CCNETWORK Regional Comprehensive Center Appalachia

Tier 2 and Tier 3 Math Intervention: An Implementation Toolkit for K–3 Teachers

How to Use This Toolkit

This toolkit is written for educators who deliver Tier 2 and Tier 3 math instruction in grades K–3, including classroom teachers, highly specialized educators, and the team members who support them. It builds on (and assumes familiarity with) two companion documents:

- *KyMTSS Implementation Guide, 2nd Edition* (2025). The system-level infrastructure—including essential elements, team structures, and decision-making processes—for Kentucky's Multi-Tiered System of Supports (KyMTSS) under Title 704 of the Kentucky Administrative Regulations (704 KAR 3:095)
- *Mathematics Improvement Plan: A Step-by-Step Resource for K–3 Classroom Teachers in Kentucky (May 2026).* A walkthrough of the Mathematics Improvement Plan template used to document any K–3 student receiving accelerated math supports under Kentucky Revised Statutes (KRS) § 158.8402 (House Bill [HB] 162, Kentucky Numeracy Counts Act, 2024)

Although those documents provide guidance at the system and plan level, this toolkit answers the harder teacher question: What does effective Tier 2 or Tier 3 math actually look like for a K-3 student; what skills do I target, what does an intervention look like, and how do I know it is working?

The toolkit is organized in six sections (described in table 1.1).

Why combine both tiers in one toolkit?

- Tier 2 and Tier 3 share most of the same DNA: alignment to K–3 standards, foundational skill progressions, evidence-based practices, decision–rule frameworks, family partnership, and fidelity expectations.
- Discussing Tier 2 and Tier 3 side-by-side helps clarify the differences between the tiers in dosage, group size, and instructional intensity of the intervention, as well as who delivers it and the level of individualization.
- Many K–3 teachers deliver Tier 2 interventions themselves and collaborate with a highly specialized educator who provides Tier 3 interventions. KyMTSS treats the tiered delivery system as one integrated continuum, not as three separate buckets.

Table 1.1. How this toolkit is organized

Section	What it includes
1. Tiered Math Intervention in KyMTSS	A summary of MTSS policy; the tiered delivery system; a side-by-side comparison of Tier 1, Tier 2, and Tier 3
2. Identifying Students for Each Tier	An understanding of how to obtain and use different data sources; written decision rules for entering Tier 2 and Tier 3; the role of student work analysis
3. K–3 Foundational Math Skill Progressions	A description of the eight core threads of math learning, mapped vertically across grades K–3
4. Evidence-Based Practices	Six practical recommendations from the 2021 Institute of Education Sciences (IES) Practice Guide, *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*
5. Progress Monitoring, Decision Rules and Modifications	Guidance on how to choose probes; chart growth; and decide when to continue, intensify, or escalate from Tier 2 to Tier 3, when to fade from Tier 3 to Tier 2, or when to exit the student
6. Family Partnership	Concrete ways to share data and support math learning at home

Section 1: Tiered Math Intervention in KyMTSS

1.1 Policy and practice context

Under the Kentucky Numeracy Counts Act (HB 162, 2024; KRS 158.8402), beginning in the 2026–27 school year, every K–3 student must be screened in math within the first 30 days. Students flagged as being at risk of not meeting grade-level benchmarks must receive a diagnostic assessment. Students whose diagnostic data confirm a need for accelerated supports must receive evidence-based intervention with documented progress monitoring and family communication, which is captured in a Mathematics Improvement Plan.

All of this happens within the KyMTSS Framework, which integrates academic, social, and behavioral supports through six essential elements (described on page 3 of the *KyMTSS Implementation Guide*).

A note on the tier badges in this toolkit

Where guidance applies to one tier and not the other, you will see a small colored badge—

TIER 2 for Tier 2

TIER 3 for Tier 3

BOTH TIERS for when the same guidance applies to both

All sections share one vocabulary, one set of tier badges, and one purpose: to make Tier 2 and Tier 3 K–3 math intervention concrete for the educator delivering it.

1.2 The KyMTSS Tiered Delivery System at a Glance

Kentucky's tiered delivery system is designed as a layered continuum of supports, not three isolated instructional plans. Every student receives high-quality Tier 1 universal instruction.

Students who need more assistance also receive Tier 2 supplemental support. A small number of students with the most significant needs or whose needs persist despite Tier 2 intervention also receive Tier 3 intensive support in addition to Tier 1. Tiers describe intensity of support and should never be used to label students. Table 2 illustrates the dimensions of the KyMTSS tiered delivery system (*KyMTSS Implementation Guide*, table 4.1, page 27) as applied to K–3 math.

