Domain: Geometry

Title: Two-Dimensional Shapes, Angles, and Symmetry

Grade 4

Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

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(2019)

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This lesson is intended to help you assess how well students are able to:

- Identify and sort quadrilaterals based on their properties and attributes
- Identify and classify angles and identify the angles in two-dimensional figures
- Identify and sort two-dimensional figures based on the absence or presence of characteristics such as parallel or perpendicular lines and angles of a specified size (acute, obtuse, right)
- Identify right triangles as a category for classification
- Identify lines of symmetry and classify symmetrical figures
- Draw lines of symmetry

Kentucky Academic Standards:

This lesson involves *mathematical content standards* within the grade, with emphasis on:

Grade 4: Geometry

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

KY.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence of absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **MP.7**

KY.4.G.3 Identify lines of symmetry.

a. Recognize a line of symmetry for a two-dimensional figure.

b. Identify line-symmetric figures and draw lines of symmetry. **MP. 5, MP.7**

This lesson involves a range of *Standards for Mathematical Practice,* with emphasis on:

MP.3 Construct viable arguments and critique the reasoning of others.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review/analyze their responses and create questions for students to consider/answer in order to improve their solutions.
- After a whole class introduction, students work collaboratively in pairs on a card matching activity.
- Students work with a partner or in small groups on collaborative discussion tasks. Throughout their work, students justify and explain their decisions to their peers.
- Toward the end of the lesson there is a whole class discussion.
- Students return to their original assessment tasks or a similar task and try to improve their own responses.

Materials required

Each pair of students will need the following resources:

- One set of shape cards (cut apart and laminated if possible)
- Word Cards Set 1 (cut apart and laminated if possible)
- Word Cards Set 2 (cut apart and laminated if possible)
- Fine tip dry erase marker
- 2 sheets of chart paper with 3 circle Venn Diagrams already drawn and labeled
- Tracing paper
- Blank Cards
- Pencil

Teacher materials:

- 2 pieces of whole group chart paper with 3 circle Venn Diagrams already drawn and labeled

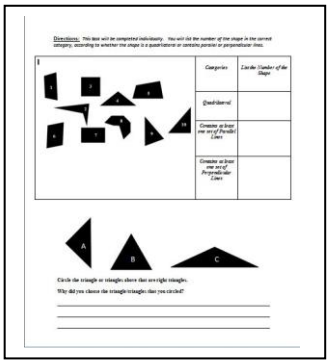
Time needed

Approximately 15 minutes before the lesson for the individual assessment task, one 55-60 minute lesson and 10 minutes for a follow-up lesson for students to revisit individual assessment task. Times given are approximated. All students need not complete all sets of cards activities. Exact timings will depend on the needs of the class.

Before the Lesson

Assessment task: Shapes, Angles, and Symmetry (15 Minutes)

Have students do this task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will be able to target your help more effectively in the follow-up lesson and develop a series of questions that will help to focus your students' thinking. Depending on your class you can have them do it all at once or in small groups (they should still work individually.)



Give each student a copy of the assessment task: Shapes, Angles, and Symmetry (2 Pages)
Teacher says: *Today we are going to work on a task "Shapes, Angles, and Symmetry". This task is to help me see ways that I can help you if you are having any problems with classifying two-dimensional shapes, angles, and lines of symmetry. If you are not sure about all of your answers, it is okay. We are going to do an activity that will help you improve. This task will be completed individually.*

- *You will list the number of the shape in the correct category, according to whether the shape is a quadrilateral or has parallel or perpendicular lines.*
- *Then you will identify if there is a right triangle by circling it, and provide reasoning for why you circled a given triangle.*
- *You will then list the number of the shape in the correct category, according to whether or not the shape has acute, obtuse, or right angles.*
- *The last part of this task requires you to draw as many lines of symmetry as possible and provide reasoning for why you drew those lines of symmetry.*

It is important that the students are allowed to answer the questions without your assistance, as far as possible. If students struggle to get started, ask questions that help them understand what they are being asked to do, but do not do the problem for them. See the *Common Issues* table.

