$, Then $\Delta ABC \sim \Delta DEF$.

Attending to the Standards for Mathematical Practice

With the aid of physical models, transparencies and geometry software, students verify whether figures are similar or not (**MP.5**, **MP.6**). As they compare similar shapes, they make generalizations about what changes and what stays the same when, and use this information to do dilations (**MP.7**). Students prepare illustrations and explanations related to the AA triangle similarity criterion, as well as by considering and discussing properties of similar triangles (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

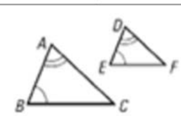
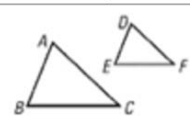
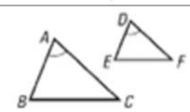
Geometry-Similarity, Right Triangles and Trigonometry

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Prove theorems involving similarity.

Standards	Clarifications
KY.HS.G.11 Understand theorems about triangles. a. Apply theorems about triangles. b. (+) Prove theorems about triangles. c. Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **MP.1**, **MP.3**	Theorems include the Pythagorean Theorem and “a line parallel to one side of a triangle divides the other two proportionally and conversely.” Students demonstrate the ability to copy a segment, copy an angle, bisect a segment, bisect an angle, construct perpendicular lines, which includes the perpendicular bisector of a line segment and construct a line parallel to a given line through a point not on the line. Triangle Similarity Postulate and Theorems:

AA Similarity Postulate	SSS Similarity Theorem	SAS Similarity Theorem
		
Two triangles are similar if they have two pairs of congruent angles.	Two triangles are similar if they have three pairs of proportional sides.	Two triangles are similar if they have two pairs of proportional sides with a congruent included angle.

Attending to the Standards for Mathematical Practice

Students identify cases where the AA triangle similarity criterion can be used (**MP.1**) and routinely use various methods of proof (formal, informal, direct and indirect) to outline their logic in order to defend their conjectures (**MP.3**).

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Define trigonometric ratios and solve problems involving right triangles.

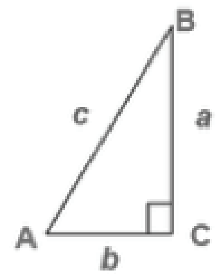
Standards

KY.HS.G.12 Understand properties of right triangles.

a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles (sine, cosine and tangent).
b. Explain and use the relationship between the sine and cosine of complementary angles.
c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

MP.3, **MP.4**

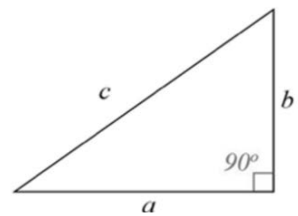
Clarifications



$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

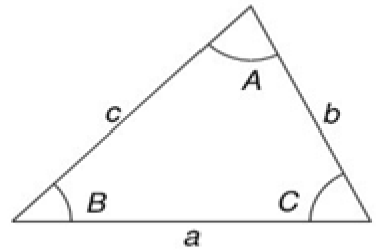
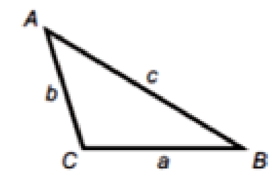
$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$



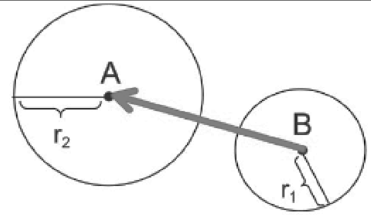
$c^2 = a^2 + b^2$

Attending to the Standards for Mathematical Practice

Given a variety of similar triangles, students compare ratios of corresponding pairs of sides in order to discover the definitions of trigonometric ratios for acute angles (**MP.3**). Students use these trigonometric ratio definitions to solve real-world problems involving right triangles, connecting their solutions to the problem posed (**MP.4**).

Geometry-Similarity, Right Triangles and Trigonometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply trigonometry to general triangles.	
Standards	**Clarifications**
KY.HS.G.13 (+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **MP.6**, **MP.7**	 Area of triangle = ½ ab sin(C)
KY.HS.G.14 (+) Understand and apply the Law of Sines and the Law of Cosines. a. Use the Law of Sines and Cosines to find unknown measurements in right and non-right triangles. b. Prove the Laws of Sines and Cosines and use them to solve problems. **MP.1**, **MP.3**	Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$ 

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Circles	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and apply theorems about circles.	
Standards	**Clarifications**
KY.HS.G.15 Verify using dilations that all circles are similar. **MP.5**, **MP.8**	
KY.HS.G.16 Identify and describe relationships among angles and segments within the context of circles involving: a. Recognize differences between and properties of inscribed, central and circumscribed angles. b. Understand relationships between inscribed angles and the diameter of a circle. c. Understand the relationship between the radius of a circle and the line drawn through the point of tangency on that radius. **MP.3, MP.5, MP.7**	Students recognize and apply relationships including the relationship between central, inscribed and circumscribed angles, inscribed angles on a diameter are right angles, the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
KY.HS.G.17 (+) Apply basic construction procedures within the context of a circle. a. Construct the inscribed and circumscribed circles of a triangle. b. Construct a tangent line from a point outside a given circle to the circle. **MP.5**, **MP.6**	Students build upon skills from other standards regarding construction procedures in the context of circles.
Attending to the Standards for Mathematical Practice	
Students compare properties of a variety of circles to verify that all circles are similar (**MP.8**). Students use technology and drawings of circles to analyze properties of angles, radii and diameters that hold true across all circles (**MP.5**) and can explain these properties (**MP.3**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Circles	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Find arc lengths and areas of sectors of circles.	
Standards	**Clarifications**
KY.HS.G.18 (+) Understand the relationship between an intercepted arc length within a circle and the radius of the circle. a. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector. b. Define the radian measure of the angle as the measure of a central angle that intercepts an arc equal in length to the radius of the circle. **MP.2**, **MP.3**	$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{2\pi}$ $\frac{\text{Area of Sector}}{\pi r^2} = \frac{\text{Central Angle}}{2\pi}$ Area of Sector = $\frac{\text{Central Angle}}{2\pi} \bullet \pi r^2$ Area of Sector =$\frac{1}{2} \bullet$ Central Angle $\bullet\ r^2$

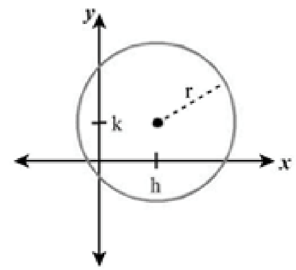
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Expressing Geometric Properties with Equations

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Translate between the geometric description and the equation for a conic section.

