



Characteristics of Highly Effective Teaching and Learning *Mathematics*

The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using effective instructional practices in developing mathematics learning for all students. The eight [Mathematics Teaching Practices \(MTP\)](#) describe the essential teaching skills derived from the research-based learning principles, as well as other knowledge of mathematics teaching that has emerged over the last two decades. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. For more information on the MPs, see the [Kentucky Academic Standards for Mathematics](#).

Section One: Learning Climate

Learning Climate: A safe environment supported by the teacher in which high, clear expectations and positive relationships are fostered; active learning is promoted.

Teacher Characteristics:

- A. Teacher creates learning environments where students are active participants as individuals and as members of collaborative groups. The teacher:
 1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies **(MTP2)**;
 2. Facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments **(MTP4)**; and
 3. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.
- B. Teacher motivates students and nurtures their desire to learn in a safe, healthy and supportive environment which develops compassion and mutual respect.
- C. Teacher cultivates cross cultural understandings and the value of diversity.
- D. Teacher encourages students to accept responsibility for their own learning and accommodates the diverse learning needs of all students.

- E. Teacher displays effective and efficient classroom management that includes classroom routines that promote comfort, order and appropriate student behaviors.
- F. Teacher provides students equitable access to technology, space, tools and time. The teacher:
 - 1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies **(MTP2)**; and
 - 2. Engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving **(MTP3)**.
- G. Teacher effectively allocates time for students to engage in hands-on experiences, discuss and process content and make meaningful connections.
- H. Teacher designs lessons that allow students to participate in empowering activities in which they understand that learning is a process and mistakes are a natural part of learning.
- I. Teacher creates an environment where student work is valued, appreciated and used as a learning tool. The teacher:
 - 1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies **(MTP2)**;
 - 2. Facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments **(MTP4)**;
 - 3. Consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships **(MTP7)**; and
 - 4. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.

Student Characteristics:

- A. Student accepts responsibility for his/her own learning. The student:
 - 1. MP.1. Makes sense of problems and persevere in solving them;
 - 2. MP.2. Reasons abstractly and quantitatively;
 - 3. MP.3. Constructs viable arguments and critique the reasoning of others;
 - 4. MP.4. Models with mathematics; and
 - 5. MP.5. Uses appropriate tools strategically.
- B. Student actively participates and is authentically engaged. The student:

1. MP.1. Makes sense of problems and persevere in solving them;
 2. MP.2. Reasons abstractly and quantitatively;
 3. MP.3. Constructs viable arguments and critique the reasoning of others; and
 4. MP.4. Models with mathematics.
- C. Student collaborates/teams with other students. The student:
1. MP.1. Makes sense of problems and persevere in solving them;
 2. MP.2. Reasons abstractly and quantitatively;
 3. MP.3. Constructs viable arguments and critique the reasoning of others; and
 4. MP.4. Models with mathematics.
- D. Student exhibits a sense of accomplishment and confidence.
- E. Student takes educational risks in class. The student:
1. MP.1. Makes sense of problems and persevere in solving them;
 2. MP.2. Reasons abstractly and quantitatively;
 3. MP.3. Constructs viable arguments and critique the reasoning of others;
 4. MP.4. Models with mathematics; and
 5. MP.6. Attends to precision.
- F. Student practices and engages in safe, responsible and ethical use of technology.

Section Two: Classroom Assessment and Reflection

Classroom Assessment and Reflection: The teacher and student collaboratively gather information and reflect on learning through a systematic process that informs instruction.

Teacher Characteristics:

- A. Teacher uses multiple methods and systematically gathers data about student understanding and ability.
- B. Teacher uses student work/data, observations of instruction, assignments and interactions with colleagues to reflect on and improve teaching practice. The teacher:
 1. Establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions **(MTP1)**;
 2. Builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems **(MTP6)**; and
 3. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.

- C. Teacher revises instructional strategies based upon student's achievement data. The teacher:
1. Establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions **(MTP1)**;
 2. Builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems **(MTP6)**; and
 3. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.
- D. Teacher uncovers students' prior understanding of the concepts and addresses students' misconceptions/incomplete conceptions. The teacher:
1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem solving; allows multiple entry points and varied solution strategies **(MTP2)**;
 2. Engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving **(MTP3)**;
 3. Facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments **(MTP4)**;
 4. Builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems **(MTP6)**; and
 5. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.
- E. Teacher co-develops scoring guides/rubrics with students and provides adequate modeling to make clear the expectations for quality performance. The teacher:
1. Establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions **(MTP1)**.
- F. Teacher guides students to apply rubrics to assess their performance and identify improvement strategies.
- G. Teacher provides regular, timely feedback to students and parents that moves learners forward. The teacher:
1. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.

