Geometry
Attributes of Shapes
Grade 3
Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

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Revised 2017
Attributes of Shapes

Mathematical goals
This lesson is intended to help you assess how well students are able to:

- Identify defining attributes of shapes
- Think abstractly and concretely
- Defend mathematical decisions

Kentucky Academic Standards
This lesson involves mathematical content in the standards from across the grade, with emphasis on:

Geometry 3.G – Reason with shapes and their attributes

This lesson involves a range of Standards for Mathematical Practice, with emphasis on:

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
6. Attend to precision
7. Look for and make use of structure

Introduction
In this lesson, students use the attributes of shapes to view shapes as members of both specific subcategories and larger, more general categories.

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work in pairs on a collaborative discussion task to sort shapes into categories with common attribute and identify the larger categories from which the subcategories “came from”. As they do this, they interpret the cards’ meanings and begin to link them together. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.
- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Materials required
Each individual student will need:

- The Attributes of Shapes Pre Lesson Assessment task.
- The task sheet Shapes which should already be cut up and neatly packaged for students.

Each pair of students will need the following resources:

- One legal sized paper for each pair of students.
• *Extension Cards* (not required but should be available to differentiate for students who find the challenge level of the Shapes task too easy).

**Time needed**
Approximately 15 minutes before the lesson for the individual assessment task, one 60 minute lesson, and 15 minutes for a follow-up lesson for students to revisit individual assessment task. Timings given are approximated. All students need not complete all sets of cards activities. Exact timings will depend on the needs of the class.

**Before the Lesson**

**Assessment task:**
Have students do the pre lesson task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will be able to target your help more effectively in the follow-up lesson.

**Framing the pre-assessment:**
Give each student a copy of the assessment task.

*Today we are going to work on a task about the attributes of shapes. This task is to help me see ways that I can help you if you are having any problems with identifying attributes of shapes. If you are not sure about all of your answers, it is okay. We are going to do an activity that will help you improve. You will identify one attribute that at least three of the shapes share. Write down the attribute and the shapes that have this property.*

It is important that the students are allowed to answer the questions without your assistance, as far as possible. If students struggle to get started ask questions that help them understand what they are being asked to do, but do not do the problem for them. See the *Common Issues* table on the following page for suggestions.

Students should not worry too much if they do not understand or cannot do everything, because in the next lesson they will engage in a task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently.

**Assessing students’ responses**
Collect students’ responses to the task. Make notes about what their work reveals about their current levels of understanding, and their different problem solving approaches and strategies. Partner students with other who displayed similar errors/misconceptions on the pre-assessment task.

We suggest that you do not score student’s work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions in the *Common Issues* table may serve as examples. These questions have been drawn from commonly identified student misconceptions.

We suggest that you write a list of your own questions, based on your students’ work, using but not limited to the ideas that follow. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written/displayed on the board at the beginning of the lesson or during the whole group discussion portion of the lesson.
Below is a list of common issues and questions/prompts that may be written on individual tasks or asked during the collaborative activity to help students clarify and extend their thinking. (Leave a couple of blank spaces for teachers to add their own common issues and suggested questions and prompts.

<table>
<thead>
<tr>
<th>Common Issues:</th>
<th>Suggested questions and prompts:</th>
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| Students have a difficult time getting started.                               | • What are the directions? Read them again please.  
• Choose four or five shapes. Within this smaller set, do these shapes share any attributes?                                                                                                                              |
| Students don’t seem to be aware of the attributes of shapes they might consider when sorting. | • What kind of shape is this? Draw another (identified shape). What qualities or attributes make this shape a (the identified shape)?  
• What kinds of attributes define shapes? For example, in what ways might a quadrilateral differ from a triangle? A rhombus from a square?  
• Does the fact that this (point to a smaller triangle) is smaller than this (point to a larger triangle) mean these two shapes belong to different categories?  
• What is the same and what is different about these two shapes?                                                                                           |
| Students are not applying their knowledge of defining attributes to sort shapes in ways that are logical. | • What is an attribute that (identify a shape) this shape has? Write that down on a scrap piece of paper and might you use that as a heading to help group other shapes. |
| Students have difficulty identifying the common attributes of shapes.        | • What are the defining attributes of shapes? Let’s write them down. Do any of these shapes share these attributes?                                                                                                           |
| Students have difficulty finding attributes that would allow a class to be divided into subclasses. | • What attribute is it that all of these shapes have in common? Are there any defining attributes that are different among the shapes you have identified?  
• What have you tried to do so far?  
• Have you tried making a list of the attributes you’ve already investigated? May I see it?  
• What is the same and what is different about these two shapes? (select two)                                                                                     |
| Students find the initial task/lesson task too easy.                         | • Distribute the set of Extension Cards.                                                                                                                                                                                                |
Suggested lesson outline

Whole class introduction (10 Minutes)

Introduce the lesson carefully:

Today we are going to look at ways to put shapes into categories by identifying their attributes.

Things other than shapes have attributes, such as people, students and animals. For example, all of you share an attribute because you are all students in this class. How might we use some of the different attributes you have to organize you into smaller groups?

Record students’ ideas on chart paper. This graphic is an example and you are encouraged to allow the students to identify other subcategories that apply to your class situation.

