

# KENTUCKY MATHEMATICS TOOLKIT TO SUPPORT STUDENTS WITH DISABILITIES



Kentucky Department of  
**EDUCATION**

# OSEEL

Office of Special Education & Early Learning



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## Table of Contents

State Systemic Improvement Plan.....	3
Guide to Using the Kentucky Mathematics Innovation Practice Profile.....	3
Measuring the Eight Mathematics Teaching Practices in the Classroom.....	6
<i>Appendix A</i> .....	10
Kentucky Mathematics Innovation Practice Profile.....	10
<i>Appendix B</i> .....	19
Kentucky Mathematics Innovation Tool (KMIT) .....	19

## **State Systemic Improvement Plan**

The Office of Special Education Programs (OSEP) developed an accountability framework, called Results Driven Accountability (RDA). RDA seeks to balance the educational results and functional outcomes for children with disabilities with the compliance requirements of the Individuals with Disabilities Education Act (IDEA). This includes states' development of a State Systemic Improvement Plan (SSIP), a comprehensive multi-year plan as part of its State Performance Plan and Annual Performance Report (SPP/APR).

Kentucky's State Identified Measurable Result (SiMR) for the SSIP is, "To increase the percentage of students with disabilities performing at or above proficient in middle school math, specifically at the 8th grade level, with emphasis on reducing novice performance, by providing professional learning, technical assistance and support to elementary and middle school teachers around implementing, scaling and sustaining Positive Behavioral Interventions and Supports (PBIS) and evidence-based practices in math."

To meet the goal of the SiMR, the Kentucky Department of Education's Office of Special Education and Early Learning (OSEEL) works with a representative group of regions, districts, and schools within the education system, called a Transformation Zone. The Transformation Zone focuses on supporting teacher practice to improve mathematics outcomes for students with disabilities.

Through the SSIP, resources have been developed to support effective mathematics instruction. The Kentucky Mathematics Toolkit to Support Students with Disabilities includes an overview of each resource and tips for use in schools and districts.

## **Guide to Using the Kentucky Mathematics Innovation Practice Profile**

Through the SSIP process, the [Eight Mathematics Teaching Practices from the National Council of Teachers of Mathematics](#) have been identified by a representative team of stakeholders from across the state as a quality standard for mathematics instruction to improve educational outcomes for students with disabilities. The stakeholder team co-created the Kentucky Mathematics Innovation Practice Profile ([Appendix A](#)) to operationalize the eight mathematics teaching practices by providing examples of use in the classroom. Practice one is pictured below for reference.

**Appendix A**

**Kentucky Mathematics Innovation Practice Profile**



**Purpose:** Based on the eight mathematics teaching practices from the National Council of Teachers of Mathematics (NCTM), this document operationalizes quality math instruction in the classroom. It can be used to support the implementation of any mathematics innovation.

*Consistently*—engages in the behavior each time there is an opportunity to demonstrate  
*Inconsistently*—misses opportunities to demonstrate the behavior

**Mathematics Teaching Practice 1: Establish mathematics goals to focus learning**

Definition	Accomplished Use	Developmental Use	Ineffective Use
<i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i>	1. Teacher <i>consistently</i> ...  a. establishes clear and detailed goals that indicate the mathematics students are learning.  b. explains how the mathematical goals contribute to enduring understandings.  c. uses these goals to adjust instruction.  d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals.  e. uses a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.	1. Teacher <i>inconsistently</i> ...  a. establishes clear and detailed goals that indicate the mathematics students are learning.  b. explains how the mathematical goals contribute to enduring understandings.  c. uses these goals to guide decision making.  d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals.  e. uses a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.	1. Teacher...  a. does not establish goals or does not clearly define goals for mathematical understandings.  b. fails to explain how the mathematical goals contribute to enduring understandings.  c. fails to use goals to guide decision making.  d. does not use concrete and semi-concrete (representational) activities or does not make any connections between activities and the mathematical goals.  e. does not use a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.

