Number and Operations Base Ten: Multi-Digit Multiplication Strategies Grade 4 Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

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Revised 2016

Multi-Digit Multiplication Strategies 4th

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Grade

Mathematical goals

This concept-based lesson is intended to help you assess how well students are able to use a variety of strategies to multiply. In particular, this unit aims to identify and help students who have difficulties with:

- Representing multiplication in multiple ways.

Kentucky Academic Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards:*

Number and Operations in Base Ten
4-NBT: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Introduction

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work in small groups (pairs) on a collaborative discussion task using number lines to show evidence of their thinking. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.

- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Materials required

Each individual student will need two copies of the worksheet *Multiplication Strategies & Representations.*
• Each small group of students will need a packet of Card Set A - D copied in **color** cut up before the lesson. *{Note: you may want to make color copies, and laminate these for use in multiple classes over multiple years.}*

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Multiplication Strategies & Representations*. Introduce the task briefly and help the class to understand the problem and its context.

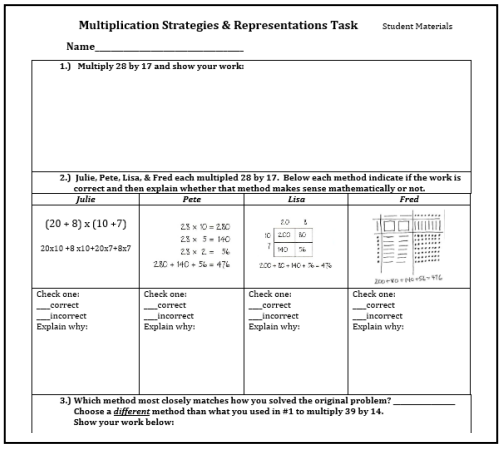
Frame the task:

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.



Multiplication Strategies & Representations Task Student Materials

Name_____

1.) Multiply 28 by 17 and show your work:

2.) Julie, Pete, Lisa, & Fred each multipled 28 by 17. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

Julie	*Pete*	*Lisa*	*Fred*
(20 + 8) x (10 +7) 20x10 +8 x10+20x7+8x7	28 x 10 = 280 28 x 5 = 140 28 x 2 = 56 280 + 140 + 56 = 476	20 5 10 200 50 7 140 56 200 + 50 + 140 + 56 = 476	200+40+140+56 = 476
Check one: ___correct ___incorrect Explain why:	Check one: ___correct ___incorrect Explain why:	Check one: ___correct ___incorrect Explain why:	Check one: ___correct ___incorrect Explain why:

3.) Which method most closely matches how you solved the original problem? _____
Choose a *different* method than what you used in #1 to multiply 39 by 14.
Show your work below:

It is important that students answer the question without assistance, as far as possible.
If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to inform you of the issues that will arise during the lesson, so that you may prepare questions carefully.

We suggest that you do not score students' work. The research shows this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written/displayed on the board at the beginning of the lesson.

It is also suggested that you plan student pairings based on their work on this initial task - pairing students homogeneously (common understandings).

Common issues: Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of multiplication.	• ***If you are multiplying 27x4, what does the 2 represent? the 7?*** • ***What would happen if you multiplied 20 x 4 and 7 x 4? Could you use those answers and to get the answer to 27x4?***
Student doesn't understand Distributive Property.	• ***How can the number(s) we are multiplying be broken apart?*** • ***What could you do with <u>those</u> numbers to solve this problem?***
Student doesn't understand the area model for multiplication.	• ***In the problem 27x14 let's look at the number 27. How many 10s are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14?*** • ***Is there a way to take those two models and fit them on a rectangle to discover 27x14 without doing any calculations?***
Student doesn't understand the base ten method for multiplication.	• ***How does the area of the base ten model compare to the numbers in the partial products method?***

	• ***How can use the units along each side of the model to get the final product?***

Suggested lesson outline

Whole Class Introduction (10 minutes)

Today we are going to do some more work on multiplication strategies. Think about the strategies we have used for multiplication and use your white boards to show at least two different ways to solve other than the standard algorithm:

14 x 14

Share your strategy with a partner. Explain your thinking. Showcase the area model, partial products, base ten representation, distributive property, etc.

Now try this problem: During the next month, you have entered a video game challenge. There are twenty-one players that will each play seventeen hours during the challenge. How many total hours will be spent playing the video game challenge?