Table 1.2. KyMTSS tiered delivery system for K–3 math

Dimension	Tier 1: Universal	Tier 2: Supplemental	Tier 3: Intensive
Group size	Whole class	Typically 3–7 students	Typically 2 or 3 students or 1:1
Frequency/dose	Daily math block	3–5 times per week for 20–30 minutes	Daily for 30–45 minutes or more
Instructional design	High-quality core curriculum aligned to Kentucky Academic Standards; grounded in an approved high-quality instructional resource and differentiated based on student data	Structured, uniform, small-group lessons with documented fidelity; targeted to known skill gaps and aligned to the scope and sequence of Tier 1 instruction where possible (i.e., preview or review of grade-level content)	Individualized intervention matched to the student's specific diagnostic profile and error patterns and aligned to the scope and sequence of Tier 1 instruction where possible (i.e., addresses critical prerequisite or foundational skills)
Person(s) delivering support	Classroom instructor	Educator, paraprofessional, or supplemental staff	Highly specialized educator with strong K–3 math content knowledge
Progress monitoring	Universal screening 3 times per year	Every 2 weeks (monthly at minimum)	Weekly
Math Improvement Plan requirement	No (in most instances); however, a Math Improvement Plan may be appropriate if a student's needs for accelerated intervention can be met in a Tier 1 setting	Yes	Yes

1.3 Common misconceptions about Tier 2 and Tier 3

TIER 2 **"Tier 2 is just small-group math."**

Tier 2 is more disciplined than general small-group math. It targets a specific, known skill gap (e.g., pairs that make 10), uses a written instructional sequence aligned to Tier 1, monitors progress every 2 weeks against the goal in section 2 of the Mathematics Improvement Plan, and exits the student when the gap closes. Pulling a group of kids for math is not considered Tier 2 unless these features are in place.

TIER 3 **"Tier 3 is just more Tier 2."**

Tier 3 is characterized by individualizing instructional design and delivery alongside changes to frequency, duration, group size, or instructor expertise. It is individualized according to diagnostic data built around the specific foundational skills the student is missing and distinguished from Tier 2 by the increased intensity of the intervention.

BOTH TIERS **"My students learn math in Tier 2 or Tier 3 instead of Tier 1."**

Tier 2 and Tier 3 should never replace core instruction in Tier 1. If the only schedulable time conflicts with Tier 1 math, the team must redesign the schedule to avoid the conflict.

Section 2: Identifying Students for Each Tier

Identification is a team decision, not the result of a single test score. Use the data sources in table 2.1 alongside the KyMTSS four-step problem-solving process (define, analyze, plan/implement, and evaluate) described on pages 19–21 of the *KyMTSS Implementation Guide*.

Table 2.1. Data sources for Tier 2 and Tier 2

Source	What it tells you about student performance	When to use it
Universal screener	• Whether the student is at, near, or below the grade-level benchmark in math • The student's composite score and broad subdomain risk flags • The effectiveness of Tier 1 and which students need closer monitoring or intervention (704 KAR 3:095) (NOTE: See the Kentucky Department of Education Numeracy Screening Assessments webpage for more information.)	Three times per year (fall, winter, and spring); fall administration should be within the first 30 days of the school year
Diagnostic analysis	• The root cause(s) and the best-fit intervention for the student (*KyMTSS Implementation Guide*, pages 23–24) • Specific skills that are intact and those that are missing • Targets to map to instruction	Within the first 45 days of the school year for at-risk students; as needed thereafter
Progress monitoring	• Information about the student's rate of improvement or responsiveness to instruction or intervention (by comparing baseline data with ongoing progress data that is collected at regular intervals) (704 KAR 3:095) (NOTE: If the student is not responding in Tier 2, consider Tier 3. If Tier 3 is not leading to improvement, modify the design, the intensity of the intervention, or both.)	Tier 2: every 2 weeks Tier 3: weekly
Classroom formative evidence (e.g., curriculum-embedded tasks and assessments, informal checklists)	• Student error patterns and mathematical reasoning; how students use representations and mathematical language (NOTE: Classroom formative evidence is often the best indicator of K–3 student understanding and is frequently embedded in curricular tasks and assessments.)	Daily to weekly; at section checkpoints; at the beginning and end of units
Team input	• Triangulates source data and interprets reasoning that short K–3 probes may not capture statistically	At each Math Improvement Plan and problem-solving meeting, with family observations shared during meetings

2.1 Sample written decision rules

Document your school and district rules on your MTSS Action Plan and revisit them annually. KyMTSS expects written decision rules; documenting them protects students from being moved between tiers based on different criteria from one meeting to the next.

TIER 2 Entry rule

A K–3 student is considered for Tier 2 math intervention when all the following are true:

- Universal screener performance falls below the school's strategic cut score (see the official benchmark cut scores on the Numeracy Screening Assessments webpage).
- Evidence (e.g., diagnostic assessments, curriculum-embedded assessments and tasks, classroom formative activities) shows a specific teachable skill gap (e.g., not yet fluent in pairs that make 10).
- Tier 1 differentiation has not closed the gap within the typical core instruction cycle.