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The practice profile can be used to support effective mathematics instruction by informing:

1. Content for training or professional learning communities (PLCs).
2. Instructional strategies to incorporate within lessons to ensure students are engaged in rigorous and authentic activities to foster a deeper understanding of mathematics.
3. Self-reflection of practice in the classroom.
4. Conversations during coaching sessions to support quality mathematics instruction.

The practice profile can be paired with the practice guides below. Each guide includes instructional strategies and processes to support students with disabilities and reflection questions to consider when designing lessons.

1. [Establish mathematics goals to focus learning;](#)
2. [Implement tasks that promote reasoning and problem-solving;](#)
3. [Use and connect mathematical representations;](#)
4. [Facilitate meaningful mathematics discourse;](#)
5. [Pose purposeful questions;](#)
6. [Build procedural fluency from conceptual understanding;](#)
7. [Support productive struggle in learning mathematics;](#)
8. [Elicit and use evidence of student thinking.](#)

## MATHEMATICS TEACHING PRACTICE 5: Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Strategy and Process for Students with Disabilities	Digital Learning Experience
<p>Preparing for responses to be a question or modeling thinking about a question</p> <ul style="list-style-type: none"> <li>• During planning, work the task and anticipate where questions might be needed to assess or advance thinking.</li> <li>• Select potential questions to have ready.</li> <li>• Use cheat sheets or anchor charts to organize questions and keep them available during teaching and learning.</li> <li>• Pose questions "just in time" as determined by purpose, student need or learning intentions/success criteria.</li> <li>• Challenge students to ask questions to one another while thinking through a task; provide cheat sheets or discussion starters to support effective questioning skills.</li> </ul> <p>Standards for Mathematical Practice (SMPs): Look-Fors and Question Stems provides more information.</p>	<p>Considerations for the digital learning experience</p> <ul style="list-style-type: none"> <li>• Place questions in the chat box "just in time."</li> <li>• Ask students to respond to questions so others can see their responses using digital tools.</li> </ul>
<p>Scaffolding questions</p> <ul style="list-style-type: none"> <li>• Anticipate where students might get stuck.</li> <li>• Prepare questions to ask students if they get stuck.</li> <li>• Consider question cards to give students as a non-verbal strategy.</li> <li>• Differentiate the questions you present to students based upon individual needs.</li> </ul> <p>Sample questions:</p> <ul style="list-style-type: none"> <li>• Does this remind you of a problem you have solved before?</li> <li>• What resources are in your toolkit that might be useful?</li> </ul>	<p>Considerations for the digital learning experience</p> <ul style="list-style-type: none"> <li>• Place questions in the chat box "just in time" to scaffold students who are stuck.</li> <li>• Have a "help" link in your electronic assignment for students to click when needed. The link can take them to one or more scaffolded questions customized for that lesson.</li> </ul>
<p>Using metacognitive questioning</p> <ul style="list-style-type: none"> <li>• Model using metacognitive questions during think-alouds.</li> <li>• Give students access to these questions to help them develop the habit of thinking about their thinking.</li> </ul> <p>Metacognitive questions:</p> <ul style="list-style-type: none"> <li>• What is the problem asking?</li> <li>• How might I get started?</li> <li>• Does this remind me of a problem I have experienced before?</li> <li>• How might I prove my answer is correct?</li> </ul>	<p>For digital learning experiences, consider encouraging students to ask metacognitive questions by having a digital "Think About Your Thinking" button on a website or within a virtual platform that reminds them to ask themselves the metacognitive questions.</p> <p>Another way to support students in learning how to use metacognitive questions is to create a video modeling a similar problem and asking metacognitive questions during the think-aloud.</p>

*Training and coaching are integral to aiding teacher growth. According to a study conducted by Joyce and Showers (2002), effective training paired with coaching can result in 95% of participants using the new skills in the classroom.*

TRAINING COMPONENTS	OUTCOMES		
	% of Participants who Demonstrate Knowledge, Demonstrate New Skills in a Training Setting and Use new Skills in the Classroom		
	Knowledge	Skill Demonstration	Use in the Classroom
Theory and Discussion	10%	5%	0%
...+Demonstration in Training	30%	20%	0%
...+ Practice & Feedback in Training	60%	60%	5%
...+ Coaching in Classroom	95%	95%	95%

Joyce, B., & Showers, B. (2002). Student achievement through staff development (3rd ed.). Alexandria, VA: ASCD.

