Operations and Algebraic Thinking: Number Puzzles

Grade 4

Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

Number Puzzles   Fourth Grade

> **This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.**

Mathematical goals

This lesson is intended to help you assess how well students are able to use "clues" about numbers including: factors, multiples, prime, composite, square, even, odd, etc. In particular, this unit aims to identify and help students who have difficulties with:

- understanding the difference between primes and composites.
- understanding the difference between factors and multiples.

Kentucky Academic Standards

This lesson involves mathematical content in the standards from across the grade, with emphasis on:
<u>Grade 4 Operations and Algebraic Thinking</u>
Cluster: Gain familiarity with factors and multiples.

This lesson involves a range of mathematical practices from the standards, with emphasis on:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Introduction

This lesson unit is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work with a partner on a collaborative discussion task to use the number puzzles and clue cards to match the mystery number match multiple representations of quantities and equations. As they do this, they interpret the cards' meanings and begin to link them together. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.
- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Materials required

- Each individual student will need two copies of the worksheet *Number Puzzles.*
- Each pair of students will need a packet of each set of the following card sets copied and cut up before the lesson:
    - Card Set A: CLUES
    - Card Set B: TASK CARDS
    - Card Set C: MYSTERY NUMBER
- A page of Hundreds Boards and Blank Graph Paper have been provided for students who may need this support. It is not required that each student get these copies.

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the Formative Assessment Lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Number Puzzle*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.

It is important that students answer the question without assistance, as far as possible.

If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on below. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues - Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't find all factor pairs for a given number.	• How can you make sure you haven't left out any factor pairs for a number? • How can you use rectangular arrays to decide if you have all the factor pairs? How can you use a factor tree to decide if you have all the factor pairs? How could you make a chart to decide if you have all the factor pairs?
Student doesn't have a strategy to find the factors of a number.	• Can you think of a way that we can model this problem? • If you were given square tiles, could you model a rectangle with 10 tiles? How does this rectangle's width and length compare to the factors of 10?
Conceptual understanding of prime numbers. Student doesn't realize that a prime number will have only one factor pair, only one rectangular array, and it will not have a factor tree if it is prime. Student should also recognize the only EVEN prime number is 2.	• How many other rectangular arrays can you find for each number? What do you notice about the number of rectangular arrays possible for each number? Which number(s) do not have a factor tree? Why not? • Why are more of the prime numbers odd than even?
Not understanding that if there are an odd number of factors for a number, the number is a square number.	• Which rectangular arrays are actually squares? How many total factors do each of your numbers have? How can you tell if a number will have a square as one of its arrays?

Suggested lesson outline

Whole-class interactive introduction (10 minutes)

Give each student a mini-whiteboard, a marker, and an eraser.

Explain to the class that in the lesson they will be working with factors and multiples.

Now project Slide 2 of the projector resource onto the board (or draw the diagrams on the board).

Ask students to write on their mini-whiteboards the answers to questions such as the following. Each time, ask students to explain their method.

Let's look at Number Puzzle 1.

There is more than one way to solve this number puzzle. Can you think what one of them might be? [Record student responses on the board.]

Let's look closer at some strategies. [Students may have suggested these and you can look closer at the strategy at that time.]

> **Puzzle 1**
> 1- This number of tiles will make a rectangle 3 tiles wide.
> 2- This number of tiles will make a rectangle 4 tiles wide.
> 3- This number is greater than 20.
> 4- This number is less than 30.
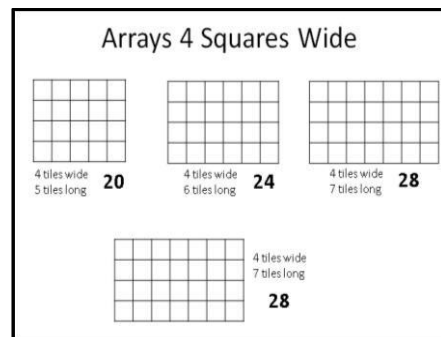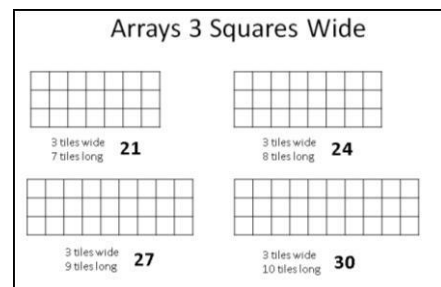
ARRAYS: [Slides 3 – 5]

Using between 20 and 30 squares, how many arrays can you make that will be 3 tiles wide? [Allow students to use graph paper or square tiles to make as many arrays as they can.]