Standards	Clarifications
KY.HS.G.19 Understand the relationship between the algebraic form and the geometric representation of a circle. a. Write the equation of a circle of given center and radius using the Pythagorean Theorem. b. (+) Derive and write the equation of a circle of given center and radius using the Pythagorean Theorem. c. (+) Complete the square to find the center and radius of a circle given by an equation. **MP.6**, **MP.8**	 $(x-h)^2 + (y-k^{2} = r^2$
KY.HS.G.20 (+) Derive the equations of conic sections. a. Derive the equation of a parabola given a focus and directrix. b. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **MP.2**, **MP.7**	Parabolas: $y-k = a(x-h)^2$ $x\text{-}h = a(y\text{-}k)^2$ Circles: $(x-h)^2 + (y-k)^2 = r^2$ Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Attending to the Standards for Mathematical Practice

Students explain the connection between the Pythagorean Theorem and the equation of a circle (**MP.8**) and use the center and radius accurately within the formula (**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry- Expressing Geometric Properties with Equations	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use coordinates to prove simple geometric theorems algebraically.	
Standards	**Clarifications**
KY.HS.G.21 Use coordinates to justify and prove simple geometric theorems algebraically. **MP.2**, **MP.6**	Students understand how to prove or disprove a figure defined by four given points in the coordinate plane is a rectangle, as well as prove or disprove the given point lies on the circle centered at the origin and containing an additional given point.
KY.HS.G.22 Justify and apply the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. **MP.3**, **MP.7**	Students understand the relationship between slope and how it relates to both parallel and perpendicular lines. Within this standard, students also understand how to find the equation of a line parallel or perpendicular to a given line that passes through a given point.
KY.HS.G.23 Find measurements among points within the coordinate plane. a. Use points from the coordinate plane to find the coordinates of a midpoint of a line segment and the distance between the endpoints of a line segment. b. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **MP.2**, **MP.8**	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
KY.HS.G.24 Use coordinates within the coordinate plane to calculate measurements of two dimensional figures. a. Compute the perimeters of various polygons. b. Compute the areas of triangles, rectangles and other quadrilaterals.★ **MP.2**, **MP.4**	Students utilize the distance formula to find distances between points in order to find the area and/or perimeter of various geometric figures.

Attending to the Standards for Mathematical Practice
Students describe the connections between geometric theorems and their algebraic formulas (**MP.2**). They intentionally manipulate coordinates appropriately, fluently selecting criterion and formulas for a given context (**MP.7**). Students use coordinate geometry to model real-world situations, posing their own real-world problems when possible (**MP.4**).

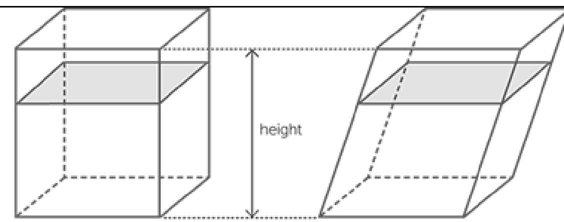
The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry- Geometric Measurement and Dimensions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

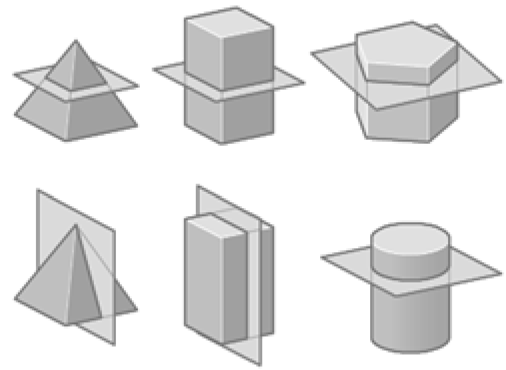
Cluster: Explain volume formulas and use them to solve problems.

Standards	Clarifications
KY.HS.G.25 Analyze and determine the validity of arguments for the formulas for the various figures and shapes. a. Finding the circumference and area of a circle. b. Finding the volume of a sphere, prism, cylinder, pyramid and cone. **MP.3**, **MP.7**	Students may use dissection arguments, Cavalieri's principle and informal limit arguments in order to find these values for these figures.
KY.HS.G.26 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **MP.2**, **MP.5**	
KY.HS.G.27 Use volume formulas to solve problems for cylinders, pyramids, cones, spheres, prisms ★ **MP.4**, **MP.6**	General Prism: $V = Bh$ Right Circular Cylinder: $V = \pi r^2 h$ Pyramid: $V = \frac{1}{3}Bh$ Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$

