

Division & Interpreting Remainders

5th Grade

Mathematics Formative Assessment Lesson

Designed and revised by Kentucky Department of Education Mathematics Specialists
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues, please contact the KDE team at:
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Created for the sole purpose of assisting teachers as they develop student understanding of Kentucky's Core Academic Standard through the use of highly effective teaching and learning.

Not intended for sale.

Division & Interpreting Remainders

5th Grade

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This concept-based lesson is intended to help you assess how well students are able to use a variety of strategies to multiply. In particular, this unit aims to identify and help students who have difficulties with:

- The traditional division algorithm.
- Representing division in multiple ways
- Interpreting Remainders

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

- 2. Reason abstractly and quantitatively.**
- 7. Look for and make use of structure.**
- 8. Look for and make use of repeated reasoning.**

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

Number and Operations in Base Ten

5-NBT: Perform operations with multi-digit whole numbers.

6-NS: Compute fluently with multi-digit numbers and find common factors and multiples.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work, and formulate questions for students to answer, to help them improve their solutions.
- During the lesson, students work in pairs and threes to match the word problem, model, and multiple strategies of the same multiplication or division problem.
- In a whole-class discussion, explain their answers.
- Finally, students return to their original assessment task, and try to improve their own responses.

Materials required

Each individual student will need two copies of the worksheet *Division Representations & Interpreting Remainders*.

- Each small group of students will need a packet of Card Set A - E copied in **color** cut up before the lesson. *{Note: you may want to make color copies, and laminate these for use in multiple classes over multiple years.}*

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Division Representations & Interpreting Remainders*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.

Mathematics Formative Assessment Lesson		Student Materials		Revised Alpha Version	
Division Representations & Interpreting Remainders Task					
1.) Divide 84 by 7 and show your work:					
2.) Sam, Julie, Pete, & Lisa each divided 84 by 7. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.					
Sam	Julie	Pete	Lisa		
$\begin{array}{r} 84 - 14 = 70 \\ 70 - 14 = 56 \\ 56 - 14 = 42 \\ 42 - 14 = 28 \\ 28 - 14 = 14 \\ 14 - 14 = 0 \\ 6 \times 2 = 12 \end{array}$	$\begin{array}{r} 7 \overline{) 84} \\ \underline{70} \\ 14 \\ \underline{14} \\ 0 \end{array}$		$\begin{array}{r} 12 \\ 7 \overline{) 84} \\ \underline{14} \\ 14 \\ \underline{14} \\ 0 \end{array}$		
Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect	Check one, Explain why: ___correct ___incorrect		
3.) Which method most closely matches how you solved the original problem? _____ Choose a different method than what you used in #1 to divide 156 by 12. Show your work below:					
4.) For each number story draw a picture and/or write a division problem. Then divide to solve the problem. Decide what to do about the remainder. Explain what you get.					
a.) It costs \$50.00 to be a member of the soccer team. The team plays 8 games per season. What is the cost per game?	b.) You expect 25 people to attend a picnic. You need to set up benches that seat 7 people each. How many benches do you need?	c.) Lynn is having a party. Pizzas cost \$8.00 each. How many pizzas can she buy with \$60.00?			
Problem & solution:	Problem & solution:	Problem & solution:			
Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?			

It is important that students answer the question without assistance, as far as possible. If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have

time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues: Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of Division.	<ul style="list-style-type: none"> • <i>If you are dividing $27 \div 4$, what does the 2 represent? the 7?</i> • <i>What would happen if you divided $20 \div 4$ and $7 \div 4$? Could you use those answers and to get the answer to $27 \div 4$?</i>
Student doesn't understand Grouping arrangements	<ul style="list-style-type: none"> • <i>How can these number(s) we are divided be broken apart?</i> • <i>What could you do with <u>those</u> numbers to solve this problem?</i>
Student doesn't understand the model and how it represents a division problem with remainders.	<ul style="list-style-type: none"> • <i>In the problem $27 \div 14$ let's look at the number 27. How many 10s are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14?</i> • <i>Is there a way to take those two models and fit them on a rectangle to discover $27 \div 14$ without doing any calculations?</i>
Students don't know what to do with the remainder nor how to interpret it.	<ul style="list-style-type: none"> • <i>What does this remainder represent?</i>

Suggested lesson outline

Collaborative Activity: matching *Card Sets Models A, B, C, D and E* (30 minutes)

Organize the class into groups of two or three students. With larger groups, some students may not fully engage in the task.