Students should not worry too much if they do not understand or cannot do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently.

Assessing students' responses

Collect students' responses to the task. Make notes about what their work reveals about their current levels of understanding and their different problem solving approaches. Partner/group students with others who displayed similar errors/misconceptions on the pre-assessment task.

We suggest that you do not score student's work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions in the *Common Issues* table may serve as examples. These questions have been drawn from commonly identified student misconceptions. This series of questions will be used in the introductory part of the lesson on the following day.

We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student or
- display a small list of questions on the board that will be of help to the majority of students or
- provide question cards for each pair of students based on their common misconception

Below is a list of common issues and questions/prompts that may be written on individual tasks, during the introductory portion of the lesson, on the board, or asked during the collaborative activity to help students clarify and extend their thinking.

Common Issues:	**Suggested questions and prompts:**
Students have a difficult time getting started.	• What are the directions? • What do you know about acute, obtuse, and right angles? • What do you know about quadrilaterals, parallel and perpendicular lines? • What do you know about right triangles? • What do you know about symmetry?
Students may struggle to be able to classify all four sided figures as quadrilaterals because they all may not be squares or rectangles, which they are most familiar with.	• What do you know about quadrilaterals? • What do all quadrilaterals have in common? • Do you know of any other words that contain "quad"?
Students may limit the classification of shapes to only one category, not understanding that they may fit into multiple categories.	• What attributes do the shapes have in common? • Do the shapes have more than one attribute in common?
Students may not recognize that any lines that meet at a right angle are perpendicular.	• What is the definition of perpendicular lines? • What makes lines perpendicular?
Students may think that most rectangles have the same number of lines of symmetry that squares do.	• What strategies can we use to determine how many lines of symmetry a shape has?
Students may struggle to recognize that the size of a shape does not determine the measure of its angles.	• How can you determine the measure of an angle?
Students may struggle to recognize that symmetry is just another attribute that some shapes have.	• What attributes can a shape have?

	• If a shape is symmetrical, can it have other attributes as well?
Students may struggle to recognize that some shapes have one line of symmetry, multiple lines of symmetry, or no lines of symmetry.	• How can you determine is shape has a line of symmetry? • Do you have any tools that could help you determine if a shape has any lines of symmetry?
Students may think that a shape appears to be symmetrical, but in reality it is not.	• How can you determine if the shape has a line of symmetry?

Suggested lesson outline

Whole Class Introduction (15 minutes)

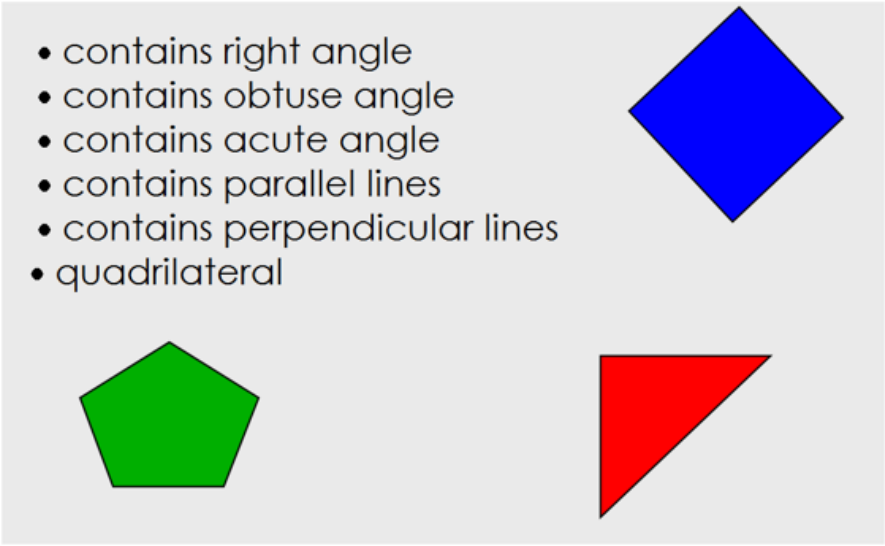
As a result of the pre-assessment given on the previous day, you, the teacher, should have developed a series of questions based on misconceptions or common issues that you identified. During this whole class introduction, students may use individual whiteboards, math notebook or journals, or just paper and pencil.

