



Multiplication Version 2

(With Distraction Cards)

Grade 3

Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

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Revised 2019

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical Goals

This lesson is intended to evaluate the understanding of your students in multiplication and how well students are able to interpret various representations of multiplication facts. It will help you to identify students who have the following difficulties:

- lack of conceptual understanding of the properties of numbers
- do not see connections from addition to multiplication
- have multiplication facts memorized, but lack conceptual understanding

Kentucky Academic Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

Operations and Algebraic Thinking 3.OA

- Represent and solve problems involving multiplication and division.
- Multiply and divide within 100.

This lesson involves a range of *Standards for Mathematical Practices*, with emphasis on:

1. Make sense of problems and persevere in solving them.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Introduction

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work in pairs on a collaborative discussion task to match representations of multiplication problems. As they do this, they interpret the cards' meanings and begin to link them together. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.
- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Materials required

Each individual student will need:

- Two copies of the assessment task *Multiplication*. (One for the initial task and one for the revision.)

Each pair of students will need the following resources:

- Card Set A and B
- Have Card Set C ready, but do not pass out.
- Have Card Set D ready, but do not pass out.
- Have Card Set E ready, but do not pass out.
- Have Card Set F ready, but do not pass out.
- Have Card Set G ready, but do not pass out.
- Have Card Set H ready, but do not pass out.

| Multiplication | |
|--|--------------|
| Using the multiplication problem at the top of each chart, create a representation of the problem that matches the labels in each of the four boxes. | |
| $4 \times 7 = 28$ | |
| Area Model | Equal Groups |
| Repeated Addition | Word Problem |
| $7 \times 8 = 56$ | |
| Area Model | Equal Groups |
| Repeated Addition | Word Problem |

Time needed

Approximately 15 minutes a day or two before the lesson (for the individual assessment task), one 40 minute lesson (30 minutes for group task and 10 minutes for whole class discussion), and 15 minutes for students to revisit individual assessment task. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the Lesson

Assessment task: Multiplication (15 minutes)

Students should have already been exposed to lessons that required them to build, draw, and write story problems for multiplication. Work should be focused around repeated addition, area models, use of arrays, equal groups, distributive property and story problems.

Have students do this task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with conceptual understanding of multiplication. You will be able to target your help more effectively in the follow-up lesson. Depending on your class you can have them do it all at once or in small groups (they should still work individually).

Give each student a copy of the assessment task *Multiplication*.

Using the multiplication problem at the top of each chart, create a model/representation of the problem that matches the labels in each of the four boxes.

It is important that the students are allowed to answer the questions without your assistance.

Students should not worry too much if they do not understand or cannot do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes about what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score student's work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions on the following page may serve as examples. These questions have been drawn from commonly identified student misconceptions.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson before students revisit initial task.

Below is a list of common issues and questions/prompts that may be written on individual initial tasks or during the collaborative activity to help students clarify and extend their thinking.

| Common Issues: | Suggested questions and prompts: |
|---|---|
| A group has trouble getting started. | <ul style="list-style-type: none"> • What information do you already know? • What do you need to find out? |
| Students have trouble drawing correct representation. Students may draw 3 groups of 6 instead of drawing 6 groups of 3. | <ul style="list-style-type: none"> • Use an example of cards that have already been matched and discuss. For example, the 3 groups of 4 card. Talk about how the picture and context of the problem would change if the card was 4 groups of 3. |
| Students are not attentive to the orientation of the array models. | <ul style="list-style-type: none"> • How is this model different if we turn it on its side? • Is the orientation of the model important? |
| Students misinterpret correct representation of each multiple in the problem. Although arranging the problem either way gives you the same answer, mathematically, it does not represent the same image. | <ul style="list-style-type: none"> • <i>How does the order of the factors affect the model?</i> • <i>Can you think of a situation that changing the order of the problem would not work? For example, if I have a party and I prepare 3 bags with 9 cookies in each bag. I will have 9 people at the party and each should get 3 cookies. Did the order matter?</i> |
| Students are not attentive to details and structure of the word problems. | <ul style="list-style-type: none"> • <i>Provide manipulatives for students.</i> • <i>Can you draw a model?</i> • <i>Can you act out what the problem is telling us?</i> |
| | <ul style="list-style-type: none"> • |
| | <ul style="list-style-type: none"> • |