TIER 3 Entry rule

A K–3 student is considered for Tier 3 math intervention when the first two criteria for Tier 2 math intervention are met *and* one of the following is true:

- The student received at least 6–8 weeks of Tier 2 intervention with documented fidelity and showed a flat or declining trend.
- The diagnostic profile shows skill gaps so significant that beginning at Tier 3 is warranted (skipping Tier 2 is appropriate when the data clearly indicate intensive needs from the start).

BOTH TIERS Escalation rule

Move a student from Tier 2 to Tier 3 math intervention when all the following are true:

- The progress-monitoring trend line is flat or below the goal line after 6–8 weeks of Tier 2 intervention with fidelity.
- Reexamination of diagnostic data shows the skill gap is more foundational than initially thought (see section 5.5).
- The team agrees that more intensity, more individualization, and a more diagnostic approach are warranted.

2.2 What student work analysis adds

K–3 diagnostic assessments may not always be precise enough at the subdomain level to be the sole determinant of a student's intervention needs. Combining data from the diagnostic assessment with curriculum-embedded assessments and formative evidence can help pinpoint the student's specific areas of need relative to the grade-level expectations. An analysis of how a student counts, talks about numbers, uses a manipulative, and explains a strategy provides the team with additional diagnostic information.

When attending Mathematics Improvement Plan team meetings, come prepared to discuss three or four concrete observations, such as the following:

- "When I ask Eli to count to 20 by ones and pause at 13, he keeps counting. He can recite the sequence but does not control where to stop."
- "Mia solves 8 + 5 by counting all 13 on her fingers, every time. She is not yet using make-10."
- "Jamal can compute 43 − 18 correctly with base-10 blocks but cannot connect it to the written algorithm."
- "Sophia can list multiplication facts up to 5 but cannot show 4 × 3 as four groups of three on an array."

Section 3: K–3 Foundational Math Skill Progressions

Children's mathematical understanding develops along predictable learning trajectories. Effective Tier 2 and Tier 3 support starts where a student's understanding becomes shaky—the lowest unstable skill—and builds upward. This section maps eight foundational threads for grades K–3, so you can determine where to begin. The map is the same for both tiers; what differs is the intensity of support once you have located the entry point.

BOTH TIERS **How to use this section**

- Find the row that matches the skill area where the student is struggling.
- Move across the row until you reach the column where the student demonstrates understanding. The next column to the right is your starting point.
- Plan instruction at that level, with the explicit intention of bridging to the student's current grade level.

3.1 Eight foundational threads for grades K–3

Table 3.1 describes the work students should encounter for each foundational thread at each grade level. Cells beginning with "Not a formal [grade] standard" indicate domains where there is no on-grade expectation yet; these cells are included to show the larger trajectory, not as intervention targets. Number, operations, and fractions dominate K–3 intervention plans in practice, but geometry is included because it is a foundational K–3 domain and interacts with fractions and measurement.

Note that these are the foundational threads most likely to cause students difficulties. They do not include all core math concepts (e.g., time and money, data and statistics). Difficulties in areas not listed often stem from gaps in the skills noted in table 3.1. Although the foundational threads are intended to provide guidance for most cases where students need intervention, there may be cases where students are at grade level for the foundational threads but still need Tier 2 or Tier 3 interventions for concepts not listed.

Table 3.1. Foundational threads for Kentucky Academic Standards for Math (KAS–Math)

Thread	Kindergarten	Grade 1	Grade 2	Grade 3
Counting and cardinality	Verbally count to 100; verbally count backward from 30; count objects to 20; recognize last number is the total (cardinality); write numbers from 0 to 20; compare quantities up to 10	Count to 120 from any starting number; read and write numerals to 120	Skip count by 5s, 10s, and 100s; count within 1,000	Skip count to support multiplicative thinking (e.g., 3, 6, 9, 12)
Subitizing and number sense (NOTE: Subitizing is not mentioned in KAS–Math but is included here as a foundational thread related to number sense.)	Perceptually subitize one to five dots; compare small sets	Conceptually subitize (e.g., see 6 as 5 + 1); compare numbers to 100 using place value	Mentally see numbers in part–whole structures (e.g., 47 as 40 + 7 or 30 + 17)	Mentally see arrays and equal groups; estimate magnitude on a number line
Compose and decompose	Compose and decompose numbers less than or equal to 10; decompose numbers into two groups in more than one way by using objects or drawings	Compose and decompose numbers 11–19 as a ten and ones or a set of ones; make 10 as a strategy (8 + 5 becomes 8 + 2 + 3); add tens and ones	Decompose for regrouping in addition and subtraction within 1,000	Decompose for distributive thinking (4 × 7 = 4 × 5 + 4 × 2); decompose fractions (3/4 = 1/4 + 1/4 + 1/4)