Using the questions developed, display the following graphic or similar graphic on the board. Ask students to focus on the green shape.

Teacher says: *On your whiteboard, list the attributes you notice about the green shape.*

Provide time to respond in writing. Have students share with a shoulder partner then call on students to share their findings aloud. Ask questions to draw on misconceptions and/or misunderstandings. Students do not have to have correct answers or answer all questions, they just need to communicate, in writing and/or verbally, their thoughts and ideas to the series of questions that were developed. It's not the time to reteach, just listen as students talk and share. Try not to judge student responses. Move to the red shape doing the same process, and finally the blue shape as time allows. If you run short on time, it's fine to only do the green and red shape.

Graphic:



Collaborative Activity: (20-30 minutes)

Strategically partner students based on pre-assessment data into groups of two or three students. With larger groups, some students may not fully engage in the task. Group students with others who displayed similar errors/misconceptions on the pre-assessment task.

Explain to students how they are to work collaboratively:

Teacher says: *You are now going to work together to place cards on the Venn diagram according to how you think the cards should be classified. The Venn diagram is already drawn and labeled for you on a sheet of chart paper. For this activity you will use the Shape Cards and Word Card Set 1. When you start this activity you will shuffle the cards and divide them as evenly as possible between all members of your group. During this activity there will be three categories, one in each circle of the Venn diagram. These three categories are as follows: 1) Shapes containing at least one acute angle 2) Shapes containing at least one obtuse angle 3) Shapes containing at least one right angle. Each time you place a card on the chart, explain your thinking clearly to your partners. If your partners disagree with your placement then challenge them to explain why. It is important that you all understand why each card is placed where it is.There is a lot of work to do today and you may not all finish. The important thing is to learn something new, so take your time.*

Give each group 8-12 minutes

Teacher says: *After you have sorted all of the cards, or when time is up, remove all of the cards from Venn diagram. You will now repeat this activity on the other piece of chart paper, using the same Shape Cards and Word Card Set 2. You will now be sorting the cards into three different categories. The three new categories are as follows: 1) Quadrilaterals 2) Shapes containing at least one set of parallel lines 3) Shapes containing at least one set of perpendicular lines. Remember to follow the same rules and instructions for sorting the cards as you did before in the first activity. There is a lot of work to do, and you may not all finish. The important thing is to learn something new, so take your time.*

Give each group 8-12 minutes

Teacher says: *Lastly you will once again clear the cards from the Venn diagram. You will not need the Venn Diagrams any more, just the cards. Shuffle the cards and give each person in the group an equal amount of cards. You will then each take turns turning over cards. When one person turns their card over, they will have to tell the other group members whether or not the shape on the card is symmetrical. If you say the card is symmetrical, you will also have to say how many lines of symmetry the shape has. You will then tell the other people in your group why you think the shape is or is not symmetrical. They will then tell you whether or not they agree with you and why. If you all agree that the shape is symmetrical, put that card in a pile. If you all agree that the card is not symmetrical, place it in another pile. Once you have taken enough turns to use all of your cards, take the cards that were determined to have symmetrical shapes and use your dry erase markers to draw all the lines of symmetry on the shapes as possible. Try to come to an agreement as a group about how many lines of symmetry each shape should have. If you are having trouble determining if a shape has lines of symmetry, you may use the tracing paper to help you. There is a lot of work to do, and you may not all finish. The important thing is to learn something new, so take your time.*

Give each group 8-12 minutes

The purpose of this structured work is to encourage students to engage with each other's explanations and take responsibility for each other's understanding. During the task, as you walk around the room, observing and questioning, ask each group of students to go tape one or two of their cards on the group chart paper in the same category in which they placed the cards on their own chart paper.