Collaborative Activity

Display or distribute the list of questions you created in response to student work on the Attributes of Shapes Pre Lesson Assessment task.

I have read your solutions to the task Attributes of Shapes and I have some questions I’d like you to think about during today’s work time. These questions will frame today’s lesson and learning.

Collaborative Work (20 minutes)

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the preassessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.
I want you to work with your partner. Today you and your partner are going to sort shapes according to defining attributes. You are also going to discuss some of the attributes shared by various shapes.

You and your partner may organize your work similar to the organizer we made for our class using labels and branches or you may choose a different model that make sense to you. You may sort your shapes into any categories you like, as long as all the shapes in any category share the defining attributes of that category.

Each time you do this, explain your thinking clearly to your partner. If your partner disagrees with your placement then challenge him or her to explain why. It is important that you both understand why each shape is placed where it is. There is a lot of work to do today and you may not all finish. The important thing is to learn something new, so take your time.

Give each pair a set of Shapes and 1 piece of legal sized paper as a work mat.

- Your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving. As you monitor the work, listen to the discussion and ask questions to help students understand concepts and clarify misconceptions. Misconceptions to look for:
  - Did some pairs “skip” categories or go directly to the specifics?
  - At any branch point, do any of the stemming categories fit into any of the other stemming categories? For example, if a student went from QUADRILATERALS to SQUARES, RECTANGLES and RHOMBI then attention needs to be given to the attributes these three stemming share and do not share.

Make a note of student approaches to the task and find out about student methods
You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. As students continue to sort shapes into categories and sub categories, circulate about the room; the purpose of circulating is to listen and monitor student methods/arguments. Use the information you gleaned from the initial, individual task to decide where to focus your attention.

Research indicates that teachers who have thoughtfully considered anticipated student responses are better able to address those misconceptions if they occur; moreover, these teachers are better able to address student misconceptions when they arise.

- Try to determine if students are making progress beyond their initial work and if they aren’t remind them of the feedback questions you have provided to them, either via whole class feedback or individual feedback.
- Are students able to reconcile the approaches and strategies of their partners to their own work if the partner’s work is different?
- Are students organizing their work in a systematic way keeping track of insights as they arise?
- Have students tried eliminating possibilities from categories?
- Once students arrive at a comfortable solution, do they reflect on their approach and communicate the approach, conclusion and reasoning in a mathematically precise way?

Support student problem solving
Try not to make suggestions that move students toward a particular sorting scheme and try not to point out the difficulties with their chosen methods. Instead, ask questions to help students clarify their thinking and evaluate their own sorting. Encourage students to use each other as a resource for learning.
If one student has placed a particular card on the chart, challenge their partner to provide an explanation.

If you find students have difficulty articulating their decisions, then you may want to use the questions from the Common Issues table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a brief whole class discussion.

If a pair finishes their solutions early or if a pair finds the initial task too easy, provide them with the Extension Cards – task them with determining where the extension cards would fit into their organizer and identify any congruent or similar attributes to the first set of shapes.

Sharing Work (10 minutes)
Once most pairs have finished with the initial card sort, structure a pair-square, in which the original pair of students pairs up with another pair, making a “square” of four students. Invite the two pairs to share the ways they sorted the shapes with one another.

Whole class discussion: comparing different approaches (10 minutes)
Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Select students or pairs who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each group shares, highlight the connections between strategies.

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions the students come up with.

- How did you decide what categories to use?
- Were there some shapes you struggled placing and why were they difficult?
- Were there shapes that fit more than one category and what attributes did you use to make that decision?
- Were there shapes that fit into more categories than others and why do you think that happened?
- What was the smallest group to which a shape belonged?

Try to avoid making evaluative comments yourself. Instead, encourage students to respond to other students’ explanations.

If some of your students were given the Extension Cards save their sharing for last.

Taking two class periods to complete all activities
If you have to divide the lesson into two class periods, you may want to have a way for students to save the work they have done with the place card sets. You may have each group tape the cards down with on their place cards. You may choose to have one or two pairs do this even if you are not dividing up the class period just to use as a visual during the class discussion or to refer to later as an anchor chart.

Extension activities
If some of your students were given the Extension Cards save their sharing for last.

Improving individual solutions to the assessment task (10 minutes)
Give the students a new copy of the original task,

Think about what you have learned during this lesson. Using what you have learned try to improve your work.
Or ask students to reflect and write about what they have learned during the lesson.

To focus your students, refer to the common issues chart. Use the questions which reflect the greatest need(s) of your students.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.
1. Some of these shapes have attributes in common that we have learned about in the past. Identify one attribute that at least three of the shapes share. Write down the attribute and the shapes that have this property.

2. Do the same thing again, but this time, choose a different attribute. It’s ok if you want to reuse any of the shapes from the first problem.

3. Find and identify at least two other shapes in the original assortment that share at least one attribute with this shape. Which attribute do they share?
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<tr>
<td>K</td>
<td>U</td>
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<tr>
<td>C</td>
<td>W</td>
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</tbody>
</table>
Attributes of Shapes – Extension Cards

1. S
2. M
3. F
4. N
5. T
6. Q