# Measuring the Eight Mathematics Teaching Practices in the Classroom

The Kentucky Mathematics Innovation Tool (KMIT) ([Appendix B](#)) is a research-based classroom walkthrough tool used to inform the system of support for teachers (e.g., training, coaching, etc.) on the [Eight Mathematics Teaching Practices](#). While the Kentucky Mathematics Innovation Practice Profile is a training and coaching tool, the KMIT measures use of the practices in the classroom. The data gathered from the KMIT can be compiled in aggregate for a school or district and used with teams of teachers during PLCs to determine training and coaching needs on the practices.

**Appendix B**  
Kentucky Mathematics Innovation Tool (KMIT)



Operational Definitions of Instruction Behavior: Definitions with -AND- must include all components to score 2 points. Definitions with -OR- must include at least one component, without missed opportunities, to score 2 points. Time of Lesson (circle one): Beginning, Middle, End Brief Description of Instructional Approach (e.g., whole group, small group, centers, number talk):	CATEGORY RATING 2 = Fully Observed 1 = Partially Observed 0 = Not Observed NA = No Opportunity to Observe
<b>1. Establish mathematics goals to focus learning.</b> <i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i> Identifies and communicates goals aligned to the standards that are specific to the lesson and clear to students (not simply stating and/or posting a standard); -AND- Communicates why the learning goal is important; -AND- Revisits goals throughout the lesson.	Score 2 1 0 NA
<b>2. Implement tasks that promote reasoning and problem solving.</b> <i>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</i> Uses engaging, high-cognitive-demand tasks including those that arise from home, community and society (Principles to Actions page 18); -AND- Implements tasks that are approached and solved in multiple ways; -AND- Uses how, why, and/or when questions to prompt students to share or reflect on their reasoning.	Score 2 1 0 NA
<b>3. Use and connect mathematical representations.</b> <i>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</i> Allocates instructional time for students to use, discuss, and make connections among representations (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -AND- Encourages students to use representations in making sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -OR- Uses representations to help students make sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations).	Score 2 1 0 NA

The purpose of the KMIT observation is to assess the quality of systems and supports available to assist teachers' use of the Eight Mathematics Teaching Practices. It is not an assessment of teachers or a teacher evaluation tool.

## Using the KMIT

**What:** The KMIT is a 20-minute observation of instruction based on the [Eight Mathematics Teaching Practices](#) from the National Council of Teachers of Mathematics.

**Who:** The KMIT can be used with teachers instructing mathematics at any level that agree to participate in classroom observations. All participating teachers should receive same-day feedback and the opportunity for training and coaching on the Eight Mathematics Teaching Practices.

**How:** Classroom observations should be scheduled in advance with teachers. Observers should conduct observations in a way that is not disturbing or distracting to classroom activities. On the same day of the observation, positive feedback is provided to teachers (e.g., email to all teachers, personal post-it notes, etc.). Observation results should be used for training and

coaching purposes only and should occur on a routine basis to inform supports (e.g., three times per year, monthly, quarterly).

**Items:** Each KMIT item includes a Mathematics Teaching Practice, the definition of the item and the behaviors that should be observed. Some items include “AND” behaviors, while others have “OR” behaviors.

**Scoring:** The KMIT is scored using the following scale:

- 2 = Fully Observed;
- 1 = Partially Observed;
- 0 = Not observed, missed opportunity;
- NA = Not observed, no opportunity.