Make a note of student approaches to the task

Your task during the small group work is to listen and watch students work - make notes of student approaches to the task, and to support student problem solving. As you monitor the work, listen to the discussion and ask questions to help students understand concepts and clarify misconceptions.

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, if you have noticed, either on the pre-assessment, or during the tasks, that students are repeatedly only classifying a shape into just one category, you may use questioning to guide students to understand that shapes often belong to two or all three categories.

Support student problem solving

Try not to make suggestions that move students toward a particular approach to the task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning. Encourage students to explain their reasoning carefully.

If one student has placed a particular card on the Venn diagram, challenge their partners to provide an explanation.

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and engineer a quick whole class discussion.

Taking two class periods to complete all activities

If you have to divide the lesson into two class periods, you may want to have a way for students to save the work they have done with the cards and Venn Diagrams. You may have each group tape the cards down on the Venn diagram. You may choose to have them do this even if you are not dividing up the class period just to use as a visual during the class discussion.

Sharing Work

If at any points students are struggling to make progress while they are completing any of the sorting tasks, you may give them the opportunity to walk around the room and observe how the other groups are sorting their cards. The purpose of this is not for the students to just copy from each other, but to look and get ideas from other groups. Students may ask other groups about their reasoning for why they categorized specific cards where they did.

Extension activities

If the students have sorted all of the cards correctly, they may notice that there are some overlapping categories that don't have any cards in them. For the first task there are two categories that doesn't have any cards. These categories are obtuse/right and obtuse/acute/right. Students that recognize that these categories have no cards should be directed to use the blank cards provided to draw shapes that could fit into these categories. In the second sorting task, there are no cards that fit into the quadrilateral/perpendicular category. Again, students that recognize that this category has no cards should be directed to use the blank cards to draw shapes that could fit into that category. For the third task, students that have correctly identified all of the lines of symmetry on the shape cards should be directed to draw shapes on the blank shape cards that have symmetry.

Whole-class discussion (10 minutes)

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions students came up with. The purpose of this discussion is to explore the processes involved in a range of different approaches. The aim is to get students to understand and share their reasoning, not just checking that everyone found the correct matches. Students will have the opportunity to share some of the work they have done in a whole-class setting.

As the teacher, you will have two pieces of whole group chart paper displayed on the board. These pieces of chart paper should have the same Venn Diagrams on them that the students had on their chart paper. When several students have had the opportunity to place one or two cards on the group chart paper, you will give them the opportunity to share their reasoning for why they placed the cards where they did. Other students are permitted to ask questions and make comments. The idea here is for students to construct a viable argument for why they placed the cards where they did and for their classmates to critique the reasoning of others. You may ask some students that have categorized cards incorrectly if they mind to share why they sorted the card incorrectly. This can be used as a teaching tool for the class to come up with reasoning for why the card was categorized incorrectly. It is important to get a student's permission before displaying their incorrect work.

It will be important that you focus on right triangles during your class discussion. Structure the discussion so that you draw out the following important concepts regarding right triangles.
1) Right triangles have to have one right angle.
2) Right triangles can only have a right angle and acute angles.

It will also be important that you focus on symmetry during your class discussion. Structure the discussion so that you draw out the following important concepts regarding right triangles.
1) Some shapes have symmetry and some shapes do not.
2) Symmetry is just another attribute that some shapes have.
3) Shapes either have no lines of symmetry, one line of symmetry or multiple lines of symmetry.

To focus your students during the whole class discussion, refer to the common issues chart. Use the questions which reflect the greatest need(s) of your students.

Ask students:

- *Which cards were difficult to place?*
- *Which cards did you and your partner struggle to place?*
- *Which cards were easy to place?"*

These questions can start great conversations if time is limited at the end.