Arrays 3 Squares Wide

3 tiles wide 7 tiles long **21**
3 tiles wide 8 tiles long **24**
3 tiles wide 9 tiles long **27**
3 tiles wide 10 tiles long **30**

Now using between 20 to 30 squares, how many arrays can you make that will be 4 tiles wide?

Are any of your arrays the same?

Could this be your mystery number?

Does it meet the criteria for all the clues?

Arrays 4 Squares Wide

4 tiles wide 5 tiles long **20**
4 tiles wide 6 tiles long **24**
4 tiles wide 7 tiles long **28**
4 tiles wide 7 tiles long **28**

[The use of a hundred's board may be beneficial to some students to "mark off" numbers that do not fit the criteria for the number puzzles. All of the solutions are for numbers of 100 or less.]

Let's look at the numbers between 20 and 30.

Which of these numbers are multiples of 3?

Which are multiples of 4?

Are there any common multiples that could be the mystery number?
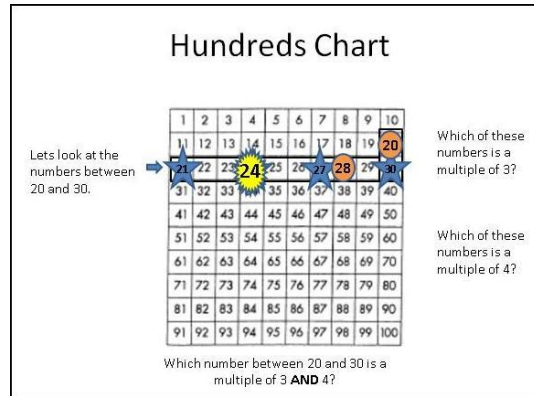
Collaborative activity (30 minutes)

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the pre-assessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

Introduce the collaborative activity carefully:

> *Today we are going to do some more work on common multiples and factors. I want you to work as a team. Take turns placing clue cards with a number puzzle to match a mystery number. Each time you do this; explain your thinking clearly to your partner. If your partner disagrees with your match then challenge him or her to explain why. It is important that you both understand why each card is matched with another one. There is a lot of work to do today and you may not all finish. The important thing is to learn something new, so take your time. When you finish with the first card set, raise your hand and I'll come and ask you to explain your thinking before moving on to a new card set.*

The teacher's task during the partner work is to make notes of student approaches to the task, and to support student problem solving through questioning.

Give each pair of students **Card Set A:** *Clues*, **Card Set B:** *Task Cards*, and **Card Set C:** *Mystery Number*.

Explain to students how they should work together:

> *Place the Clue Cards and Mystery Numbers face up so each partner can see the clues and numbers.*
>
> *Choose one Number Puzzle to solve at a time.*
>
> *Find the four clues that match the Number Puzzle your pair is solving. Divide the Clue Cards among the partners.*
>
> *Solve for the Mystery Number by using a strategy your pair is comfortable using. Each partner member should help solve for the mystery.*
>
> *After your pair has reached a consensus, match the Number Puzzle Card to the Mystery Number Card.*
>
> *Set these two cards aside and place the Clue Cards face up with the other clues.*
>
> *Choose another Number Puzzle to solve.*

To remind students about these instructions, you could use slide 7, *Instructions for Working Together*. While students are working, you have two tasks: to find out about students' work and to support their reasoning.

Find out about students' work
As you move around the room listen to students' explanations.

Note any difficulties that emerge for more than one pair of students; these can be discussed later in the lesson.

Your tasks during the partner work are to make a note of student approaches to the task, and to support student problem solving.

You can then use this information to focus a whole-class discussion toward the end of the lesson. In particular, notice any common mistakes. For example, students may know how to find a mystery number represented by two of the clues, but have difficulty finding the answer when all four clues are used.

Make a note of student approaches to the task

Try not to make suggestions that move students toward a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors as they find matches for all the cards.

Wrap up the lesson when students are completely satisfied with their own work by having them glue or tape their cards together in solution sets that include number puzzles and mystery cards that match.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each pair to visit another pair's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

> *If you are staying at your desk, be ready to explain the reasons for your pair's matches.*

> *If you are visiting another pair of students, write your card placements on a piece of paper. Go to another pair's desk and check to see which matches are different from your own.*

> *If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.*

> *When you return to your own desk, you need to consider, as a pair, whether to make any changes to your work.*

Students may now want to make changes.