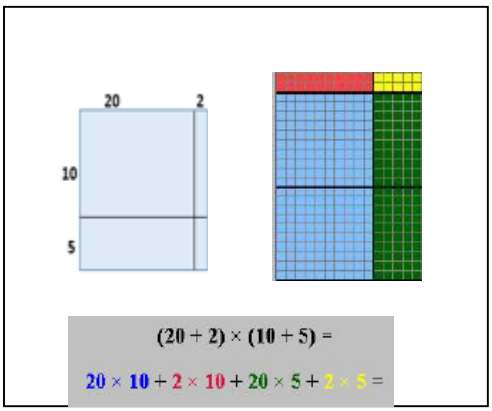
Follow the same directions as above.

Collaborative Activity: matching *Card Sets Models A, B, C, D, and E*. (30 minutes)

Strategically group students based on pre assessment data into pairs. With larger groups, some students may not fully engage in the task. Consider grouping students who displayed like misconceptions together. While this may seem counterintuitive, this will allow each student to more confidently share their thinking.

Give each group *Card Sets A*, *B*, and *C* – Word problem, base ten model, and area model.

Introduce the collaborative activity carefully:



I want you to work as a team. Take turns to match a word problem card with both a base ten model and area model card.

> ***Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.***
>
> ***There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.***

As the teacher, your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes.

Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

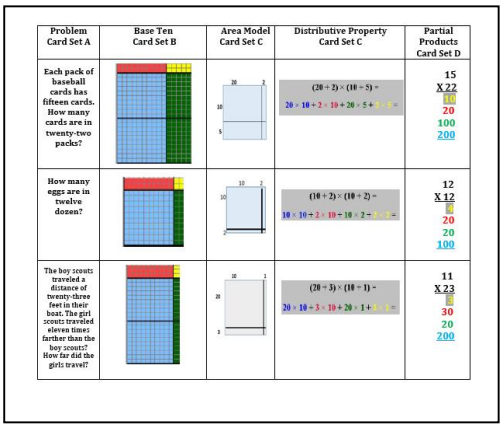
Students will correct their own errors once the Partial Product cards are added.

For students struggling to get started:

There is more than one way to tackle this task.

Can you think what one of them might be? [focusing on either the base ten strategy or the area model.]
What is the story problem asking you to find?
What information is there? Is there any information missing? What do you already know?

How can you calculate products with the base ten model? With the area model?



Problem Card Set A	Base Ten Card Set B	Area Model Card Set C	Distributive Property Card Set C	Partial Products Card Set D
Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?			(20 + 2) × (10 + 5) = 20 × 10 + 2 × 10 + 20 × 5 + 2 × 5 =	15 X 22 10 20 100 200
How many eggs are in twelve dozen?			(10 + 2) × (10 + 2) = 10 × 10 + 2 × 10 + 10 × 2 + 2 × 2 =	12 X 12 2 20 20 100
The boy scouts traveled a distance of twenty-three feet in their boat. The girl scouts traveled eleven times farther than the boy scouts? How far did the girls travel?			(20 + 3) × (10 + 1) = 20 × 10 + 3 × 10 + 20 × 1 + 3 × 1 =	11 X 23 3 30 20 200

If one student has placed a particular card, challenge their partner to provide an explanation.

> ***Maria placed this base ten card with this area model. Martin, why has Maria placed it here?***

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue**,** then you may want to write a couple of questions on the board and organize a whole class discussion.

Placing *Card Sets D and E: Distributive Property and Partial Products*

As students finish placing the word problems, base ten models, and area model cards, hand out *Card Sets D, E: Distributive Property and Partial Products* These provide students with different ways of interpreting the situation.

Do not collect the card sets they have been using. An important part of this task is for students to make connections between all the different representations of multiplication problems.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. Groups should have 8 different clusters of cards with 6 cards in each. The original cards show the correct matches on each row of the table as they are originally arranged.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

> ***If you are staying at your desk, be ready to explain the reasons for your group's matches.***
>
> ***If you are visiting another group, make note of your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.***
>
> ***If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.***
>
> ***When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.***

Students may now want to make changes.

Whole Class Discussion: comparing different approaches (10 minutes)

Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Select students or groups who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each group shares, highlight the connections between strategies.

The discussion offers students the opportunity to learn from each other and for you to address some of the common misconceptions observed in the initial task and during the collaborative activity.

Students should be expected to use this time to compare their solutions, discuss misconceptions and eventually evaluating their own responses based on correct answers. As part of the culture in your mathematics classroom students need to feel safe to share their solution strategies and ask questions of the teacher and each other.

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions the students come up with.