A 2-point score means the teacher:

- Demonstrated **all “and”** components of the operational definition and any **subsequent** uses of each “and” component was used **correctly** and **without missed opportunities**;
- Demonstrated **any “or”** components **without any missed opportunities**.

Examples:

<b>1. Establish mathematics goals to focus learning.</b> <i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i>	Score <b>2</b> 1 0 NA
Identifies and communicates goals aligned to the standards that are specific to the lesson and clear to students (not simply stating and/or posting a standard); -AND-	2
Communicates why the learning goal is important; -AND-	2
Revisits goals throughout the lesson.	2

<b>5. Pose purposeful questions.</b> <i>Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</i>	Score <b>2</b> 1 0 NA
Uses strategies to ensure every student is thinking of responses (including wait time and other accountability strategies); -AND-	2
Asks questions that require students to explain and or connect mathematical ideas, representations, or strategies; -AND-	2
Asks questions that build on, but do not take over or funnel, student thinking.	NA

A 1-point score means the teacher:

- Demonstrated **at least one** of the “and” components but **did not demonstrate all** of the “and” components;
- Demonstrated **all “and”** components **but also missed** opportunities for using any components.

Examples:

<b>3. Use and connect mathematical representations.</b> <i>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</i>	Score 2 <b>1</b> 0 NA
Allocates instructional time for students to use, discuss, and make connections among representations (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -AND-	2
Encourages students to use representations in making sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -OR- Uses representations to help students make sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations).	0

<b>4. Facilitate meaningful mathematical discourse.</b> <i>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</i>	Score 2 <b>1</b> 0 NA
Provides every student the opportunity to share, listen to, honor, and critique the reasoning of others; -AND-	2
Makes explicit connections among student approaches and reasoning.	1

<b>6. Build procedural fluency from conceptual understanding.</b> <i>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</i>	Score 2 <b>1</b> 0 NA
Encourages, and provides time for, students to use their own strategies or algorithms; -AND-	1
Makes explicit connections among concrete, semi-concrete, and abstract representations; -OR- Asks students to compare different strategies or algorithms; -OR- Asks when a strategy or algorithm is appropriate.	0

A 0-point score means the teacher:

- Missed the opportunity to implement an essential function. For “*and*” components, if the observer noted more “0” scores than “1” or “2” scores, the score would be a zero.

Examples:

<b>2. Implement tasks that promote reasoning and problem solving.</b> <i>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</i>	Score 2 1 <b>0</b> NA
Uses engaging, high-cognitive-demand tasks including those that arise from home, community, and society ( <i>Principles to Actions</i> page 18); -AND-	1
Implements tasks that are approached and solved in multiple ways; -AND-	0
Uses how, why, and/or when questions to prompt students to share or reflect on their reasoning.	0

<b>7. Support productive struggle in learning mathematics.</b> <i>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</i>	Score 2 1 <b>0</b> NA
Provides time for students to grapple with tasks; -AND-	0
Discusses the value of making multiple attempts and persistence; -OR- Facilitates discussion on mathematical error(s), misconception(s), or struggle(s) and how to overcome them; -OR- Asks questions that scaffold students’ thinking without stepping in to do the work for them.	NA

An NA score means the teacher did not have the opportunity to implement the item.

Example:

<b>8. Elicit and use evidence of student thinking</b> <i>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</i>	Score 2 1 0 <b>NA</b>
Identifies evidence of student understanding or misconceptions, attending to important representations or processes; -OR- Provides students with opportunities to reflect on their work to capture understanding or misconceptions.	NA



**Observer Training:** Contact [veronica.sullivan@education.ky.gov](mailto:veronica.sullivan@education.ky.gov) in the Office of Special Education and Early Learning to learn more about training available on the KMIT.