Improving individual solutions to the assessment task (10 minutes)

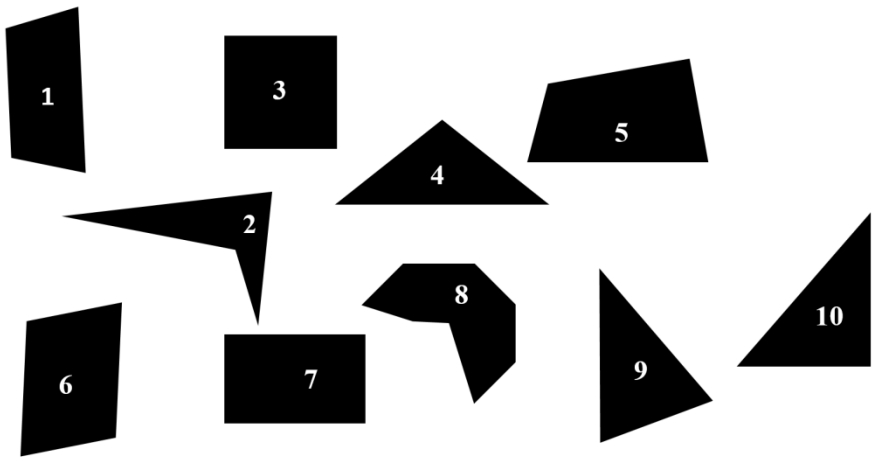
Give the students their original pre-assessment task.

Teacher says: *Think about what you have learned during this lesson. Using what you have learned try to improve your work. You can use the blank assessment task to make any changes that you wish, taking into consideration what you have learned during this lesson.*

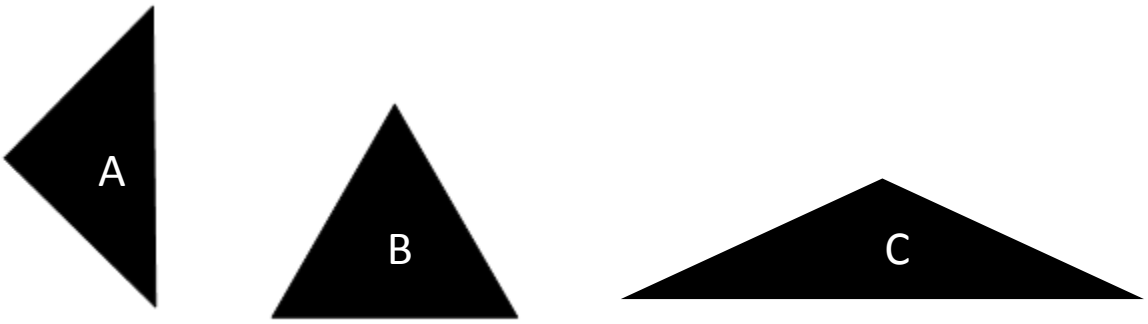
Assessment: Shapes, Angles, and Symmetry

Name: _____

Directions: ***This task will be completed individually. You will list the number of the shape in the correct category, according to whether the shape is a quadrilateral or contains parallel or perpendicular lines.***



Categories	*List the Number of the Shape*
Quadrilateral	
Contains at least one set of Parallel Lines	
Contains at least one set of Perpendicular Lines	



Circle the triangle or triangles above that are right triangles.

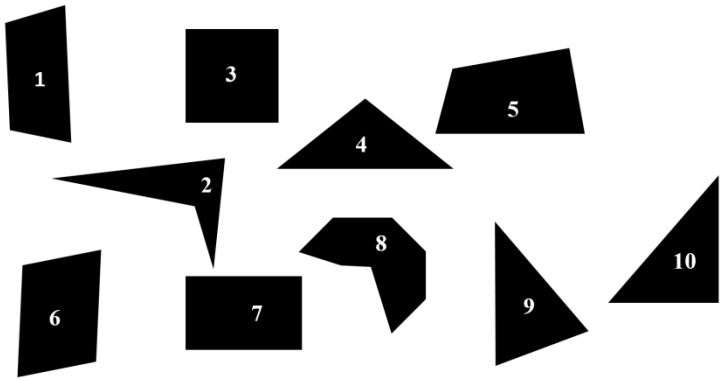
Why did you choose the triangle/triangles that you circled?