Thread	Kindergarten	Grade 1	Grade 2	Grade 3
Addition and subtraction	Fluently add and subtract within 5 with symbols; add and subtract within 10 with objects; understand addition and subtraction as joining and separating	Demonstrate fluency within 10; add and subtract within 20; solve all 11 word-problem types with unknowns in all positions	Demonstrate fluency within 20; add and subtract within 100; solve one- and two-step addition/ subtraction word problems within 100 with unknowns in all positions	Fluently add and subtract within 1,000; transition focus to multiplication and division
Base-10 understanding	Compose and decompose numbers less than or equal to 10; decompose numbers into two groups in more than one way by using objects or drawings	Recognize that the numbers from 11 to 19 are composed of 1 ten and a set of ones; understand place value to 100 (i.e., tens and ones; 10 ones = 1 ten)	Understand place value to 1,000 (i.e., hundreds, tens, ones; comparisons; rounding readiness)	Complete multidigit arithmetic using place value strategies and algorithms; round to 10 or 100
Multiplicative thinking	*Not a formal kindergarten standard—watch for repeated copies and equal sharing in play*	*Not a formal Grade 1 standard—watch for equal groups within addition word problems*	Use equal groups, arrays, repeated addition (foundations for multiplication)	Multiply and divide within 100; use properties of operations; use factors, equal groups, arrays, area model

Thread	Kindergarten	Grade 1	Grade 2	Grade 3
Fractions and measurement	Compare measurable attributes; classify and sort objects	Compare lengths with units; create equal shares (halves and fourths) of shapes	Use standard length units (cm, in); partition shapes into halves, thirds, and fourths	Understand fractions as numbers on a number line (denominators 2, 3, 4, 6, and 8); equivalence; area and perimeter
Geometry	Identify and describe 2D shapes (e.g., squares, triangles, circles, rectangles, hexagons) and 3D shapes (e.g., cubes, cones, cylinders, spheres) regardless of orientation/size; compose simple shapes; describe relative positions of objects	Distinguish defining from nondefining shape attributes; compose 2D shapes; partition circles and rectangles into halves and fourths	Draw shapes with specified attributes (e.g., polygons, angles); partition rectangles into rows and columns; partition shapes into halves, thirds, and fourths	Classify quadrilaterals by shared attributes; partition shapes into equal parts and express each part as a unit fraction

3.2 Bridging back to grade level

Several consistent practices keep instruction connected to the student's current grade-level standards at both tiers.

- Anchor each target to a current-grade KAS standard. Write it on the lesson plan (e.g., "The skill 'make 10 within 20' unlocks the grade 2 fluency standard KY.2.OA.2").
- Use representations and manipulatives that show up in core instruction (e.g., 10 frames, number lines, base-10 blocks, bar models) so the student recognizes them when they return to Tier 1.
- Move to the next higher skill once the student demonstrates an appropriate understanding of the prerequisite. The point is to reduce the gap with current-grade work, not to stay below grade level for the school year.
- Plan for and emphasize coherence with Tier 1 instruction whenever possible.

Section 4: Evidence-Based Practices

BOTH TIERS The following six recommendations come from the 2021 IES Practice Guide, *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*. At both Tier 2 and Tier 3, these practices separate strong intervention from less effective, "more of the same" intervention. The difference between the tiers is intensity (e.g., group size, dose) and individualization of content, not which practices are used.

4.1 Recommendation 1: Systematic instruction

Build skills incrementally. Begin with the easiest case of the new content, model it explicitly, support guided practice with corrective feedback, and gradually fade supports as the student gains proficiency. Sequence the next concept so that it builds logically on what was just taught. Include cumulative review of previously taught skills so they do not erode.

BOTH TIERS **Try a four-move explicit teaching cycle.**

1. I do (model): "Watch how I solve 8 + 5 by making a 10."
2. We do (guided): "Let's try 8 + 6 together. What do we take from the 6 to make 10?"
3. You do (independent): "Now try 8 + 7 on your own; I'll watch."
4. We review (cumulative): "Yesterday we worked on adding from 9. Quick check: What's 9 + 4?"

Fading in practice. The "I do, we do, you do" cycle highlights how to "fade" (or gradually reduce scaffolds) at the cycle level. You can also fade within a stage, such as moving from full manipulatives to crossed-out manipulatives to drawings to symbols alone; or moving from full sentence frames to partial frames to student-generated explanations. The goal is to remove scaffolds as soon as the student can succeed without them and add them back the moment errors return.

4.2 Recommendation 2: Mathematical language

Teach precise math vocabulary on purpose. Use the same words across Tier 1, Tier 2, and Tier 3 and in recommended home-based activities. Avoid imprecise terms like *borrowing*; instead, use *regrouping*. Additionally, avoid introducing "fun" names (such as the "flip-flop" property instead of the commutative property) to prevent confusion when future instruction does not use the "fun" name and students are expected to know the formal vocabulary. Require students to use math vocabulary when they explain their reasoning.

BOTH TIERS Try a math word wall (post in every space where the student learns math, adding words as they are introduced in instruction).