Whole-Class Discussion (10 minutes)
Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Select students or partners who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each pair shares, highlight the connections between strategies.

Ask: How does student A's strategy connect to student B's strategy?

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions the students come up with.

Possible questions to ask:

> *Which cards were easiest/hardest to match? Why?*
> *What might be a different way to explain?*
> *Did anyone do the same or something different?*
> *How would you explain in words your model?*

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Number Puzzles*, as well as a second blank copy of the task.

> *Look at your original responses and think about what you have learned this lesson.*
>
> *Using what you have learned, try to improve your work.*

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solution

Assessment Task: *Number Puzzles*

For questions 1 and 2 student responses will vary depending on how they solve multiplication problems. Be sure that the responses have correct interpretations of each model.

Question 1:

# of shifts	1	2	3	4	6	8	12	24
# of hours	24	12	8	6	4	3	2	1

Question 2: The dimensions of the sides of the rectangles should be the factors for 32. Students should drawn rectangles with the following dimensions:

1x32, 2x16, 4x8 (or the reverse 32x1, 16x2, 8x4)

Question 3: Numbers with exactly three factors would be square numbers. Square numbers between 12 and 40 are: 16, 25, & 36. Only 25 has just three factors.

Question 4: only one because prime numbers only have two factors – one and itself.

For example, 7 is prime and has only factors 1 and 7 so the only array would be 1x7 (or 7x1).

Question 5: All prime numbers are odd, except for the number 2. So ODDS have more prime numbers.

Collaborative Activity Solutions:	
NUMBER PUZZLE 1: **24**	NUMBER PUZZLE 5: **100**
Clue 1 – A – This number will make a rectangle 3 tiles wide.	Clue 1 – E – Add the digits of this number and the sum is odd.
Clue 2 – B – This number has exactly 8 factors.	Clue 2 – F – This number is a multiple of 5.
Clue 3 – C – 1 factor of this number is 4.	Clue 3 – G – This number has an odd number of factors.
Clue 4 – D – This number is not a multiple of 5 or 7.	Clue 4 – H – This number is a square number.
NUMBER PUZZLE 2: **45**	NUMBER PUZZLE 6: **47**
Clue 1 – A – This number will make a rectangle 3 tiles wide.	Clue 1 – E – Add the digits of this number and the sum is odd.
Clue 2 – E – Add the digits of this number and the sum is odd.	Clue 2 – I – This number is not even and is less than 50.
Clue 3 – I – This number is not even and is less than 50.	Clue 3 – J – The product of the digits of this number is greater than 20.
Clue 4 – F – This number is a multiple of 5.	Clue 4 – D – This number is not a multiple of 5 or 7.
NUMBER PUZZLE 3: **36**	NUMBER PUZZLE 7: **30**
Clue 1 – A – This number will make a rectangle 3 tiles wide.	Clue 1 – A – This number will make a rectangle 3 tiles wide.
Clue 2 – H – This number is a square number.	Clue 2 – E – Add the digits of this number and the sum is odd.
Clue 3 – C – 1 factor of this number is 4.	Clue 3 – B – This number has exactly 8 factors.
Clue 4 – G – This number has an odd number of factors.	Clue 4 – F – This number is a multiple of 5.
NUMBER PUZZLE 4: **72**	NUMBER PUZZLE 8: **29**
Clue 1 – A – This number will make a rectangle 3 tiles wide.	Clue 1 – D – This number is not a multiple of 5 or 7.
Clue 2 – E – Add the digits of this number and the sum is odd.	Clue 2 – I – This number is not even and is less than 50.
Clue 3 – C – 1 factor of this number is 4.	Clue 3 – E – Add the digits of this number and the sum is odd.
Clue 4 – D – This number is not a multiple of 5 or 7.	Clue 4 – Blank (write a fourth clue) ***Answers will vary.***

These materials were adapted from *Connected Mathematics: Prime Time,* and *Investigations in Number Data & Space: Mathematical Thinking.*

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the [Math Assessment Project](#).

NUMBER PUZZLES

Name_____

1. A restaurant is open 24 hours a day. The manager wants to divide the day into work shifts of equal length. Show the different ways this can be done. The shifts should not overlap, and all shifts should be a whole number of hours long.

2. Sammie's Game Store wants to rent a space of 32 square units. Find all the possible ways Sammie can arrange the squares. How are the rectangles you found related to the factors of 32?

3. Lewis has chosen a mystery number. His number is larger than 12 and smaller than 40 and it has exactly three factors. What could his number be?