Attending to the Standards for Mathematical Practice

As students analyze volume formulas, they looking for relationships between the shapes and the related formulas (**MP.7**). Students critique different explanations or justifications for the formulas (**MP.3**). Students recognize various situations for which these formulas would apply and use them to solve real-world problems, posing their own real-world problems when possible (**MP.4**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Geometric Measurement and Dimensions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Visualize relationships between two-dimensional and three-dimensional objects.	
Standards	**Clarifications**
KY.HS.G.28 Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects. **MP.5**, **MP.7**	Students recognize visually the two dimensional shapes created via the cross sections of three dimensional solid figures. Examples include, but are not limited to 
Attending to the Standards for Mathematical Practice	
Students use technology to identify the result of cutting a three- dimensional object and the result of rotating two-dimensional objects (**MP.5**). As students analyze two-dimensional and three-dimensional shapes, they gain insights into the structure of specific shapes (**MP.7**). For instance, students consider the two-dimensional figures that result from removing the top of a shoe box or from slicing an orange. Students compare and contrast the two-dimensional cross sections of an orange when sliced at different locations or angles verses slicing. For an extension, students can compare their conjectures from circles when slicing a cone at different locations or angles.	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry-Modeling with Geometry	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply geometric concepts in modeling situations.	
Standards	**Clarification/Illustration**
KY.HS.G.29 Use geometric shapes, their measures and their properties to describe objects in real world settings. **MP.1**, **MP.4**	Students use geometric shapes to model objects, for example, modeling a tree trunk or a human torso as a cylinder).★
KY.HS.G.30 Apply concepts of density based on area and volume in modeling situations, using appropriate units of measurement. **MP.4**, **MP.6**	Students explore scenarios where they find the area of regions and the volume of solid figures. In the process, they appropriately use units of measurement, for example, persons per square mile, BTUs per cubic foot
KY.HS.G.31 Apply geometric methods to solve design problems. ★ **MP.1**, **MP.4**	Students practice modeling techniques in this standard using a variety of strategies and practices, for example, designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios

Attending to the Standards for Mathematical Practice

Students recognize various situations for which geometric knowledge would apply and do so to solve real-world problems (**MP.4**). As students use geometric methods to solve design problems, they continually reflect on whether their method and process makes sense for the problem and revise, as needed, until a viable solution has been found (**MP.1**). Students also select appropriate theorems and formulas and report units with appropriate accuracy (**MP.6**).

Kentucky Academic Standards for Mathematics: Conceptual Category Statistics and Probability

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data	Making Inferences and Justifying Conclusions	Conditional Probability and the Rules of Probability	Using Probability to Make Decisions
• Summarize, represent and interpret data on a single count or measurement variable. • Summarize, represent and interpret data on two categorical and quantitative variables. • Interpret linear models.	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments and observational studies.	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model.	• Calculate expected values and use them to solve problems. • Use probability to evaluate outcomes of decisions.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

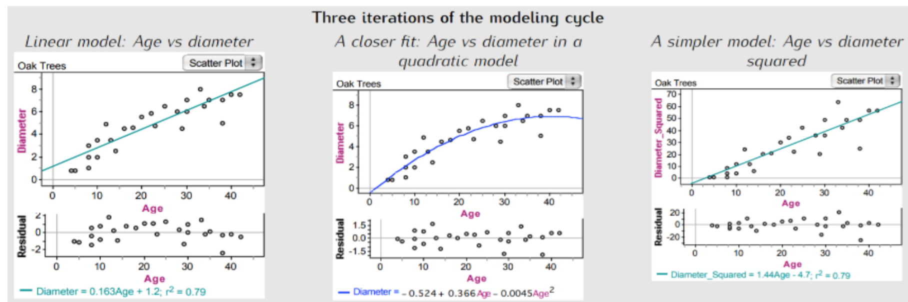
Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Statistics and Probability-Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Summarize, represent and interpret data on a single count or measurement variable.	
Standards	**Clarifications**
KY.HS.SP.1 Represent the distribution of data with plots on the real number line (stem plots, dot plots, histograms and box plots). **MP.4**, **MP.5**	Students create appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.2 Use statistics appropriate to the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. **MP.2**, **MP.6**	Students use raw data and data from appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.3 Interpret differences in shape, center and spread in the context of the distributions of the numerical data, accounting for the presence and possible effects of extreme data points (outliers). **MP.1**, **MP.7**	Students analyze contextual situations as they interpret differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.4 (+) When appropriate, fit a normal distribution to a numerical data set for given mean and standard deviation and then estimate population percentages using the Empirical Rule and recognize that there are data sets for which such a procedure is not appropriate. **MP.1**, **MP.3**	Students use the empirical rule (68%-95%-99.7% rule), calculators and/or tables to estimate areas under the normal curve, recognizing when data sets are skewed this can be problematic.
Attending to the Standards for Mathematical Practice	
Students use technology to visualize data using stem plots, dot plots, histograms and box plots (**MP.5**). After the data have been collected, students are precise about choosing the appropriate analyses and representations to reveal the variability in the data (**MP.6**). Students analyze quantitative data and classify any observation(s) that deviate(s) considerably from the majority of data within a distribution as potential outliers (**MP. 7**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability- Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: **Summarize, represent and interpret data on two categorical and quantitative variables.**

Standards	Clarifications
KY.HS.SP.5 Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal and conditional relative frequencies (probabilities) in the context of the data, recognizing possible associations and trends in the data. **MP.2**, **MP.7**	Students use frequency tables to both calculate probabilities, as well as determine relationships between the variables represented in those tables.
KY.HS.SP.6 Represent data on two quantitative variables on a scatter plot and describe how the explanatory and response variables are related. a. Calculate an appropriate mathematical model, or use a given mathematical model, for data to solve problems in context. b. Informally assess the fit of a model (through calculating correlation for linear data, plotting, calculating and/or analyzing residuals). **MP.3**, **MP.4**, **MP.5**	Emphasize linear, quadratic and exponential models as illustrated below. 

Attending to the Standards for Mathematical Practice

Students discover structures or patterns in data to answer statistical questions using tables or appropriate representations (**MP.7**). Students informally determine whether a selected model is appropriate for a set of data and use technology when appropriate to do so (**MP 5**). Students draw and discuss conclusions about a statistical question (**MP.3**) using appropriate mathematical models.

Statistics and Probability-Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Interpret linear models.

Standards	Clarifications
KY.HS.SP.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **MP.1**, **MP.2**	Students demonstrate interpreting slope in the context of a given situation when examining two variable statistics as being "for each additional known unit increase in an explanatory variable, we expect or predict a known unit increase (or decrease) in the response variable." Students demonstrate interpreting intercept in the context of a given situation when examining two variable statistics as being "the predicted known unit of a response variable when the explanatory variable is zero known units."
KY.HS.SP.8 Understand the role and purpose of correlation in linear regression. a. Use technology to compute correlation coefficient of a linear fit. b. Interpret the meaning of the correlation within the context of the data. c. Describe the limitations of correlation when establishing causation. **MP.5**, **MP.6**	a. Students use technology to perform the calculation of: $r = \frac{\Sigma(x-\overline{x})}{\sqrt{\Sigma(x-\overline{x})^2}\,\sqrt{\Sigma(y-\overline{y})^2}}$ b. Students understand correlation measures linear associations between two quantitative variables addressing the direction (positive or negative) and the relative strength of the given association. c. Students understand one of the most common misinterpretations of correlation is to think of it as a synonym for causation. A high correlation between two variables (suggesting a statistical association between the two) does not imply one causes the other.