- **Counting and numbers.** Cardinality, subitize, greater than, less than, equal to, digit, place value, ten, hundred
- **Operations.** Addend, sum, starting number (minuend), number taking from (subtrahend), difference, factor, product, quotient, regrouping, partial sum, partial product
- **Representations.** Number line, 10 frame, bar model, array, area model, unit fraction, equivalent

4.3 Recommendation 3: Provide students with multiple representations of the math

Move students deliberately across three forms of the same math: concrete (something you can hold, something that is 3D); representational (2D depictions like drawings, 10 frames, and number lines; sometimes called semi-concrete or pictorial); and abstract (numerals and equations; symbolic). Do not skip stages. Do not stay too long in the concrete. Table 4.1 provides an example of a Concrete–Representational–Abstract (CRA) approach to a specific problem.

Table 4.1. CRA template: A worked example for the problem "23 – 17"

Stage	What the student does
Concrete (C)	Build 23 with 2 ten sticks and 3 unit blocks. To subtract 17, the student cannot take 7 units from 3 units; trade 1 ten stick for 10 units (regroup). Now there are 13 units; take 7 away. Take 1 ten stick away. Read the answer: 6.
Representational (R)	Draw 2 sticks and 3 dots. Cross out 1 stick and redraw it as 10 dots. Cross out 7 dots and 1 stick. Count what's left.
Abstract (A)	Write 23 − 17 in vertical form. Mark the regrouping (2 tens and 3 ones become 1 ten and 13 ones). Carry out the standard algorithm. Read the answer: 6. Restate verbally: "Twenty-three minus 17 is 6 because I regrouped 1 ten into 10 ones to subtract 7."

Note: The IES Practice Guide (2021) refers to the middle stage as "concrete and semi-concrete representations." This is the same stage as Representational. We use Representational throughout this guide because it matches the CRA acronym many K–3 teachers know.

Tip. When you move from C to R, keep the picture next to the manipulative for a few problems. When you move from R to A, keep the picture next to the equation. The transition between the stages is where the learning happens.

4.4 Recommendation 4: Use the number line

The number line is the single most important representation in grades K–3. It builds understanding of magnitude, ordering, operations, and (in grade 3) fractions. Use it in every relevant Tier 2 or Tier 3 session once students can count meaningfully.

- Open number lines: for adding and subtracting (jumps of 10 and 1)
- Vertical number lines: for tracking goals (fluency progress) and measurement
- Fraction number lines: for placing unit fractions, comparing 1/2 to 2/4, and locating sums like 1/4 + 1/4 + 1/4 + 1/4 = 4/4 = 1

4.5 Recommendation 5: Word problems and schemas

Teach the underlying structure of word problems explicitly. Never teach keyword strategies because they fail on multistep and comparison problems. Use the schema names in table 4.2; students can sort problems by schema long before they can solve them all.

Table 4.2. Common schemas for math word problems

Schema	Plain language description	Example representation
Add to (join)	Start with some; add more; end with all	Bar model with two parts and a whole
Take from (separate)	Start with all; take some away; end with what is left	Bar model with a whole and a removed part
Put together and take apart	Two parts and a whole; no action over time	Bar model (two parts make one whole)
Compare	How much more or less is one quantity than another	Two stacked bar models with a difference bar
Equal groups (grades 2 and 3)	Some number of items is partitioned into equal-size groups; the total, group-size, or number-of-groups can be unknown	Array or grouped circles

4.6 Recommendation 6: Brief, timed activities for fluency

After conceptual understanding is in place, short (1–5 minute) fluency routines build automatic recall of facts and procedures. Keep them positive, predictable, and tied to the student's growth—never to peer comparison. A typical structure consists of a 1-minute warmup, a 1-minute timed set, and a 1-minute self-check to chart progress.

Section 5: Progress Monitoring, Decision Rules, and Modifications

Progress monitoring compares baseline student data with ongoing progress data at regular intervals to check for continuous progress (704 KAR 3:095). As described on page 25 of the *KyMTSS Implementation Guide*, progress monitoring enables teachers to make data-informed decisions about a student's response to an intervention.

Effective progress-monitoring tools include alternate forms of problems of equal and controlled difficulty, specify acceptable performance levels, define benchmarks for end-of-year performance, and are valid and reliable for students with intensive needs. Such tools, when matched with clear decision rules, allow teachers to modify interventions as necessary to meet their students' needs.

5.1 Pick the right probe

Progress-monitoring probes should always align with a defined target as described in table 5.1. The progress-monitoring probe must (a) match the specific skill you are teaching, (b) be sensitive to small amounts of growth in a short time, and (c) be brief enough to deliver in 1–5 minutes. The same types of probes work for both tiers; what differs is the cadence.