4. How many rectangles can you build with a prime number of square tiles?

5. Which group of numbers – evens or odds- has more prime numbers? Why?

A – This number will make a rectangle 3 tiles wide.	**F – This number is a multiple of 5.**
B – This number has exactly 8 factors.	**G – This number has an odd number of factors.**
C – 1 factor of this number is 4.	**H – This number is a square number.**
D – This number is not a multiple of 5 or 7.	**I – This number is not even and is less than 50.**
E – Add the digits of this number and the sum is odd.	**J – The product of the digits of this number is greater than 20.**
Blank ClueCard	

NUMBER PUZZLE 1: Clue 1 – A Clue 3 – C Clue 2 – B Clue 4 – D	**NUMBER PUZZLE 5:** Clue 1 – E Clue 3 – G Clue 2 – F Clue 4 – H
NUMBER PUZZLE 2: Clue 1 – A Clue 3 – I Clue 2 – E Clue 4 – F	**NUMBER PUZZLE 6:** Clue 1 – E Clue 3 – J Clue 2 – I Clue 4 – D
NUMBER PUZZLE 3: Clue 1 – A Clue 3 – C Clue 2 – H Clue 4 – G	**NUMBER PUZZLE 7:** Clue 1 – A Clue 3 – B Clue 2 – E Clue 4 – F
NUMBER PUZZLE 4: Clue 1 – A Clue 3 – C Clue 2 – E Clue 4 – D	**NUMBER PUZZLE 8:** Clue 1 – D Clue 3 – E Clue 2 – I Clue 4 – Blank (write a fourth clue)

36	45
24	30
72	47
100	29

HUNDREDS BOARDS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

| 1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Blank Graph Paper
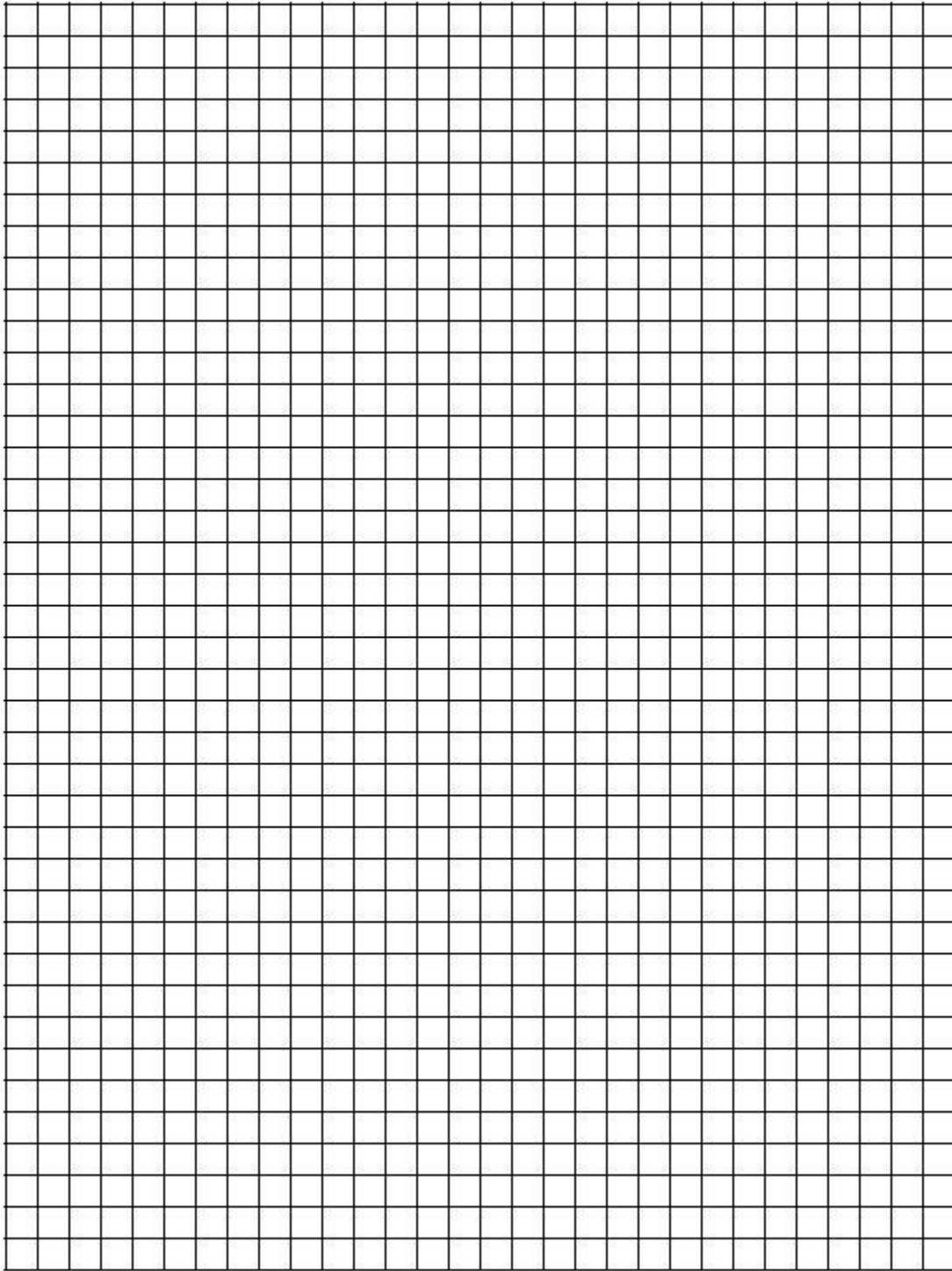
Number Puzzles