Attending to the Standards for Mathematical Practice

Students interpret the results to a statistical question and relate the results to the context of the data (**MP.1**, **MP.2**). Students use technology to compute correlation coefficients (**MP.5**). Students recognize that correlation is an indication of a linear relationship between two quantitative variables and not simply another word for association (**MP.6**).

Statistics and Probability-Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand and evaluate random processes underlying statistical experiments.	
Standards	**Clarifications**
KY.HS.SP.9 Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. **MP.1**, **MP.3**	Students use sample statistics (mean and standard deviation) obtained from random samples to help estimate population parameters which are unknown values.
KY.HS.SP.10 Decide if a specified model is consistent with the results from a simulation. **MP.3**, **MP.6**	If a model shows a spinning coin falls heads-up with probability of 0.5, would a result of 5 tails in a row cause you to question the model?
Attending to the Standards for Mathematical Practice	
Students follow the progression of the statistical problem-solving process to investigate answers to a statistical question (**MP.3**). Students justify their conclusions, communicate them to others (orally and in writing) and critique the conclusions of others (**MP.3**). Students are precise about choosing the appropriate analyses and representations that account for the variability in the data (**MP.6**).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability- Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Make inferences and justify conclusions from sample surveys, experiments and observational studies.	
Standards	**Clarifications**
KY.HS.SP.11 Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **MP.3**, **MP.8**	Students understand a random selection of 100 students from your school will allow you to draw some conclusions about all the students in the school, whereas taking your class as a sample will not allow that generalization. Students recognize experiments involve imposing treatments on units/subjects, whereas observational studies do not.
KY.HS.SP.12 Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. **MP.4**, **MP.7**	KY.HS.SP.12 differs from KY.HS.SP.9 in that results from non-random samples (Voluntary Response and Convenience) generate biased results when compared with more appropriate, random samples of the same population.
KY.HS.SP.13 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. **MP.3**, **MP.8**	Hypotheses can be tested to determine if significant differences between two treatments exist using statistical data. If significance exists, claims and conclusions can be made about the treatment.

Attending to the Standards for Mathematical Practice
Students compare and contrast the different roles randomization plays in data collection (**MP.8**). Students look for patterns in the variability around the structure (**MP.7**).

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand independence and conditional probability and use them to interpret data.	
Standards	**Clarifications**
KY.HS.SP.14 Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as, • as unions, "A or B," that are mutually exclusive events and • as unions, "A or B," that are non-mutually exclusive events and • as intersections, "A and B," and • as complements of other events, "not A." to calculate basic probabilities. **MP.1**, **MP.2**	A union of two events, "A or B," includes *all* elements in both events notated by: $A \cup B$. Addition Rule for mutually exclusive events: If A and B are mutually exclusive, P(A or B) = P(A) + P(B). Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. An intersection, "A and B," of two events includes all overlapping elements notated by: $A \cap B$. A complement for any event A, P(not A) = 1 – P(A).
KY.HS.SP.15 Understand the concept of independence. a. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their individual probabilities, P(A) x P(B) b. (+) Determine whether two events are independent and provide a justification to support the decision. c. Recognize and explain the concept of independence in everyday language and everyday situations. **MP.1**, **MP.6**	a. Events *A* and *B* are independent if and only if P(*A* and *B*) = *P*(*A*)*P*(*B*).
KY.HS.SP.16 Understand the concept of conditional probability. a. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*). b. (+) Interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A* and the conditional probability of *B* given *A* is the same as the probability of *B*.	a. For any two events *A* and *B,* P(*A* given *B*) = $\frac{P(A \text{ and } B)}{P(B)}$.

Standards	Clarifications
c. Recognize and explain the concept of conditional probability in everyday language and everyday situations. d. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A and* interpret the answer in terms of the model. **MP.1**, **MP.3**	
KY.HS.SP.17 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. **MP.2**, **MP.4**	Students collect their own data or use data obtained from a random sample of their choosing and construct two-way frequency tables from their sample in order to determine independence and calculate probabilities.

Attending to the Standards for Mathematical Practice
Students encounter chance events in real contexts, including situations involving both dependent and independent events, are able to determine the difference between the contexts and fluently select and use appropriate formulas (**MP.1**). Students consider whether the occurrence of one event affects the probability of the other event (**MP.2**) in order to determine if two events are independent. Students analyze and discuss a variety of sources such as contingency tables to provide a context for conditional probability (**MP.3**). Students consider how conditions or assumptions affect the computation of a probability (**MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Use the rules of probability to compute probabilities of compound events	
Standards	**Clarifications**
KY.HS.SP.18 (+) Apply the General Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B), in a uniform probability model and interpret the answer in terms of the model. **MP.1**, **MP.2**	Consider an experiment where two cards are drawn without replacement. Define events A and B: A = 1^{st} card drawn is a king B = 2^{nd} card drawn is a king P(B|A) is the probability that the second card is a king given the first card drawn was a king. In that case, there will be 3 kings left out of 51 cards, so P(B|A) = 3/51 P(A and B) = P(1^{st} is a king and 2^{nd} is a king) P(A and B) = P(1^{st} king) • P(2^{nd} is a king, given 1^{st} is a king) P(A and B) = (4/52) (3/51) P(A and B) = P(A) • P(B|A)
KY.HS.SP.19 Use permutations and combinations to compute probabilities. a. Distinguish between situations that can be modeled using counting techniques, including Fundamental Counting Principle, permutations and combinations. b. Perform calculations using the appropriate counting technique, including simple probabilities. c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **MP.1**, **MP.8**	Permutations are calculated when order matters. Combinations are calculated when order does not matter. Number of permutations of *n* items taken *r* at a time: ${}_nP_r = \frac{n!}{(n-r)!}$ Number of combinations of *n* items taken *r* at a time: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Attending to the Standards for Mathematical Practice
Students recognize and solve real-world problems using the Fundamental Counting Principle, Permutations and Combinations (**MP.1**)**.** Students identify patterns to generalize a formula for calculating permutations and combinations (**MP.8**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