_____

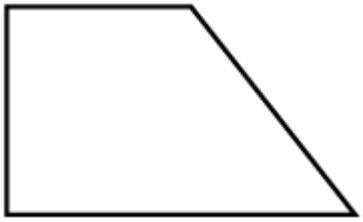
_____

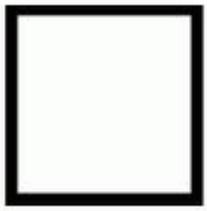
_____

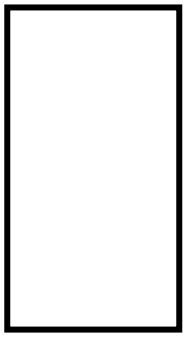
Directions: ***List the number of the shape in the correct category, according to whether or not the shape has acute, obtuse, or right angles.***



Categories	*List the Number of the Shape*
Contains at least one Acute Angle	
Contains at least one Obtuse Angle	
Contains at least one Right Angle	







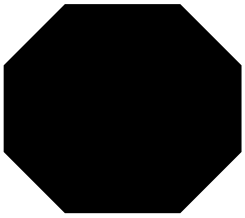
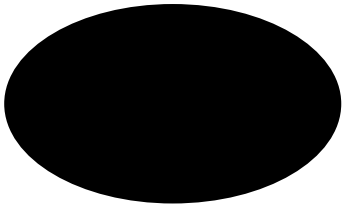
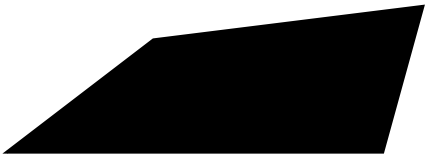
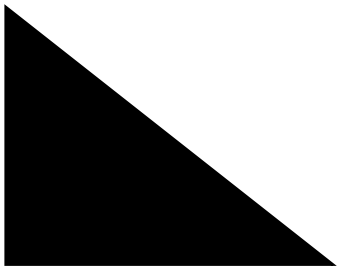
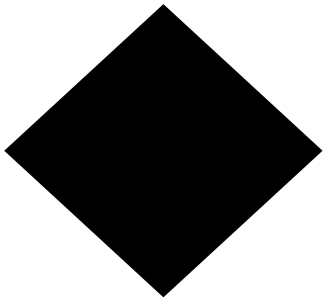
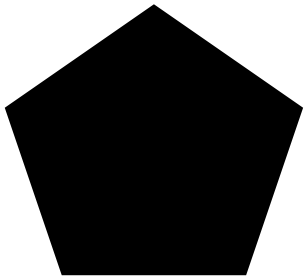
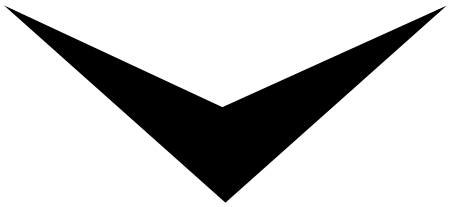
In the shapes above, draw as many lines of symmetry as possible. Explain how you know that you drew lines of symmetry.