Table 5.1. Example progress-monitoring probes for K–3 math

Target	Example probe
Counting and cardinality (kindergarten)	Oral counting to 20; asking "how many?" with sets of 0–20; quantity-discrimination probes
Pairs that make 10 (grade 1)	1-minute missing-addend probe with 10 frame supports
Adding within 20 (grades 1–2)	1- to 2-minute basic-facts probe (mixed addends within 20)
Add and subtract within 100 (grade 2)	Computation probe (mixed two-digit problems with and without regrouping)
Multidigit addition fluency (grade 3)	Computation probe within 1,000 with regrouping; weekly digits correct
Multiplication within 100 (grade 3)	1- to 2-minute multiplication-facts probe; weekly target-set growth
Fractions on a number line (grade 3)	Brief fraction-placement task with four to six items per week

5.2 Apply the recommended cadence

Progress monitoring varies in intensity, depending on the tier:

- **TIER 2** Progress monitoring occurs every other week (monthly at minimum). Review student progress trends every six to eight data points (about every 12–16 weeks).
- **TIER 3** Progress monitoring occurs weekly. Review student progress trends every six to eight data points (about every 6–8 weeks).

5.3 Reassess after six to eight data points

Use the decision tree in table 5.2 to determine next steps, including any necessary modifications to the intervention.

Table 5.2. Decision tree after six to eight progress-monitoring data points

Pattern	Action
Positive: trend line meets or exceeds goal line	Continue. If the student met the goal early, set a more ambitious goal aimed at a grade-level benchmark. Plan to exit the student from additional support when mastery is sustained.
Questionable: highly variable data, no clear trend	Check implementation fidelity (attendance, dosage, implementation). If fidelity is intact, hold the plan for four more data points; if it is not, fix implementation first.
Poor: flat or falling trend, gap widening (see section 5.5)	Work through the National Center on Intensive Intervention Taxonomy (see section 5.5) in order: • Strength • Dosage • Alignment • Transfer • Comprehensiveness • Behavior • Individualization Change one dial at a time. If you are in Tier 2, this is when to consider escalating to Tier 3.

5.4 Escalate to Tier 3 or fade to Tier 2

Plan the bridges between tiers from day 1 to ensure that interventions are responsive to students' changing instructional needs. Consider the following recommendations when planning:

- **TIER 2** Escalate to Tier 3 when both of the following are true:
  - » After six to eight fidelity-checked data points, the trend line is flat or below the goal line.
  - » The team has worked through the National Center on Intensive Intervention (NCII) Taxonomy, or the diagnostic data show a more foundational gap than first thought.
- **TIER 3** Fade to Tier 2 when both of the following are true:
  - » Progress is at or above the goal line for 4 consecutive weeks.
  - » The student demonstrates the target skill in the Tier 1 classroom.
    (NOTE: Ensure that the team has a maintenance plan, such as a continued daily number talk and quarterly progress check.)

5.5 Next steps: When intervention results are below expectations

TIER 3 When a Tier 3 plan is not working—or when a student receiving Tier 2 intervention is improving but not at the necessary rate—the NCII Taxonomy of Intervention Intensity provides seven concrete dials to turn to improve intervention effectiveness. Use table 5.3 to walk through the dials in order and change only one at a time, so you can tell which dial matters.

Table 5.3. NCII Taxonomy of Intervention Intensity dials

Dial	Question to ask the team	What changes if the answer is "no"
1. Strength	Is the program or practice supported by strong K–3 math evidence?	Swap in a stronger program or routine.
2. Dosage	Are we delivering enough opportunities to respond and receive corrective feedback?	Add minutes or sessions or shrink group size.
3. Alignment	Is the intervention matched to this student's specific gap (not just a general program)?	Rediagnose; adjust target.
4. Attention to transfer	Are we explicitly bridging the skill to Tier 1 work?	Add bridging tasks and shared representations.
5. Comprehensiveness	Are we using the full set of explicit-instruction principles (e.g., modeling, faded supports, cumulative review)?	Strengthen the instructional design.
6. Behavioral support	Are attention, self-regulation, or motivation interfering with learning during the math block?	Add behavioral supports within the math intervention; coordinate with the student's social, emotional, and behavioral plan.

Dial	Question to ask the team	What changes if the answer is "no"
7. Individualization	Are we adjusting based on the student's own data in response to ongoing progress monitoring?	Move to data-based individualization (DBI) cycles. (NOTE: For more information on NCII's DBI framework, see What is Data-Based Individualization?)

Section 6: Family Partnership

Family partnership is the math-specific application of the KyMTSS essential element Family, School, and Community Partnerships (see the *KyMTSS Implementation Guide* to learn more about this essential element), and it is a significant amplifier of both Tier 2 and Tier 3 interventions. A family that knows the skill, can see the data, and has a good at-home routine reinforces what's happening in school in ways that meaningfully accelerate student gains. Strong school–family partnerships that specifically address math have been shown to positively influence children's math achievement (Nelson et al., 2024; Berkowitz et al., 2015). Although this section recommends specific aspects of family partnership when students are receiving Tier 2 or Tier 3 interventions, there should be consistent communication with all families about Tier 1 instruction as well. This ensures that families have the context to understand how to best support their child when intervention is required.

6.1 Share data without jargon

Use plain language and concrete examples. Instead of saying, "Marcus is at the 25th percentile on multidigit fluency," try saying, "Marcus knows how to add three-digit numbers. What we're working on is speed—making the math automatic so it stops feeling hard." Send a chart with the goal line and the trend line; explain what the lines mean.