## Appendix A

# Kentucky Mathematics Innovation Practice Profile



**Purpose:** Based on the eight mathematics teaching practices from the National Council of Teachers of Mathematics (NCTM), this document operationalizes quality math instruction in the classroom. It can be used to support the implementation of any mathematics innovation.

*Consistently*—engages in the behavior each time there is an opportunity to demonstrate

*Inconsistently*—misses opportunities to demonstrate the behavior

### Mathematics Teaching Practice 1: Establish mathematics goals to focus learning

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i></p>	<p>1. Teacher <i>consistently</i> ...</p> <ul style="list-style-type: none"> <li>a. establishes clear and detailed goals that indicate the mathematics students are learning.</li> <li>b. explains how the mathematical goals contribute to enduring understandings.</li> <li>c. uses these goals to adjust instruction.</li> <li>d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals.</li> <li>e. uses a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.</li> </ul>	<p>1. Teacher <i>inconsistently</i> ...</p> <ul style="list-style-type: none"> <li>a. establishes clear and detailed goals that indicate the mathematics students are learning.</li> <li>b. explains how the mathematical goals contribute to enduring understandings.</li> <li>c. uses these goals to guide decision making.</li> <li>d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals.</li> <li>e. uses a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.</li> </ul>	<p>1. Teacher...</p> <ul style="list-style-type: none"> <li>a. does not establish goals or does not clearly define goals for mathematical understandings.</li> <li>b. fails to explain how the mathematical goals contribute to enduring understandings.</li> <li>c. fails to use goals to guide decision making.</li> <li>d. does not use concrete and semi-concrete (representational) activities or does not make any connections between activities and the mathematical goals.</li> <li>e. does not use a concrete—semi-concrete (representational)—abstract learning progression to meet these goals.</li> </ul>

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## Mathematics Teaching Practice 2: Implement tasks that promote reasoning and problem solving

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</i></p>	<p>2. Teacher <i>consistently</i>...</p> <p>a. provides opportunities for students to explore and solve problems that build on and extend their current mathematical understanding.</p> <p>b. selects tasks that provide multiple entry points.</p> <p>c. poses tasks that require a high level of cognitive demand.</p> <p>d. provides opportunities for students to discuss tasks without taking over student thinking.</p> <p>e. selects tasks that allow students to make sense of and solve using varied approaches and strategies.</p>	<p>2. Teacher <i>inconsistently</i> ...</p> <p>a. provides opportunities for students to explore and solve problems that attempt to build on and extend their current mathematical understanding.</p> <p>b. selects tasks that provide multiple entry points.</p> <p>c. poses tasks that require a high level of cognitive demand.</p> <p>d. provides opportunities for students to discuss tasks or sometimes take over student thinking.</p> <p>e. selects tasks that allow students to make sense of and solve using those approaches and strategies presented in class.</p>	<p>2. Teacher...</p> <p>a. does not provide the opportunity for students to explore and solve problems or provides opportunities that do not build on and extend their current mathematical understanding.</p> <p>b. does not select tasks that provide multiple entry points.</p> <p>c. poses tasks that require a low level of cognitive demand.</p> <p>d. does not provide opportunities for students to discuss tasks or often takes over student thinking.</p> <p>e. does not select tasks that allow students to make sense of and solve using varied approaches and strategies or provides one specific strategy or approach for students to use to solve tasks.</p>

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**Mathematics Teaching Practice 3: Use and connect mathematical representations**

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</i></p>	<p>3. Teacher <i>consistently</i>...</p> <p>a. selects tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems.</p> <p>b. allocates instructional time for students to use, discuss, and make connections among representations.</p> <p>c. introduces and models a variety of representations that provide students with choices for making sense of problems.</p> <p>d. asks students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.</p>	<p>3. Teacher <i>inconsistently</i> ...</p> <p>a. selects tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems.</p> <p>b. allocates instructional time for students to use, discuss, and make connections among representations.</p> <p>c. introduces and models a variety of representations that provide students with choices for making sense of problems.</p> <p>d. asks students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.</p>	<p>3. Teacher...</p> <p>a. does not select tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems.</p> <p>b. does not allocate sufficient instructional time for students to use, discuss, and make connections among representations.</p> <p>c. introduces and models a single representation that does not provide students with choices for making sense of problems or introduces and models representations not aligned to the learning goal.</p> <p>d. does not ask students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.</p>