4th Grade
Formative Assessment Lesson

What is the Mystery Number?
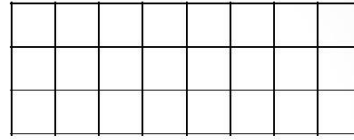
Puzzle 1

1-This number of tiles will make a rectangle 3 tiles wide.

2-This number of tiles will make a rectangle 4 tiles wide.

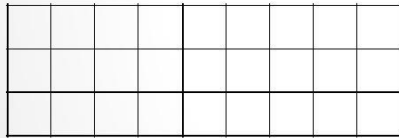3- This number is greater than 20.
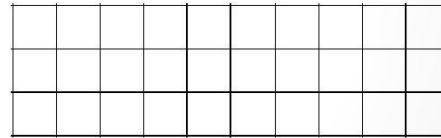
4- This number is less than 30.

Arrays 3 Squares Wide
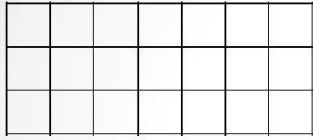
3 tiles wide
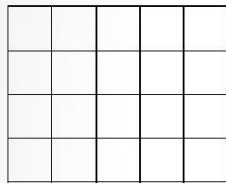7 tiles long **21**

3 tiles wide
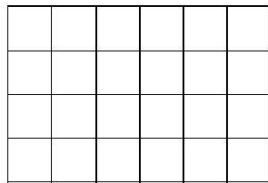8 tiles long **24**

3 tiles wide
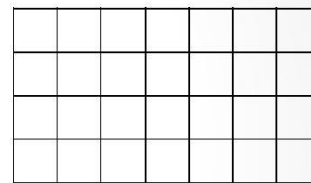9 tiles long **27**

3 tiles wide
10 tiles long **30**

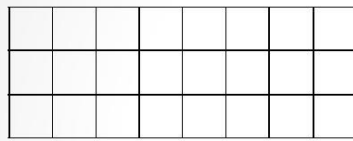# Arrays 4 Squares Wide

4 tiles wide
5 tiles long **20**
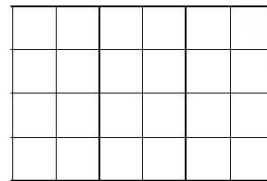
4 tiles wide
6 tiles long **24**

4 tiles wide
7 tiles long **28**

Are Any Arrays the Same?

3 tiles wide
8 tiles long **24**

4 tiles wide
6 tiles long **24**

Could 24 be our mystery number?

Hundreds Chart

Lets look at the numbers between 20 and 30.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Which of these numbers is a multiple of 3?

Which of these numbers is a multiple of 4?

Which number between 20 and 30 is a multiple of 3 **AND** 4?

Instructions for Working Together

Place the Clue Cards and Mystery Numbers face up so each partner can see the clues and numbers. Solve one Number Puzzle at a time.

Find the four clues that match the Number Puzzle your pair is solving. Divide the Clue Cards among the pair of students.

Solve for the Mystery Number by using a strategy your pair is comfortable with using. Each partner should help solve for the mystery.

After your pair has reached a consensus, match the Number Puzzle Card to the Mystery Number Card.

Set these two cards aside and place the Clue Cards face up with the other clues.

Choose another Number Puzzle to solve.