- H. Teacher allows students to use feedback to improve their work before a grade is assigned.
- I. Teacher facilitates students in self- and peer-assessment.
- J. Teacher reflects on work and makes adjustments as learning occurs.

Student Characteristics:

- A. Student recognizes what proficient work looks like and determines steps necessary for improving his/her work.
- B. Student develops and/or uses scoring guides periodically to assess his/her own work or that of peers.
- C. Student uses teacher feedback to improve his/her work.
- D. Student reflects on work and makes adjustments as learning occurs.

Section Three: Cognitive Complexity and Student Engagement

Cognitive Complexity and Student Engagement: A teacher supports and encourages a student's commitment to initiate and complete complex, inquiry-based learning requiring creative and critical thinking with attention to problem-solving.

Teacher Characteristics:

- A. Teacher utilizes the Kentucky Academic Standards for Mathematics in the development of course outlines/maps. The teacher:
 - 1. Establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions **(MTP1)**;
 - 2. Engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies **(MTP2)**; and
 - 3. Consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships **(MTP7)**.
- B. Teacher scaffolds instruction to help students reason and develop problem-solving strategies. The teacher:

1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving, and allows multiple entry points and varied solution strategies **(MTP2)**;
 2. Engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem-solving **(MTP3)**; and
 3. Consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships **(MTP7)**.
- C. Teacher orchestrates effective classroom discussions, questioning, and learning tasks that promote higher-order thinking skills. The teacher:
1. Engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allow multiple entry points and varied solution strategies **(MTP2)**;
 2. Engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving **(MTP3)**;
 3. Facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments **(MTP4)**;
 4. Uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships **(MTP5)**;
 5. Builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems **(MTP6)**; and
 6. Uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning **(MTP8)**.
- D. Teacher provides meaningful learning opportunities for students. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
 2. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 3. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 4. builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. **(MTP6)**

5. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
- E. Teacher challenges students to think deeply about problems and encourages/models a variety of approaches to a solution. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
 2. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 3. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 4. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP5)**
 5. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
 6. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**
- F. Teacher integrates a variety of learning resources with classroom instruction to increase learning options. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
 2. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 3. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**
- G. Teacher structures and facilitates ongoing formal and informal discussions based on a shared understanding of rules and discourse.
- H. Teacher integrates the application of inquiry skills into learning experiences. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**

2. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 3. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP5)**
 4. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
- I. Teacher clarifies and shares with students learning intentions/targets and criteria for success. The teacher:
1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**

Student Characteristics:

- A. Student articulates and understands learning intentions/targets and criteria for success.
- B. Student reads with understanding a variety of texts. The student:
1. MP.1. Makes sense of problems and persevere in solving them.
 2. MP.2. Reasons abstractly and quantitatively.
- C. Student applies and refines inquiry skills. The student:
1. MP.1. Makes sense of problems and persevere in solving them.
 2. MP.2. Reasons abstractly and quantitatively.
 3. MP.3. Constructs viable arguments and critique the reasoning of others.
 4. MP.4. Models with mathematics.
 5. MP.5. Uses appropriate tools strategically.

Section Four: Instructional Relevance

Instructional Relevance: A teacher's ability to facilitate learning experiences that are meaningful to students and prepare them for their futures.

Teacher Characteristics:

- A. Teacher designs learning opportunities that allow students to participate in empowering activities in which they understand that learning is a process and mistakes are a natural part of the learning. The teacher:

1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 3. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP5)**
 4. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
- B. Teacher links concepts and key ideas to students' prior experiences and understandings, uses multiple representations, examples and explanations. The teacher:
1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 3. builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. **(MTP6)**
 4. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP7)**
- C. Teacher incorporates student experiences, interests and real-life situations in instruction.
- D. Teacher selects and utilizes a variety of technology that support student learning.
- E. Teacher works with other teachers to make connections between and among disciplines. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
- F. Teacher makes lesson connections to community, society, and current events. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**

Student Characteristics:

- A. Student poses and responds to meaningful questions. The student:
 - 1. MP.1. Makes sense of problems and persevere in solving them.
 - 2. MP.3. Constructs viable arguments and critique the reasoning of others.