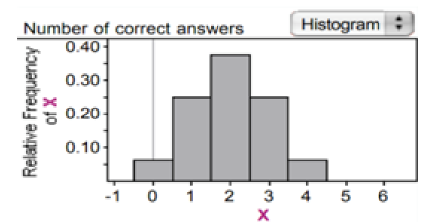
Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Calculate expected values and use them to solve problems.

<table>
<tr><th>Standards</th><th>Clarifications</th></tr>
<tr><td>KY.HS.SP.20 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays as for data distributions.
MP.3, MP.6</td><td>Students realize random variables are different from the variables used in other high school domains and random variables are functions of the outcomes of a random process and thus have probabilities attached to their possible values.
A possible example of a probability distribution:

Number Correct (x)	Probability
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$
4	$\frac{1}{16}$



</td></tr>
<tr><td>KY.HS.SP.21 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions.
MP.1, MP.8</td><td>The expected value/mean of a discrete random variable is
$\mu = E(x) = \Sigma xp(x)$.</td></tr>
<tr><td>KY.HS.SP.22 (+) Develop a probability distribution for a random variable.
a. Find an expected value based on a sample space in which theoretical probabilities can be calculated.
b. Find an expected value based on a sample space in which empirical probabilities can be calculated.
MP.2, MP.8</td><td>a. (+) Theoretical probability is given by the number of ways a particular event can occur divided by the total number of possible outcomes.
b. (+) The empirical probability of an event is given by number of times an event occurs divided by the total number of incidents observed.</td></tr>
</table>

Statistics and Probability-Using Probability to Make Decisions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use probability to evaluate outcomes of decisions.	
Standards	**Clarifications**
KY.HS.SP.23 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values. c. Use calculated expected values to make fair decisions and formulate strategies. **MP.6**, **MP.8**	Students use expected values to play a role in decision making in a variety of contexts.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Kentucky Academic Standards for Mathematics: Calculus (+)

Calculus instructional time should focus on 3 critical areas:

1. Conceptual understanding and procedural fluency of limits, derivatives and integration.
2. Applications of derivatives and integrals.
3. Working with functions in a variety of ways: graphical, numerical, analytical and verbal.

Calculus Overview

Limits	Function Behavior	Continuity	Understanding the Derivative	Applications of the Derivative	Understanding Integration	Applications of Integration
• Understanding the concept of the limit of a function.	• Describe the asymptotic and unbounded behavior of functions.	• Develop an understanding of continuity as a property of functions.	• Demonstrate an understanding of the derivative.	• Apply differentiation techniques.	• Understand and apply the Fundamental Theorem of Calculus.	• Apply techniques of integration. • Use integration to solve problems.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

(+): Calculus standards are not required standards for all Kentucky students; therefore, all Calculus standards would be considered (+) standards.

TWO plus signs (++) indicate a standard that is optional even for calculus.

Calculus-Limits

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand the concept of the limit of a function.

Standards	Clarifications
KY.HS.C.1 (+) Understand limits. a. Apply limits to a variety of functions, including piecewise functions. b. (++) Prove that the limit of a function exists, based upon the definition of a limit. **MP.2, MP.3**	
KY.HS.C.2 (+) Demonstrate an understanding of limits by estimating and finding the limit of a function at a point graphically, numerically and algebraically. **MP.5, MP.8**	Include analysis of limits in piecewise functions. Algebraic techniques include but are not limited to factoring, multiplying by the conjugate and finding the lowest common denominator.
KY.HS.C.3 (+) Apply properties and theorems of limits, including limits of indeterminate forms. **MP.2, MP.3**	Include sums, differences, products, quotients, composition of functions, special limits, Squeeze Theorem and L'Hospital's Rule.
KY.HS.C.4 (+) Communicate understanding of limits using precise mathematical symbols and language. **MP.3, MP.6**	Use of limits to predict the function value for an undefined value (hole in the graph). Apply the definition of a limit to margin of error. For example, if the weight of a golf ball needs to be within a certain range (ϵ), then the radius of the ball must be to a certain level of accuracy (δ).

Attending to the Standards for Mathematical Practice

Students can use technology to examine the graph of a function and determine whether or not the limit of the function exists at a point (**MP.5**). Students can use a table to find the value of a function for points that approach a given point, leading to conjectures about the limit of the function (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Function Behavior	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Describe the asymptotic and unbounded behavior of functions.	
Standards	**Clarifications**
KY.HS.C.5 (+) Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **MP.2, MP.5**	
KY.HS.C.6 (+) Discuss the end behavior of functions; identify representative functions for each type of end behavior using precise mathematical symbols and language. **MP.2, MP.6**	$\lim_{x\to\infty} f(x) = 4$ implies a horizontal asymptote of $y = 4$ $\lim_{x\to\infty} f(x) = \infty$ implies right hand end behavior is positive infinity NOTE: odd functions result in end behavior similar to lines (opposite directions); even functions result in end behavior similar to parabolas (same direction)

Attending to the Standards for Mathematical Practice
Students use analytic methods to identify vertical and horizontal asymptotes (**MP.2**). Students use technology to examine the graph of a function, to determine the values for which it is defined and to convergence for increasingly large values in the domain (**MP.5**).