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*Inconsistently*—misses opportunities to demonstrate the behavior

## Mathematics Teaching Practice 4: Facilitate meaningful mathematical discourse

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</i></p>	<p>4. Teacher <i>consistently</i> ...</p> <p>a. engages students in purposeful discourse of mathematical ideas, reasoning, and approaches.</p> <p>b. facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>c. makes connections among student approaches and reasoning clearly and explicitly.</p> <p>d. selects and sequences “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion.</p>	<p>4. Teacher <i>inconsistently</i> ...</p> <p>a. engages students in purposeful discourse of mathematical ideas, reasoning, and approaches, or regular discourse may not always be purposeful.</p> <p>b. facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>c. makes connections among student approaches and reasoning or make vague connections among student approaches and reasoning.</p> <p>d. selects and sequences “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion.</p>	<p>4. Teacher...</p> <p>a. does not engage students in discourse of mathematical ideas, reasoning, and approach.</p> <p>b. does not facilitate discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>c. does not make connections among student approaches and reasoning.</p> <p>d. does not select and sequence “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion.</p>

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## Mathematics Teaching Practice 5: Pose purposeful questions

Definition	Accomplished Use	Developmental Use	Ineffective Use
<i>Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</i>	<p>5. Teacher <i>consistently</i> ...</p> <p>a. advances student understanding by asking questions that build on, but do not take over or funnel, student thinking.</p> <p>b. assesses thinking by asking questions that require explanation and justification.</p> <p>c. asks intentional questions that make mathematical ideas and relationships more visible for students.</p>	<p>5. Teacher <i>inconsistently</i> ...</p> <p>a. advances student understanding by asking questions that build on, but do not take over or funnel, student thinking.</p> <p>b. assesses thinking by asking questions that require explanation and justification.</p> <p>c. asks intentional questions that make the structure of mathematics more accessible for student learning.</p>	<p>5. Teacher ...</p> <p>a. asks questions that take over or funnel student thinking.</p> <p>b. asks questions that simply gather information and do not probe thinking or require explanation and justification.</p> <p>c. does not ask questions that make the structure of mathematics more accessible for student learning.</p>

*Consistently*—engages in the behavior each time there is an opportunity to demonstrate

*Inconsistently*—misses opportunities to demonstrate the behavior

## Mathematics Teaching Practice 6: Build procedural fluency from conceptual understanding

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</i></p>	<p>6. Teacher <i>consistently</i> ...</p> <p>a. provides students with opportunities to use their own reasoning strategies and methods for solving problems.</p> <p>b. asks students to discuss and explain why the procedures that they are using work.</p> <p>c. connects student-generated strategies and methods to more efficient procedures as appropriate.</p> <p>d. uses an intentional concrete-semi-concrete (representational)—abstract sequence to develop student understanding of the structure within procedural fluency.</p> <p>e. provides students with regular opportunities for practice of procedures that are brief, engaging, and purposeful.</p>	<p>6. Teacher <i>inconsistently</i> ...</p> <p>a. provides students with opportunities to use their own reasoning strategies and methods for solving problems.</p> <p>b. asks students to discuss and explain why the procedures that they are using work.</p> <p>c. connects student-generated strategies and methods to more efficient procedures as appropriate.</p> <p>d. uses a concrete-semi-concrete (representational)—abstract sequence to develop some student understanding of the structure within procedural fluency.</p> <p>e. provides students with opportunities for practice of procedures that are brief, engaging, and purposeful.</p>	<p>6. Teacher ...</p> <p>a. does not provide students with opportunities to use their own reasoning strategies and methods for solving problems or provide a preferred strategy.</p> <p>b. does not ask students to discuss and explain why the procedures that they are using work.</p> <p>c. does not connect student-generated strategies and methods to more efficient procedures as appropriate.</p> <p>d. does not develop student understanding of the structure within procedural fluency, i.e., by using automaticity practice too soon.</p> <p>e. provides students with opportunities for rote practice of procedures that are not purposeful.</p>