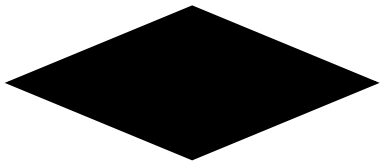
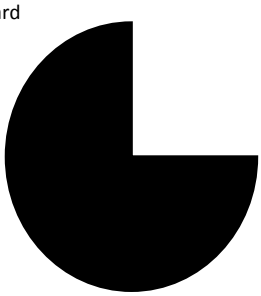
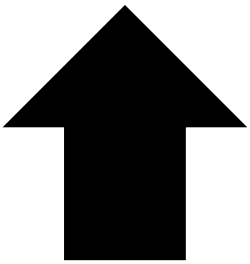
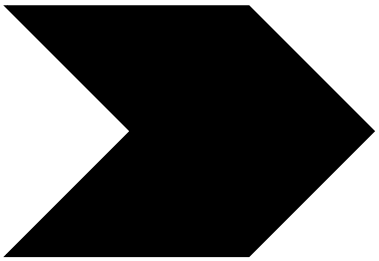
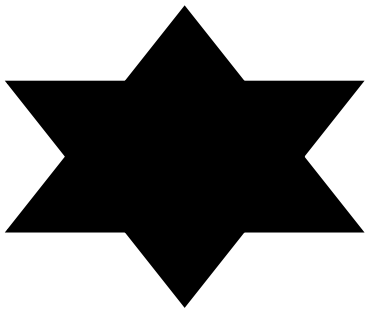
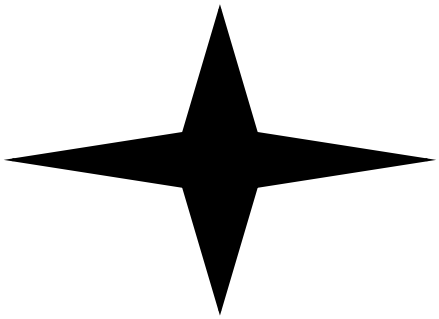
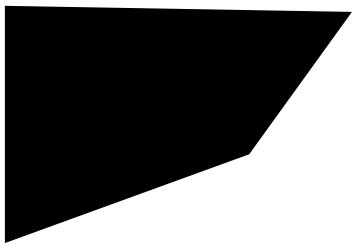
_____

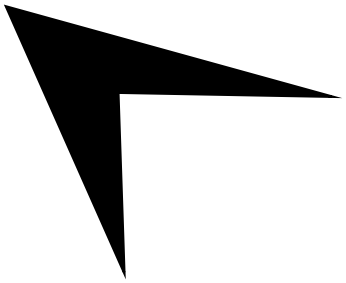
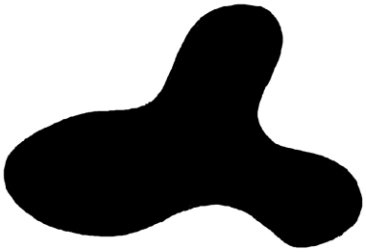
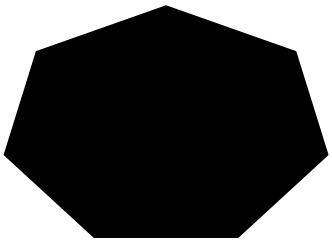
_____