Calculus-Continuity

Standards for Mathematical Practice

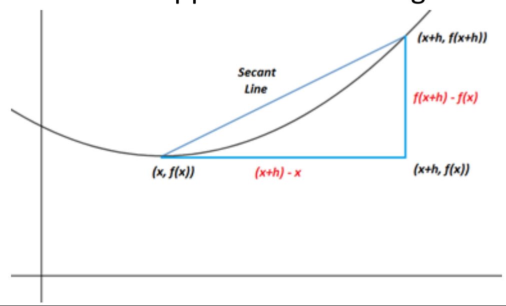
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Develop an understanding of continuity as a property of functions.

Standards	Clarifications
KY.HS.C.7 (+) Understand and use the limit definition of continuity to determine whether a given function is continuous at a specific point. **MP.2, MP.3**	If a function is continuous at $x = c$, then $\lim_{x \to c} f(x) = f(c)$.
KY.HS.C.8 (+) Define and identify different types of discontinuity – removable (hole) or non-removable (jump, asymptote) – in terms of limits. **MP.3, MP.6**	Non-removable discontinuity is identified by vertical asymptotes (infinite discontinuity) and jumps (non-agreement of left- and right-hand limits). Removable discontinuity is represented by a hole in the graph (agreement of left- and right-hand limits). Include analysis of special limits, such as $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
KY.HS.C.9 (+) Understand and apply continuous function theorems. a. Apply the Intermediate Value Theorem to continuous functions. b. Apply the Extreme Value Theorem to continuous functions. **MP.2, MP.3**	a. Intermediate Value Theorem illustration: Sarah's mom measures her height every year on her birthday. On her 10th birthday, Sarah was 48 inches tall and on her 11th birthday she measured 52 inches. Her cousin said, "You were the same height as me sometime during this year." How tall is Sarah's cousin? Justify your answer. b. The Extreme Value Theorem is contingent on the concept of continuity, but will not be addressed in sequence until the concept of derivatives and critical numbers is established.
KY.HS.C.10 (+) Communicate an understanding of continuity using precise mathematical symbols and language. **MP.2, MP.6**	Continuity on a closed interval $[a, b]$ requires continuity on the open interval (a, b), $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$

Attending to the Standards for Mathematical Practice

Students explain why a function is continuous or continuous at a point or over an interval (**MP.3**). Students use technology to examine the graph of a function and determine whether it is continuous in a given interval (**MP.5**).

Calculus-Understanding the Derivative	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Demonstrate an understanding of the derivative.	
Standards	**Clarifications**
KY.HS.C.11 (+) Define derivatives. a. Define the derivative of a function as the limit of the difference quotient. b. Understand this limit of the difference quotient can be interpreted as an instantaneous rate of change or the slope of a tangent line. **MP.5, MP.8**	The difference quotient $\frac{f(x+h)-f(x)}{h}$ represents the slope of the secant line between $(x, f(x))$ and $(x+h, f(x+h))$ as shown below. The secant line approaches the tangent line as *h* approaches 0. 
KY.HS.C.12 (+) Use average rate of change to estimate the derivative from a table of values or a graph. **MP.2, MP.8**	
KY.HS.C.13 (+) Understand the derivative as a function. **MP.2, MP.5**	Analysis of the derivative as a function implies characteristics of the original function. This analysis can be done analytically or graphically. Students move fluently between graphs of functions and derivatives using each to predict what the other would look like. (For example, positive/negative values of derivative imply increasing/decreasing of original function.) Differentiability of original function implies the derivative is a continuous function.
KY.HS.C.14 (+) Apply the definition of derivative to find derivative values and derivative functions.	Include the formal definition of a derivative: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

Standards	Clarifications
MP.2, MP.3	The alternate form of this formal definition is to calculate the derivative at one particular value.
KY.HS.C.15 (+) Explain why differentiability implies continuity yet continuity does not imply differentiability. **MP.3, MP.6**	
KY.HS.C.16 (+) Understand and apply the Mean Value Theorem, including numerical, graphical and algebraic representations. **MP.2, MP.5**	
KY.HS.C.17 (+) Understand the relationship between the concavity of a function and the sign of the second derivative. **MP.2, MP.3**	
KY.HS.C.18 (++) Understand Rolle's Theorem as a special case of the Mean Value Theorem. **MP.2, MP.3**	

Attending to the Standards for Mathematical Practice
Students use analytic strategies to determine characteristics of functions as they relate to derivatives (**MP.2**) and technology to confirm the analytic results (**MP.5**). Students use tables of values to examine the average rate of change of a function over smaller and smaller intervals, leading to the derivative as the instantaneous rate of change (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Derivatives	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply differentiation techniques.	
Standards	**Clarifications**
KY.HS.C.19 (+) Efficiently find derivatives of functions with and without technology. **MP.2, MP.5**	Functions include linear, quadratic, polynomial, exponential, logarithmic (including bases other than e), trigonometric (including inverses), square root and other root functions. Efficiently finding a derivative involves selecting the most appropriate formula. For example, the derivative of $f(x) = x/4$ can be found using the quotient rule, but it is more efficient to use the power rule to find the derivative of $f(x) = \frac{1}{4}x$.
KY.HS.C.20 (+) Understand and use derivative rules for sums, differences, products and quotients of two functions and calculate the derivative of a composite function using the chain rule. **MP2, MP.3**	
KY.HS.C.21 (+) Use implicit differentiation to find a derivative in an equation of two variables. **MP.1, MP.2**	Include a variety of functions (such as polynomial, root, logarithmic, exponential and trigonometric. Implicit differentiation can be used to explore rules such as exponential and logarithmic for bases other than e.
KY.HS.C.22 (+) Use implicit differentiation to find the derivative of the inverse of a function. **MP.2, MP.3**	
KY.HS.C.23 (+) Understand the relationship between the increasing and decreasing behavior of a function and the sign of the first derivative of the function. **MP.1, MP.2**	