6.2 Model at-home routines

Teachers can use the examples in table 6.1 to engage parents/guardians in brief math activities with their children. These routines are easy to implement and provide family members with an opportunity to help children strengthen their mathematical understanding.

Table 6.1. At-home math routines

Routine	How to support parent/guardian implementation
Number talks at dinner	• Provide parents/guardians with one short (2-minute) prompt per night, such as "How many ways can you make 8?" or "What's 6 + 7, and how did you do it?" • Encourage parents/guardians to praise the strategy, not the answer.
Math games	• Select a game from the list of Related Instructional Resources in Appendix B and share the directions with parents/guardians. • Highlight a game at a family event at school.
Math noticing	• Encourage parents/guardians to point out math in the world: "How many minutes until dinner?" "How many quarters in a dollar?" "If we split this pizza into six pieces, what fraction do you get?"

Parents/guardians may also benefit from suggestions of mathematical games to play with their child. The Kentucky Family Math Resources website has examples of activities families can do at home.

6.3 Two-way communication

Ask parents/guardians what they notice: "Has Marcus talked about math at home? What does he do confidently? What does he avoid?" Their answers will inform the next problem-solving meeting and often surface barriers (e.g., homework time, language, prior experience) that other data alone will not.

Appendix A: Glossary

Term	Plain language definition
Cardinality	The understanding that the last number named when counting a set tells the total quantity
Concrete–Representational–Abstract (CRA)	An instructional sequence that moves from physical manipulatives to drawings and other representations to abstract symbols and equations (Note that the 2021 IES Practice Guide refers to the middle stage as "semi-concrete.")
Data-based individualization (DBI)	NCII's iterative five-step process for tailoring intervention to a student who is not responding: (1) implement a validated intervention, (2) monitor progress, (3) collect diagnostic data when needed, (4) adapt using the Taxonomy of Intervention Intensity, and (5) continue monitoring (Note that the Taxonomy is the framework for the adaptation step within DBI.)
Diagnostic assessment	A formal or informal student assessment using valid and reliable tools, given to guide instruction and tailor interventions according to individual student academic and behavioral strengths and needs to accelerate progress toward proficiency; provides data about students' knowledge and skills and helps the problem-solving team identify the intervention that is the best match for a student or group of students
Fidelity	The degree to which an intervention is delivered as it was designed
Kentucky Academic Standards (KAS)	Standard codes appear as KY.[grade].[domain].[standard] (e.g., KY.2.OA.1)
Kentucky's Multi-Tiered System of Supports (KyMTSS)	The integrated framework for academic, behavioral, and social-emotional supports (704 KAR 3:095)
National Center on Intensive Intervention (NCII) Taxonomy of Intervention Intensity	Seven dimensions—strength, dosage, alignment, attention to transfer, comprehensiveness, behavioral support, and individualization—used to intensify interventions when students are not responding; used inside DBI cycles
Progress monitoring	Brief, repeated measures aligned to the intervention target; used to evaluate whether the intervention is working by comparing baseline data collected prior to invention with ongoing progress data
Schema-based instruction	Teaching students to identify the underlying mathematical structure of word problems (e.g., join, take from, compare, equal groups) rather than relying on keywords

Term	Plain language definition
Subitizing	Recognizing the quantity of a small set without counting
Tier 1, Tier 2, Tier 3	Levels in the tiered delivery system—Tier 1 is universal instruction for all; Tier 2 is supplemental small-group support; Tier 3 is intensive, individualized intervention
Tier 2 Lesson Plan	A documented, repeatable instructional sequence shared across the group at Tier 2 that the group works through together
Universal screener	A valid and reliable tool to assess learning and achievement as part of a systematic process for analyzing students' performance at certain points during the academic year; may include validated indicators to evaluate the effectiveness of Tier 1 universal instruction and determine which students need closer monitoring or intervention

Appendix B: Resources, References, and Related Research

Kentucky Policy and Standards

- KRS 158.8402—Kentucky Numeracy Counts Act, HB 162
- 704 KAR 3:095—Kentucky Multi-Tiered System of Supports (KyMTSS) administrative regulation
- Kentucky Academic Standards (KAS) for Mathematics
- Kentucky Department of Education (KDE)—Numeracy Screening Assessments
- KY Numeracy Counts—Kentucky Mathematics Improvement Plan Requirements

Kentucky Multi-Tiered System of Supports

- KyMTSS website
- KyMTSS Implementation Guide (KDE)
- KyMTSS Self-Assessment Tool (KDE)

Related Instructional Resources

- Bedtime Math
  - » Free daily math story problems by age band (preK, grades K–2, grade 3 and higher) via website, email, or app
  - » Designed for short parent/guardian–child math conversations at home
- Development and Research in Early Math Education (DREME) Family Math, Stanford University
  - » Free early-math activities, "Math Snacks," and simple card and number games for home and school (infant–age 8; English and Spanish)
  - » Supports the family routines in section 6 of this toolkit
- Center for Instructional and Behavioral Research in Schools (CIBRS)
  - » Free instructional videos, including two about the six principles in the Institute of Education Sciences (IES) Practice Guide, *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* (2021); short videos on evidence-based strategies; and certificate-earning online courses