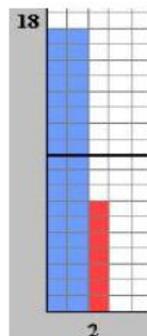
Give each group *Card Sets A and B – Models & Groupings*.

Introduce the lesson carefully:

I want you to work as a team. Take it in turns to match a Model card a Grouping card.

Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.



Quotient: 2 groups of 18

$$\boxed{?} = 18 \times 2 + 7$$

Remainder: 7 left over

As the teacher, your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes.

Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors once the other sets of cards are added.

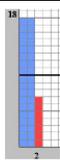
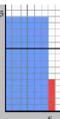
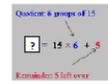
For students struggling to get started:

There is more than one way to tackle this task.

Can you think what one of them might be? [

How can you calculate quotients with the model? with the groupings?

This Groupings card shows $18 \times 2 + 7$? What would the original division problem be for this model? Does that division problem match any of the other cards on the table?

Model Set A	Groupings Set B	Problem Set C	Remainder Set D	Interpreting Remainders Set E
		Sally scooped out forty-three pieces of hard candy to buy at the store. She wants to divide the candy evenly among eighteen people. How many pieces of candy will each person get?	2	Ignore the remainder.
		You are organizing a trolley ride for ninety-five total students, teachers & parents. If each trolley can seat fifteen people, how many trolleys do you need?	7	Round up the remainder.

If one student has placed a particular card, challenge their partner to provide an explanation.

Maria placed this Grouping card with this Model. Martin, why has Maria placed it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a whole class discussion.

Placing Card Sets C, D & E: Word Problems, Remainder, & Interpreting Remainders

As students finish placing the Model & Groupings cards, hand out *Card Sets C, D, & E: Word Problems, Remainder, & Interpreting Remainders*. These provide students with different ways of interpreting the situation.

Do not collect the card sets they have been using. An important part of this task is for students to make connections between all the different representations of multiplication problems.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. Groups should have 7 different clusters of cards with 5 cards in each. The original cards show the correct matches on each row of the table they are originally arranged in.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, make note of your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.

Students may now want to make changes.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Division Representations & Interpreting Remainders* as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then write your list of questions on the board.

Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solutions

Assessment Task: *Division Representations & Interpreting Remainders*

Question 1: Each of the responses from Sam, Julie, Pete, Lisa, & Fred are all correct. Sam used repeated subtraction, Julie used a variation of partial quotients, Pete drew base-10 blocks to show equal shares, Lisa used the traditional division algorithm. Be sure that the responses have correct interpretations of each model, but answers may vary in the way each is described.

Question 2: Student should choose a strategy to solve $156 \div 12 = 13$ correctly.

Question 3:

a.) $\$50.00 \div 8 = \$6 \text{ R } 2$ (or $\$6.25$) Report remainder as a decimal because it is money.

b.) $25 \div 7 = 3 \text{ R } 4$

Round the remainder up because the 4 people will need a bench to sit on.

c.) $\$60.00 \div \$8.00 = 7 \text{ R } 4$

Ignore the remainder because you don't have enough left over to buy a whole pizza.

Rationale for the card sets

Sally scooped out forty-three pieces of hard candy to buy at the store. She wants to divide the candy evenly among eighteen people. How many pieces of candy will each person get?

**$43 \div 18 = 2 \text{ remainder } 7$ but each person only gets 7
to keep the each person's share fair, each only gets 2 pieces**

You are organizing a trolley ride for ninety-five total students, teachers & parents. If each trolley can seat fifteen people, how many trolleys do you need?

**$95 \div 15 = 6 \text{ remainder } 5$ but you need 7 trolleys
however you need another whole trolley to make sure everyone has a ride**

Mr. Jones bought ninety-five new pencils to give his class of nineteen students. How many pencils will each student get?

$95 \div 19 = 5$ no remainder

The soccer team bought their coach a \$55.00 sweatshirt. The fifteen players split the bill evenly. How much did each pay?

**$55 \div 15 = 3 \text{ remainder } 10$ or 3.67
However since this is money the remainder can be written as a decimal so each player pays \$3.67**

Compact discs are on sale for \$13.00 including tax. How many can you buy with \$84.00?