Standards	Clarifications
KY.HS.C.24 (+) Use the first derivative to analyze curves and identify relative extrema. **MP.2, MP.3**	The Extreme Value Theorem is useful in optimization problems involving closed intervals since absolute extrema may occur at endpoints. (For example, consider using wire to create a circle and square of maximum value. The maximum area is obtained by using all the wire on the circle).
KY.HS.C.25 (+) Understand the relationship of concavity to the second derivative. **MP.2, MP.5**	
KY.HS.C.26 (+) Use the second derivative to find points of inflection. **MP.2, MP.3**	Points of inflection must be defined values for the function.
KY.HS.C.27 (+) Use the second derivative to analytically locate intervals on which a function is concave up, concave down or neither. **MP.2, MP.3**	
KY.HS.C.28 (+) Describe how graphical characteristics of a given function, the first derivative of that function and the second derivative of that function interrelate. **MP.2, MP.5**	
KY.HS.C.29 (+) Use derivatives to express rate of change in a variety of contexts. **MP.2, MP.4**	Examples include but are not limited to exponential growth (population) and decay (half-life), logistic growth, continuous interest and Newton's Law of Cooling.
KY.HS.C.30 (+) Use derivatives to solve a variety of problems including related rates, optimization, tangent line approximations and growth and decay models. **MP.1, MP.4**	Related rate examples include but are not limited to relating variables using the Pythagorean Theorem, relating variables using trigonometric relationships and relating variables using geometric formulas. Tangent line approximations (linearization): ● Tangent lines make good approximations of function values close to the point of tangency. ● Tangent line approximations will be an overestimate if the function is concave down. ● Tangent line approximations will be an underestimate if the function is concave up. Growth and Decay:

Standards	Clarifications
	● Use the derivative to calculate the rate of change of growth or decay at a specific time.
KY.HS.C.31 (+) Use differentiation to solve problems involving velocity, speed and acceleration. **MP.1, MP.2**	
KY.HS.C.32 (+) Understand and apply differential equations. a. Verify solutions to differential equations and use them to model real-world problems with and without technology. b. Solve separable differential equations and use them in modeling real-world problems with and without technology. **MP.1, MP.4**	Solving separable equations requires integration, however, students establish patterns for recognizing what makes a solution work. ● Students create differential equations by starting with the answers ● Students understand what makes the differential equation separable.

Attending to the Standards for Mathematical Practice
Students use derivatives to identify and describe the characteristics of a function (**MP.2, MP.6**). Contextual questions about optimal or extreme values can be identified by representing problem situations in a variety of ways (**MP.4**) and applying appropriate tools and techniques to solve the questions that are posed (**MP.1**).

Calculus-Understanding Integration	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Demonstrate understanding of a definite integral.	
Standards	**Clarifications**
KY.HS.C.33 (+) Understand the definite integral of a function over an interval. Interpret a definite integral as a limit of Riemann Sums and as net accumulation of change. **MP.2, MP.5**	
KY.HS.C.34 (+) Write a Riemann sum that represents the definition of a definite integral. **MP.2, MP.3**	
KY.HS.C.35 (+) Calculate the values of Riemann Sums over equal subdivisions to approximate definite integrals of functions represented graphically and numerically (using tables). Use left-hand sums, right-hand sums, midpoint sums and trapezoidal sums. **MP.2, MP.3**	For strictly increasing functions, a right-hand sum overestimates and left-hand sum underestimates. For strictly decreasing, the opposite is true.
KY.HS.C.36 (+) Recognize differentiation and integration as inverse operations. **MP.2, MP.8**	Integration rules can be established by reversing derivative rules. Many integration rules can be developed using implicit derivatives and/ or substitution.
KY.HS.C.37 (+) Understand how the Fundamental Theorem of Calculus connects differentiation and integration and use it to evaluate definite and indefinite integrals and to represent particular antiderivatives. **MP.2, MP.3**	Include understanding and applying the Second Fundamental Theorem of Calculus.
KY.HS.C.38 (+) Perform analytical and graphical analysis of functions using the Fundamental Theorem of Calculus. **MP.2, MP.5**	Use integration capabilities of graphing utilities to verify solutions obtained by applying the Fundamental Theorem of Calculus.

Standards	Clarifications
KY.HS.C.39 (+) Understand and use the definite integral of a function over an interval and understand its use as a mathematical tool. **MP.1, MP.2**	

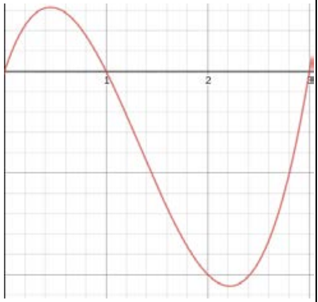
Attending to the Standards for Mathematical Practice
Students understand how graphical displays of functions (**MP.5**) and the application of limits (**MP.2**) lead to the concept of integration.

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Integration	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Apply techniques of integration.	
Standards	**Clarifications**
KY.HS.C.40 (+) Find antiderivatives of a variety of basic functions including power, exponential, logarithmic and trigonometric and apply basic properties of definite integrals. **MP.2, MP.7**	
KY.HS.C.41 (+) Use substitution techniques and change of limits of integration to find antiderivatives. **MP.2, MP.3**	Combining substitution techniques with basic rules allows for a broad spectrum of additional functions to be integrated. Substitution is the derivative equivalent of the chain rule and may be used to develop basic integration rules.
KY.HS.C.42 (+) Find particular antiderivatives given initial conditions. **MP.1, MP.2**	

Attending to the Standards for Mathematical Practice
When applying techniques of integration represent problem situations (**MP.1**)**,** students identify whether an available techniques (**MP.7**) is applicable to a given integral expression (**MP.2**)**.**

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Calculus-Applications of Integration	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Define trigonometric ratios and solve problems involving right triangles.	
Standards	**Clarifications/Illustrations**
KY.HS.C.43 (+) Model, solve and interpret applications of antiderivatives including finding area, velocity, acceleration and volume of a solid. **MP.1, MP.4**	Include area under a curve and area between two curves. Students calculate intersection points and note when functions “switch” requiring two integrals. Students calculate horizontal area. Students calculate volume using the dish, washer and shell methods.
KY.HS.C.44 (+) Apply integration to solve problems including particle motion and exponential growth and decay. **MP.1, MP.4**	Include particle motion problems, such as the velocity function below. • Where does particle change direction? • When is it moving to the left? • When is it moving to the right? • How far does it move to the left? • How far does it move to the right? • What is the displacement of the particle? • What is the total distance traveled? • If the particle started at x = 5, where is it at the end of the first 3 seconds? 
KY.HS.C.45 (+) Describe the application of integration to a variety of problems using precise mathematical language and notation. **MP.4, MP.6**	Use definite integrals to represent displacement, total distance traveled and average value of a function. Integrals are solutions to differential equations, such as $\frac{dy}{dx} = ky$ is the solution to $y = Ce^{kt}$.