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## Mathematics Teaching Practice 7: Support productive struggle in learning mathematics

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</i></p>	<p>7. Teacher <i>consistently</i> ...</p> <p>a. gives students time to struggle with tasks.</p> <p>b. asks questions that purposefully scaffold students’ thinking without stepping in to do the work for them.</p> <p>c. helps students develop a growth mindset by facilitating discussions on mistakes, misconceptions and struggles, and acknowledging students for their efforts (rather than an innate ability).</p> <p>d. provides specific feedback that supports students with moving learning forward to develop mathematical ideas and/or persevering to solve problems.</p>	<p>7. Teacher <i>inconsistently</i> ...</p> <p>a. gives students time to struggle with tasks.</p> <p>b. asks questions that purposefully scaffold students’ thinking without stepping in to do the work for them.</p> <p>c. helps students develop a growth mindset through facilitating discussions on mistakes, misconceptions, and struggles.</p> <p>d. provides specific feedback that supports students with moving learning forward to develop mathematical ideas and/or persevering to solve problems.</p>	<p>7. Teacher ...</p> <p>a. does not give students time to struggle with tasks.</p> <p>b. does not ask questions that purposefully scaffold students’ thinking or step in to do the work for them.</p> <p>c. contributes to students’ fixed mindsets through a lack of discussions around mistakes, misconceptions, and struggles.</p> <p>d. does not provide specific feedback to support students with moving learning forward with mathematical ideas and/or persevering to solve problems.</p>



*Consistently*—engages in the behavior each time there is an opportunity to demonstrate  
*Inconsistently*—misses opportunities to demonstrate the behavior

## Mathematics Teaching Practice 8: Elicit and use evidence of student thinking

Definition	Accomplished Use	Developmental Use	Ineffective Use
<p><i>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</i></p>	<p>8. Teacher <i>consistently</i> ...</p> <p>a. elicits appropriate evidence of student understanding at strategic points during instructional time.</p> <p>b. uses this evidence to adjust instruction to support and extend student learning.</p> <p>c. provides students opportunities to reflect on their work to capture understanding and misconceptions.</p>	<p>8. Teacher <i>inconsistently</i> ...</p> <p>a. elicits appropriate evidence of student understanding during instructional time.</p> <p>b. uses this evidence to adjust instruction to support and extend student learning.</p> <p>c. provides students opportunities to reflect on their work to capture understanding and misconceptions.</p>	<p>8. Teacher ...</p> <p>a. does not gather evidence of student understanding during instructional time or may only gather evidence at the end of instructional time.</p> <p>b. does not use this evidence to adjust instruction to support and extend student learning.</p> <p>c. does not provide students opportunities to reflect on their work to capture understanding and misconceptions.</p>