**$84 \div 13 = 6 \text{ remainder } 6$
So you could only buy 6 CDs and you have 6 dollars left over**

There are eighty-four girls in a basketball league and six girls on each team. How many teams are there?

$84 \div 6 = 14$ no remainder

The twelve cheerleaders each want a piece of pink ribbon to wear for the breast cancer march. There is eighty-seven inches of ribbon. How much ribbon should each girl get?

**$87 \div 12 = 7 \text{ remainder } 3$ or $7 \frac{3}{12}$ which reduces to $7 \frac{1}{4}$
However since the ribbon is cut in inches each girl would get $7 \frac{1}{4}$ inches of ribbon.**

These materials were adapted from *Everyday Mathematics, Uncovering Student Misconceptions in Mathematics*, and the *National Library of Virtual Manipulatives*.

When teaching these multiplication and division strategies, *Teaching Student Centered Mathematics* by Van de Walle will be a useful resource.

Division Representations & Interpreting Remainders Task

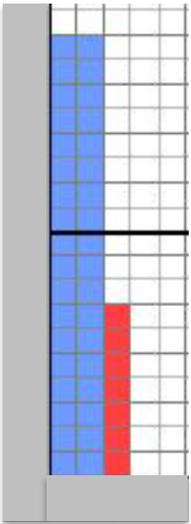
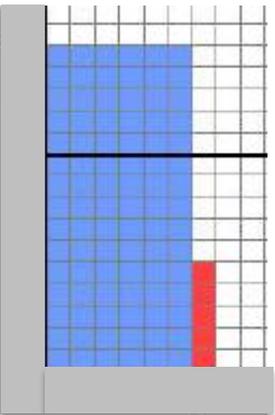
1.) Sam, Julie, Pete, & Lisa each divided 84 by 7. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

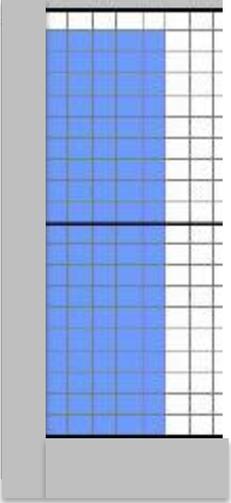
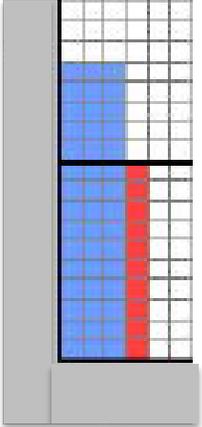
<i>Sam</i>	<i>Julie</i>	<i>Pete</i>	<i>Lisa</i>
$84 - 14 = 70$ $70 - 14 = 56$ $56 - 14 = 42$ $42 - 14 = 28$ $28 - 14 = 14$ $14 - 14 = 0$ $6 \times 2 = 12$	$\begin{array}{r} 7 \overline{) 84} \\ - 70 \\ \hline 14 \\ - 14 \\ \hline 0 \end{array}$ $10 + 2 = 12$		$\begin{array}{r} 12 \\ 7 \overline{) 84} \\ - 7 \\ \hline 14 \\ - 14 \\ \hline 0 \end{array}$
Check one, Explain why: <input type="checkbox"/> correct <input type="checkbox"/> incorrect	Check one, Explain why: <input type="checkbox"/> correct <input type="checkbox"/> incorrect	Check one, Explain why: <input type="checkbox"/> correct <input type="checkbox"/> incorrect	Check one, Explain why: <input type="checkbox"/> correct <input type="checkbox"/> incorrect

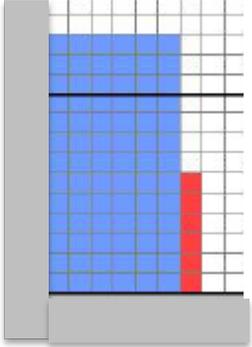
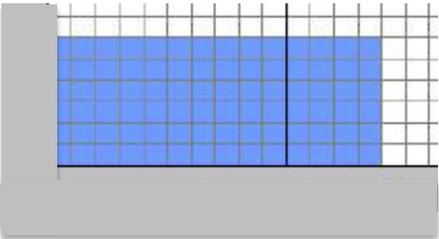
2.) Choose one of the methods in #1 to divide 156 by 12. Show your work below:

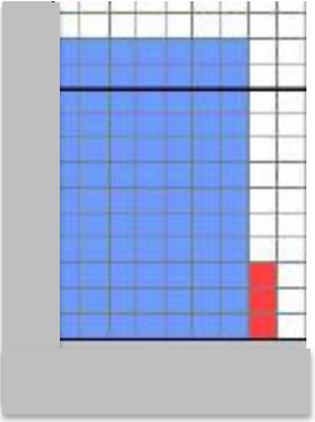
3.) For each number story draw a picture and/or write a division problem. Then divide to solve the problem. Decide what to do about the remainder. Explain what you get.

<i>a.) It costs \$50.00 to be a member of the soccer team. The team plays 8 games per season. What is the cost per game?</i>	<i>b.) You expect 25 people to attend a picnic. You need to set up benches that seat 7 people each. How many benches do you need?</i>	<i>c.) Lynn is having a party. Pizzas cost \$8.00 each. How many pizzas can she buy with \$60.00?</i>
Problem & solution:	Problem & solution:	Problem & solution:
Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?	Circle what you did with the remainder: ignored it, reported it as a fraction or decimal, rounded the answer up Why?

Model Set A	Groupings Set B	Problem Set C	Solution to Problem Set D	Interpreting Remainders Set E
	<p>Quotient: 2 groups of 18</p> $\boxed{?} = 18 \times 2 + 7$ <p>Remainder: 7 left over</p>	<p>Sally scooped out forty-three pieces of hard candy to buy at the store. She wants to divide the candy evenly among eighteen people. How many pieces of candy will each person get?</p>	<p>2</p>	<p>Ignore the remainder.</p>
	<p>Quotient: 6 groups of 15</p> $\boxed{?} = 15 \times 6 + 5$ <p>Remainder: 5 left over</p>	<p>You are organizing a trolley ride for ninety-five total students, teachers & parents. If each trolley can seat fifteen people, how many trolleys do you need?</p>	<p>7</p>	<p>Round up the remainder.</p>

Model Set A	Groupings Set B	Problem Set C	Solution to Problem Set D	Interpreting Remainders Set E
	<p>Quotient: 5 groups of 19</p> $\boxed{?} = 19 \times 5 + 0$ <p>Remainder: 0 left over</p>	<p>Mr. Jones bought ninety-five new pencils to give his class of nineteen students. How many pencils will each student get?</p>	<p>5</p>	<p>There is no remainder.</p>
	<p>Quotient: 3 groups of 15</p> $\boxed{?} = 15 \times 3 + 10$ <p>Remainder: 10 left over</p>	<p>The soccer team bought their coach a \$55.00 sweatshirt. The fifteen players split the bill evenly. How much did each pay?</p>	<p>3.67</p>	<p>Write the remainder as a decimal.</p>

Model Set A	Groupings Set B	Problem Set C	Solution to Problem Set D	Interpreting Remainders Set E
	<p>Quotient: 6 groups of 13</p> $\boxed{?} = 13 \times 6 + 6$ <p>Remainder: 6 left over</p>	<p>Compact discs are on sale for \$13.00 including tax. How many can you buy with \$84.00?</p>	<p>6</p>	<p>Ignore the remainder.</p>
	<p>Quotient: 14 groups of 6</p> $\boxed{?} = 6 \times 14 + 0$ <p>Remainder: 0 left over</p>	<p>There are eighty-four girls in a basketball league and six girls on each team. How many teams are there?</p>	<p>14</p>	<p>There is no remainder.</p>

Model Set A	Groupings Set B	Problem Set C	Solution to Problem Set D	Interpreting Remainders Set E
	<p data-bbox="600 435 894 472">Quotient: 7 groups of 12</p> $? = 12 \times 7 + 3$ <p data-bbox="600 662 869 695">Remainder: 3 left over</p>	<p data-bbox="1003 363 1396 704">The twelve cheerleaders each want a piece of pink ribbon to wear for the breast cancer march. There is eighty-seven inches of ribbon. How much ribbon should each girl get?</p>	<p data-bbox="1493 526 1612 586">7 $\frac{1}{4}$</p>	<p data-bbox="1713 477 2007 613">Write the remainder as a fraction.</p>