- IES Practice Guide, Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades (2021)
  - » Free downloadable practice guide and summary
  - » Informs the six recommendations in section 4 of this toolkit
- IRIS Center, Vanderbilt University
  - » Free, self-paced professional learning modules with optional professional development hours, including "RTI/MTSS: Mathematics," "High-Quality Mathematics Instruction," and explicit, systematic instruction
- Kentucky Center for Mathematics (KCM)
  - » Statewide hub of free K–4 numeracy resources, on-demand professional learning, and Numeracy Counts supports for Kentucky teachers
- Kentucky Center for Mathematics Fluency Readiness Assessments
  - » Free assessments aligned to KAS for grades K–3
  - » Used to pinpoint the foundational skills and understandings that underpin fluency
- Kentucky Numeracy Project (KNP) Instructional Resources
  - » Free, searchable bank of proven K–4 differentiated numeracy activities (e.g., lesson plans, student pages, printables) that can be filtered by KAS, grade level, fluency benchmark, task group, or setting
  - » Ideal for building Tier 2 and Tier 3 lessons
- KNP Family Math
  - » Free Kentucky-specific family math activities and guidance for teachers to share with parents/guardians, supporting the home routines in section 6 of this toolkit
- National Center on Intensive Intervention (NCII)—Mathematics
  - » Free sample intervention lessons and activities across six K–8 skill areas, plus the DBI process and progress-monitoring guidance referenced in section 5 of this toolkit
- NCII Taxonomy of Intervention Intensity
  - » Free tool and worksheet for adjusting the "dials" of intervention intensity (see table 5.3 in this toolkit) to intensify and individualize support
- The Math Learning Center
  - » Free math apps and web, iPad, and Chrome virtual manipulatives (e.g., number frames, 10 frames, number line, number rack, number pieces, base-10 blocks)

References and Related Research

K–3 mathematics intervention

- Nelson, G., & McMaster, K. L. (2019). The effects of early numeracy interventions for students in preschool and early elementary: A meta-analysis. *Journal of Educational Psychology, 111*(6), 1001–1022. https://doi.org/10.1037/edu0000334
  - » Examined 34 studies on the effectiveness of early numeracy interventions for young

students, including students with disabilities or those at risk for math difficulty (MD); average weighted effect size was moderate (g = 0.64);
  - » Found that interventions emphasizing counting with one-to-one correspondence showed larger effects
- Myers, J. A., Witzel, B. S., Powell, S. R., Li, H., Pigott, T. D., Xin, Y. P., & Hughes, E. M. (2022). A meta-analysis of mathematics word-problem-solving interventions for elementary students who evidence mathematics difficulties. *Review of Educational Research, 92*(5), 695–742. https://doi.org/10.3102/00346543211070049
  - » Supports Recommendation 5 in this toolkit (to provide explicit instruction in problem structure/schemas)

Family engagement and math outcomes

- Nelson, G., Carter, H., Boedeker, P., Knowles, E., Buckmiller, C., & Eames, J. (2024). A meta-analysis and quality review of mathematics interventions conducted in informal learning environments with caregivers and children. *Review of Educational Research, 94*(1), 112–152. https://doi.org/10.3102/00346543231156182
  - » Examined 25 preK–3 studies with 83 effect sizes that yielded a statistically significant summary effect (g = 0.26, 95% CI [0.07, 0.45]) on children's math achievement
  - » Showed larger effects when caregiver training included follow-up support
  - » Supports section 6 of this toolkit
- Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science, 350*(6257), 196–198. https://doi.org/10.1126/science.aac7427
  - » Randomized controlled trial of the Bedtime Learning Together parent/guardian–child math app with 587 first-graders
  - » Landmark study of structured at-home math interaction

Math games for K–3

- Nelson, G., Boedeker, P., Devlin, B. L., Lesner, T., Drake, K. R., Sutherland, M., Wilke, E. J., Cook, M., & Schmitt, S. A. (2025). Investigating main effects and moderators of linear number board games: A meta-analytic review. *Review of Educational Research*. https://doi.org/10.3102/00346543251383552
  - » Examined 18 preK–2 studies with 123 effect sizes that yielded a statistically significant summary effect (g = 0.21, 95% CI [.08, 0.34], p = .004) on children's mathematics outcomes.
  - » Showed brief play with linear number board games improves counting, number recognition, and magnitude understanding
- Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*(2), 375–394. https://doi.org/10.1111/j.1467-8624.2007.01131.x

- Seminal experiment that showed about 1 hour of play with a linear number board game produced broad, lasting gains, but 1 hour of play with squares varying in color rather than number did not lead to improvement on any measure; provided foundational evidence for linear number board games

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