_____

_____

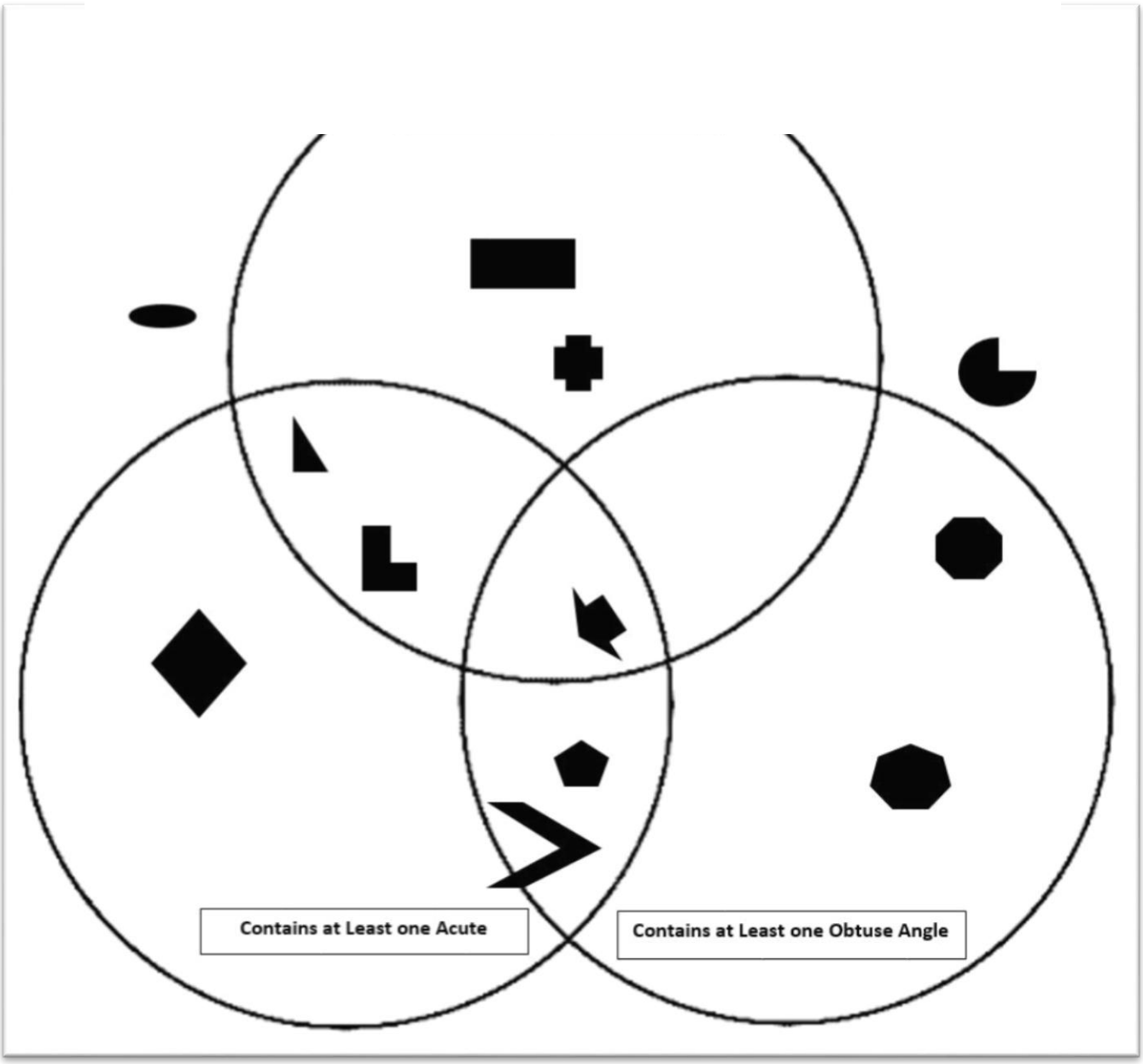
Shape Card 	Shape Card	
Shape Card	Shape Card 	Shape Card 
Shape Card	Shape Card 	
Shape Card 	Shape Card 	Shape Card 

Shape Card 	Shape Card	Shape Card
Shape Card 	Shape Card 	Shape Card 
Shape Card 	Shape Card 	Shape Card
Shape Card 	Shape Card	Shape Card

Shape Card 	Shape Card 	Shape Card 

Venn Diagram Labels

Sort # 1 **Shapes Containing at Least One Acute Angle**	Sort # 1 **Shapes Containing at Least One Obtuse Angle**
Sort # 1 **Shapes Containing At Least One Right Angle**	Sort # 2 **Quadrilaterals**
Sort # 2 **Shapes Containing at Least One Set of Parallel Lines**	Sort # 2 **Shapes Containing at Least One Set of Perpendicular Lines**

Word Card Set 1 **An angle that measures 90°**	Word Card Set 1 **An angle that measures less than 90°**	Word Card Set 1 **An angle that measures more than 90°**
Word Card Set 2 **Four Sided Polygon**	Word Card Set 2 **Lines that never touch or intersect**	Word Card Set 2 **Lines that meet at a right angle**
Word Card Set 1 and 2 **A triangle with a right angle**		

Sample Shape Sort 1: Acute, Right, and Obtuse Angles