- B. Student uses appropriate tools and techniques to gather, analyze, and interpret quantitative and qualitative data. The student:
 - 1. MP.4. Models with mathematics.
 - 2. MP.5. Uses appropriate tools strategically.

- C. Student develops descriptions, explanations, predictions, and models using evidence. The student:
 - 1. MP.3. Constructs viable arguments and critique the reasoning of others.
 - 2. MP.4. Models with mathematics.
 - 3. MP.7. Looks for and makes use of structure.
 - 4. MP.8. Looks for and expresses regularity in repeated reasoning

- D. Student works collaboratively to address complex, authentic problems which require innovative approaches to solve. The student:
 - 1. MP.1. Makes sense of problems and persevere in solving them.
 - 2. MP.2. Reasons abstractly and quantitatively.
 - 3. MP.3. Constructs viable arguments and critique the reasoning of others.
 - 4. MP.4. Models with mathematics.
 - 5. MP.7. Looks for and makes use of structure.
 - 6. MP.8. Looks for and expresses regularity in repeated reasoning

- E. Student communicates knowledge and understanding in a variety of real-world forms. The student:
 - 1. MP.1. Makes sense of problems and persevere in solving them.
 - 2. MP.7. Looks for and makes use of structure.
 - 3. MP.8. Looks for and expresses regularity in repeated reasoning

- F. Student communicates knowledge and understanding for a variety of purposes. The student:
 - 1. MP.3. Constructs viable arguments and critique the reasoning of others.

Section Five: Knowledge of Content

Knowledge of Content: A teacher's understanding and application of the current theories, principles, concepts and skills of a discipline.

Teacher Characteristics:

- A. Teacher demonstrates an understanding and in-depth knowledge of content and maintains an ability to convey this content to students. The teacher:
 1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
 3. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 4. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 5. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP5)**
 6. builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. **(MTP6)**
 7. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
 8. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**

- B. Teacher maintains on-going knowledge and awareness of current content developments. The teacher:
 1. maintains awareness of current developments in the field of mathematics and can explain what these developments mean to each group of stakeholders.
 2. maintains an awareness of current developments in the field of mathematics education.
 3. uses current developments to build pedagogical content knowledge.
 4. engages in professional dialogue with colleagues regarding the research implications for sound practice that develops students' advanced thinking, robust fluency and flexible use of mathematical content.

- C. Teacher designs and implements courses/lessons/units that are aligned to the Kentucky Academic Standards for Mathematics. The teacher:
 1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**

- D. Teacher uses and promotes the understanding of appropriate content vocabulary. The teacher:
1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
- E. Teacher provides essential supports for all students. The teacher:
1. establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. **(MTP1)**
 2. builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. **(MTP6)**
 3. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**
 4. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**
- F. Teacher accesses a rich repertoire of instructional practices, strategies, resources and applies them appropriately. The teacher:
1. engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. **(MTP2)**
 2. engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. **(MTP3)**
 3. facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **(MTP4)**
 4. uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **(MTP5)**
 5. builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. **(MTP6)**
 6. consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. **(MTP7)**

7. uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. **(MTP8)**

Student Characteristics:

- A. Student demonstrates growth in content knowledge. The student:
 1. MP.2. Reasons abstractly and quantitatively.
 2. MP.3. Constructs viable arguments and critique the reasoning of others.
 3. MP.4. Models with mathematics.
 4. MP.7. Looks for and makes use of structure.
 5. MP.8. Looks for and expresses regularity in repeated reasoning
- B. Student uses and seeks to expand appropriate content vocabulary.
- C. Student connects ideas across content areas. The student:
 1. MP.1. Makes sense of problems and persevere in solving them.
 2. MP.2. Reasons abstractly and quantitatively.
 3. MP.7. Looks for and makes use of structure.
 4. MP.8. Looks for and expresses regularity in repeated reasoning
- D. Student uses ideas in realistic problem solving situations. The student:
 1. MP.1. Makes sense of problems and persevere in solving them.
 2. MP.2. Reasons abstractly and quantitatively.
 3. MP.4. Models with mathematics.

Bibliography:

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