Attending to the Standards for Mathematical Practice
Students recognize that a variety of applied problems can be represented using integral expressions (**MP.4**) and identify appropriate integration strategies (**MP.2**) to solve these problems (**MP.1**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Appendix A: Tables

Table 1

Common Addition and Subtraction Situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = **?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + **?** = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **?** + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = **?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - **?** = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? **?** – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[3]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = **?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + **?** = 5, 5 – 3 = **?**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Lucy have than Julie? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + **?** = 5, 5 – 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = **?**, 3 + 2 = **?**	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = **?**, **?** + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes students in grade 1 work with but do not need to master until grade 2.

[1] Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

[2] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2
Common Multiplication and Division Situations[1]

	Unknown Product	**Group Size Unknown**	**Number of Groups Unknown**
	3 × 6 = **?**	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: you need 3 lengths of string, each 6 inches long. How much string will you need all together?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: you have 18 inches of string which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: you have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with 6 apples in each row. How many apples are there? Area example: what is the area of a 3 cm by 6 cm triangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: a rectangle has area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example: a rectangle has area of 18 square centimeters. If one side is 6 cm long, how long is the side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: a rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: a rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue? Measurement example: a rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	***a × b = ?***	***a × ? = p and p ÷ a = ?***	***? × b =p and p ÷ b = ?***

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: the apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3

Properties of Operations

The variables *a*, *b* and *c* stand for arbitrary numbers in a given number system.
The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	$(a \times b) \times c = a\,x\,(b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4
Properties of Equality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5
Properties of Inequality

The variables *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c then a > c
If a > b, then b < a
If a > b, then –a < -b
If a > b, then a ± c > b ± c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a ÷ c > b ÷ c
If a > b and c < 0, then a ÷ c < b ÷ c

Table 6
Fluency Standards across All Grade Levels

Grade	Coding	Fluency Standards
K	**KY.K.OA.5**	Fluently add and subtract within 5.
1	**KY.1.OA.6**	Fluently add and subtract within 10.
2	**KY.2.OA.2** **KY.2.NBT.5**	Fluently add and subtract within 20. Fluently add and subtract within 100.
3	**KY.3.OA.7** **KY.3.NBT.2**	Fluently multiply and divide within 100. Fluently add and subtract within 1000.
4	**KY.4.NBT.**	Fluently add and subtract multi-digit whole numbers using an algorithm.
5	**KY.5.NBT.5**	Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.
6	**KY.6.NS.2** **KY.6.NS.3** **KY.6.EE.2**	Fluently divide multi-digit numbers using an algorithm. Fluently add, subtract, multiply and divide multi-digit decimals using an algorithm for each operation. Write, read and evaluate expressions in which letters stand for numbers.
7	**KY.7.NS.1d** **KY.7.NS.2c**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.
8	**KY.8.EE.7**	Solve linear equations in one variable.
Algebra	**KY.HS.A.2** **KY.HS.A.19**	Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. Solve quadratic equations in one variable.
Functions	**KY.HS.F.4** **KY.HS.F.8**	Graph functions expressed symbolically and show key features of the graph both with and without technology (i.e., computer, graphing calculator).★ Understand the effects of transformations on the graph of a function.
Geometry	**KY.HS.G.21** **KY.HS.G.11c** **KY.HS.G.12c**	Use coordinates to justify and prove simple geometric theorems algebraically. Use similarity criteria for triangles to solve problems in geometric figures. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Appendix B: Writing and Review Committees

The writing team, composed of current mathematics teachers, represented both rural and urban settings – including representation from several different regions of the state. While these teachers taught a variety of courses and grade levels throughout their careers, the selected committee members were currently teaching courses related to the standards development process: K-8 mathematics, Algebra I, Geometry, Algebra II and Calculus. Additionally, the selected writers served in many roles in their schools, mathematics community and a wide variety of professional organizations. To ensure fidelity to the standards, the writing committee provided feedback at all stages of the development process. The writing and review committee members listed below represented Kentucky's best as evidenced by their countless qualifications.

Mathematics Advisory Panel (AP) Members

Jenny Bay-Williams, University of Louisville
Sherry Bertram, McCracken County
Amanda Boyle , Pulaski County
Teresa Chinn, Ohio County
Al Cornish, Community Member
Maggie Doyle, Clark County
Angela England, Boone County
Tiffany Goble, Clark County
Jeani Gollihue, Russell Independent
Nick Harris, Madison County
Matthew Hawkins, Hart County
Deron Hitch, Campbell County
Ted Hodgson, Northern Kentucky University
Patricia Hubbard, Mason County
Chandra (Love) Welte, Grant County
Amy Newsome, Pike County
Marsha Reddick, Taylor County
Jeff Richie, Knott County
Alissa Riley, Christian County
Molly Imes Ross, Calloway County
Beverly Rutledge, Carter County
Reva Slone, Johnson County
Joanna Stevens, Lincoln County
Amy Stokes-Levine, Jefferson County
Jonathan Thomas, University of Kentucky

Diana Taylor, Community Member
Casey Watson, Jefferson County

<u>Standards and Assessments Review and Development Committee (RDC) Members</u>

Krista Althauser, Eastern Kentucky University
Christine Bickett, Bullitt County
Scott Castle, Community Member
Bonny Davenport, Henderson County
Jodi Grannis, Fleming County
Dan McGee, Northern Kentucky University
Amanda Mullins, Scott County
Jeanne Reed, Johnson County
Amy Rose, Fulton County
Forrest Spillman, Somerset Independent
Adam Tilley, Community Member
Chad White, Bracken County