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**Appendix B**

**Kentucky Mathematics Innovation Tool (KMIT)**



<p><b>Operational Definitions of Instruction Behavior:</b>                      Definitions with -AND- must include all components to score 2 points.                      Definitions with -OR- must include at least one component, without missed opportunities, to score 2 points.  <b>Time of Lesson</b> (circle one): Beginning, Middle, End  <b>Brief Description of Instructional Approach</b> (e.g., whole group, small group, centers, number talk):</p>	<p><b>CATEGORY RATING</b>                      2 = <i>Fully Observed</i>                      1 = <i>Partially Observed</i>                      0 = <i>Not Observed</i>                      NA = <i>No Opportunity to Observe</i></p>
<p><b>1. Establish mathematics goals to focus learning.</b>  <i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i></p>	<p><b>Score</b>                      2 1 0 NA</p>
<p>Identifies and communicates goals aligned to the standards that are specific to the lesson and clear to students (not simply stating and/or posting a standard); -AND-</p>	
<p>Communicates why the learning goal is important; -AND-</p>	
<p>Revisits goals throughout the lesson.</p>	
<p><b>2. Implement tasks that promote reasoning and problem solving.</b>  <i>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</i></p>	<p><b>Score</b>                      2 1 0 NA</p>
<p>Uses engaging, high-cognitive-demand tasks including those that arise from home, community and society (Principles to Actions page 18); -AND-</p>	
<p>Implements tasks that are approached and solved in multiple ways; -AND-</p>	
<p>Uses how, why, and/or when questions to prompt students to share or reflect on their reasoning.</p>	
<p><b>3. Use and connect mathematical representations.</b>  <i>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</i></p>	<p><b>Score</b>                      2 1 0 NA</p>
<p>Allocates instructional time for students to use, discuss, and make connections among representations (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -AND-</p>	
<p>Encourages students to use representations in making sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations); -OR-                      Uses representations to help students make sense of mathematics (physical models/concrete, pictures/semi-concrete, symbols/abstract, verbal, real-life/contextual situations).</p>	

<b>4. Facilitate meaningful mathematical discourse.</b> <i>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</i>	Score 2 1 0 NA
Provides every student the opportunity to share, listen to, honor, and critique the reasoning of others; <b>-AND-</b>	
Makes explicit connections among student approaches and reasoning.	
<b>5. Pose purposeful questions.</b> <i>Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</i>	Score 2 1 0 NA
Uses strategies to ensure every student is thinking of responses (including wait time and other accountability strategies); <b>-AND-</b>	
Asks questions that require students to explain and or connect mathematical ideas, representations, or strategies; <b>-AND-</b>	
Asks questions that build on, but do not take over or funnel, student thinking.	
<b>6. Build procedural fluency from conceptual understanding.</b> <i>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</i>	Score 2 1 0 NA
Encourages, and provides time for, students to use their own strategies or algorithms; <b>-AND-</b>	
Makes explicit connections among concrete, semi-concrete, and abstract representations; <b>-OR-</b>	
Asks students to compare different strategies or algorithms; <b>-OR-</b>	
Asks when a strategy or algorithm is appropriate.	
<b>7. Support productive struggle in learning mathematics.</b> <i>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</i>	Score 2 1 0 NA
Provides time for students to grapple with tasks; <b>-AND-</b>	
Discusses the value of making multiple attempts and persistence; <b>-OR-</b>	
Facilitates discussion on mathematical error(s), misconception(s), or struggle(s) and how to overcome them; <b>-OR-</b>	
Asks questions that scaffold students' thinking without stepping in to do the work for them.	
<b>8. Elicit and use evidence of student thinking</b> <i>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</i>	Score 2 1 0 NA
Identifies evidence of student understanding or misconceptions, attending to important representations or processes; <b>-OR-</b>	
Provides students with opportunities to reflect on their work to capture understanding or misconceptions.	

Notes/Comments:	<p><b>1. Establish mathematics goals to focus learning.</b>  <i>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</i></p>
	<p><b>2. Implement tasks that promote reasoning and problem solving.</b>  <i>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</i></p>
	<p><b>3. Use and connect mathematical representations.</b>  <i>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</i></p>
	<p><b>4. Facilitate meaningful mathematical discourse.</b>  <i>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</i></p>
	<p><b>5. Pose purposeful questions.</b>  <i>Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</i></p>
	<p><b>6. Build procedural fluency from conceptual understanding.</b>  <i>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</i></p>
	<p><b>7. Support productive struggle in learning mathematics.</b>  <i>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</i></p>
	<p><b>8. Elicit and use evidence of student thinking.</b>  <i>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</i></p>

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