- *Were there certain problems that were more difficult?*
- *Which strategy do you find most useful?*
- *Which strategy do you find most difficult?*
- *How can one strategy help you understand another strategy?*

Try to avoid making evaluative comments yourself. Instead, encourage students to respond to other students' explanations.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Multiplication Strategies & Representations* as well as a second blank copy of the task.

> ***Look at your original responses and think about what you have learned this lesson.***
>
> ***Using what you have learned, try to improve your work.***

If you have not added questions to individual pieces of work then write your list of questions on the board.
Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solutions

Assessment Task: *Multiplication Strategies & Representations*
Question 1: 28 x 17 = 476 Students show work, but strategies may vary.

Question 2: Each of the responses from Julie, Pete, Lisa, & Fred are all correct. Julie used the distributive property, Pete used clustering with the distributive property & some partial products, Lisa used a box method that shows the partial products in the area model, Fred drew a base-10 block area model representation. Be sure that the responses have correct interpretations of each model, but answers may vary in the way each is described.

Question 3: Student should state which person's strategy most closely matched their own work in question #1. They should then use a different strategy to solve 39 x 14 = 546 correctly.

These materials were adapted from *Everyday Mathematics*, *Uncovering Student Misconceptions in Mathematics*, and the *National Library of Virtual Manipulatives*.

When teaching these multiplication and division strategies, *Teaching Student Centered Mathematics* by Van de Walle will be a useful resource.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

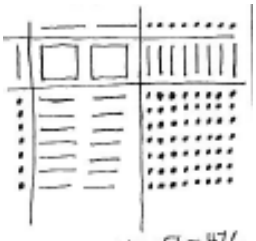
Multiplication Strategies & Representations Task

Student Materials

Name_____

1.) Multiply 28 by 17 and show your work:

2.) Julie, Pete, Lisa, & Fred each multipled 28 by 17. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

Julie	***Pete***	***Lisa***	***Fred***
(20 + 8) x (10 +7) 20x10 +8 x10+20x7+8x7	28 × 10 = 280 28 × 5 = 140 28 × 2 = 56 280 + 140 + 56 = 476	20 8 10 200 80 7 140 56 200 + 80 + 140 + 56 = 476	 200+80+140+56=476
Check one: ____correct ____incorrect Explain why:	Check one: ____correct ____incorrect Explain why:	Check one: ____correct ____incorrect Explain why:	Check one: ____correct ____incorrect Explain why:

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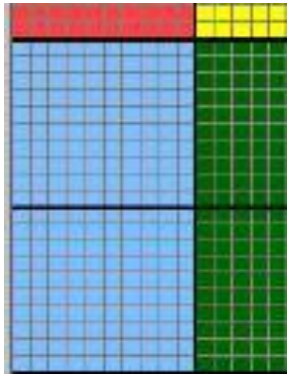
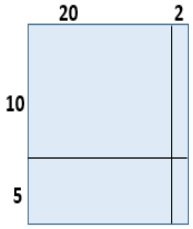
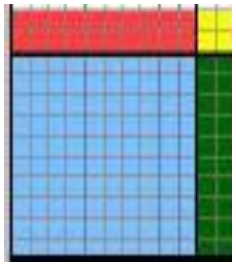
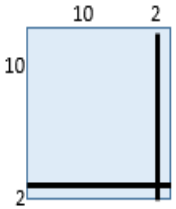
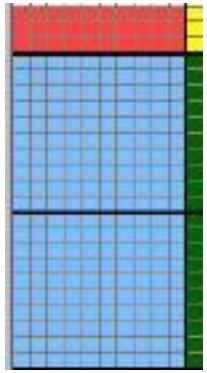
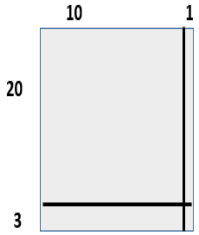
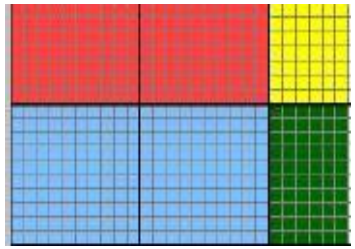
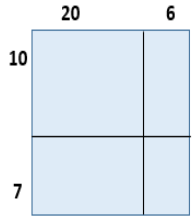
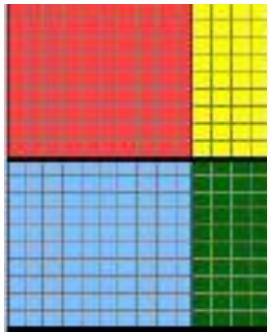
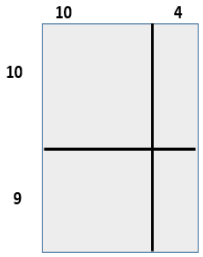
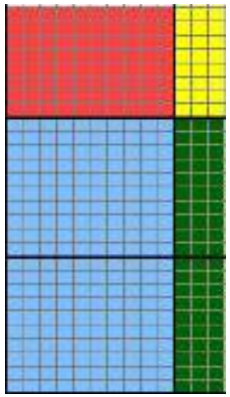
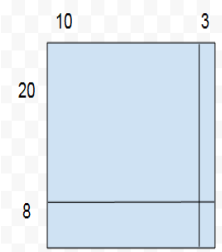
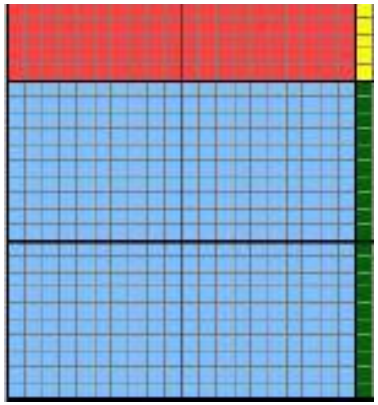
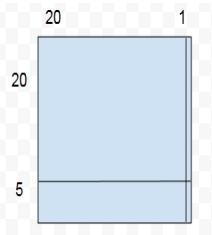
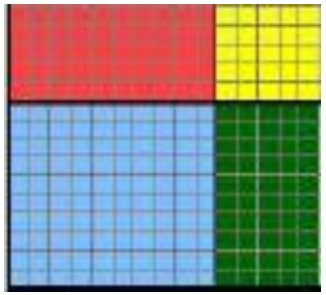
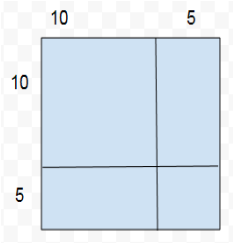
Problem Card Set A	Base Ten Card Set B	Area Model Card Set C	Distributive Property Card Set D	Partial Products Card Set E
Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?			$(20 + 2) \times (10 + 5) =$ $20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$	15 X 22 10 20 100 200
How many eggs are in twelve dozen?			$(10 + 2) \times (10 + 2) =$ $10 \times 10 + 2 \times 10 + 10 \times 2 + 2 \times 2 =$	12 X 12 4 20 20 100
The boy scouts traveled a distance of twenty-three feet in their boat. The girl scouts traveled eleven times farther than the boy scouts? How far did the girls travel?			$(20 + 3) \times (10 + 1) =$ $20 \times 10 + 3 \times 10 + 20 \times 1 + 3 \times 1 =$	11 X 23 3 30 20 200

Problem Card Set A	Base Ten Card Set B	Area Model Card Set C	Distributive Property Card Set D	Partial Products Card Set E
The tree house Scott is building needs twenty-six boards and each board needs seventeen nails. How many nails does Scott need to buy?			$(10 + 7) \times (20 + 6) =$ $10 \times 20 + 7 \times 20 + 10 \times 6 + 7 \times 6 =$	26 X 17 42 140 60 200
An opossum sleeps an average of nineteen hours per day. How many hours does it sleep in a 2-week time period?			$(10 + 9) \times (10 + 4) =$ $10 \times 10 + 9 \times 10 + 10 \times 4 + 9 \times 4 =$	14 X 19 36 90 40 100
Cam bought thirteen different colored folders and each had twenty-eight dots. How many total dots are on her folders?			$(20 + 8) \times (10 + 3) =$ $20 \times 10 + 8 \times 10 + 20 \times 3 + 8 \times 3 =$	13 X 28 24 80 60 200

Problem Card Set A	Base Ten Card Set B	Area Model Card Set C	Distributive Property Card Set D	Partial Products Card Set E
Bags of Reese's cups have twenty-one individually wrapped peanut butter cups. How many cups are in twenty-five bags?			$(20+5)\times(20+1)=$ $20\times20+5\times20+20\times1+5\times1=$	21 X 25 5 100 20 400
The zoo has fifteen monkeys who eat fifteen bananas each day. How many bananas do they need each day for the monkeys?			$(10+5)\times(10+5)=$ $10\times10+5\times10+10\times5+5\times5=$	15 X 15 25 50 50 100