Solution to Card Sort

Full Sort – the blank spaces are intentional to create distractors

Note: The initial expressions may not be in the order below.

| | | | | | | | | |
|--------------|--|--|--|--|--|--|--|--|
| 6×4 | | | | | | | | |
| 5×9 | | | | | | | | |
| 6×3 | | | | | | | | |
| 5×3 | | | | | | | | |
| 3×4 | | | | | | | | |
| 4×8 | | | | | | | | |
| 7×8 | | | | | | | | |
| 4×9 | | | | | | | | |
| | | | | | | | | |

Initial expressions (left column):

- 6×4
- 5×9
- 6×3
- 5×3
- 3×4
- 4×8
- 7×8
- 4×9

Final expressions (right column):

- 18
- 15
- 12
- 32
- 56
- $(2 \times 3) + (4 \times 3)$
- $(2 \times 3) + (3 \times 3)$
- $(2 \times 3) + (4 \times 2)$

Equations (top row):

- $3 + 3 + 3 + 3 + 3$
- $6 \text{ sets of } 3$
- $\text{Noah made 6 shots in his basketball game. Each shot was worth 3 points. How many points did Noah score?}$
- $(2 \times 3) + (4 \times 3)$

Equations (middle row):

- $3 + 3 + 3 + 3 + 3$
- $5 \text{ sets of } 3$
- $\text{Katrina has 3 oranges. Renae has five times as many oranges as Katrina. How many oranges does Renae have?}$
- $(2 \times 3) + (3 \times 3)$

Equations (bottom row):

- $4 + 4 + 4$
- $3 \text{ sets of } 4$
- $\text{Connie walked for 3 hours. She walked at the rate of 4 miles per hour. How many miles did Connie walk?}$
- $(2 \times 3) + (3 \times 3)$

Equations (far right):

- $8 + 8 + 8 + 8 + 8 + 8$
- $7 \text{ sets of } 8$
- $\text{Sarah has 8 bracelets. Madison has 7 times as many bracelets as Sarah. How many bracelets does Madison have?}$
- $(2 \times 8) + (2 \times 8)$

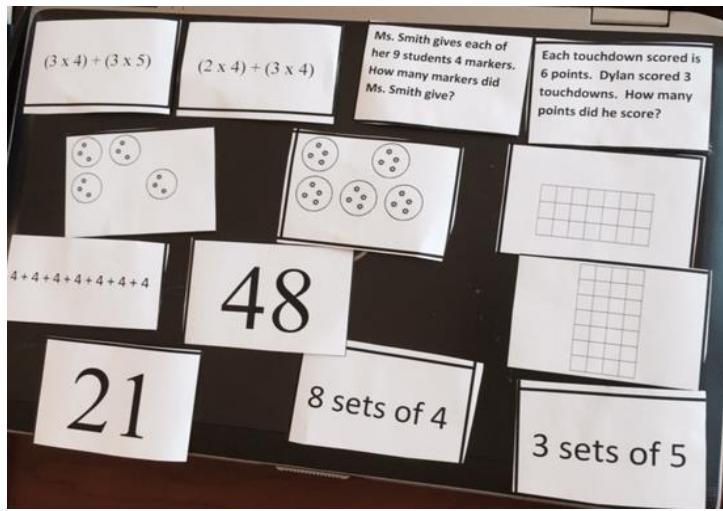
Equations (far left):

- $8 + 8 + 8 + 8 + 8 + 8 + 8$
- $7 \text{ sets of } 8$
- $\text{Joanna planted 7 rows of trees. She planted 8 trees in each row. How many trees did Joanna plant?}$
- $(5 \times 8) + (2 \times 8)$

Equations (bottom left):

- $4 + 4 + 4 + 4 + 4 + 4$
- 48
- 21
- $8 \text{ sets of } 4$
- $3 \text{ sets of } 5$

Distractor Cards



Suggested lesson outline

Whole Class Introduction (10 minutes)

Materials- whiteboard, dry erase markers, and an eraser

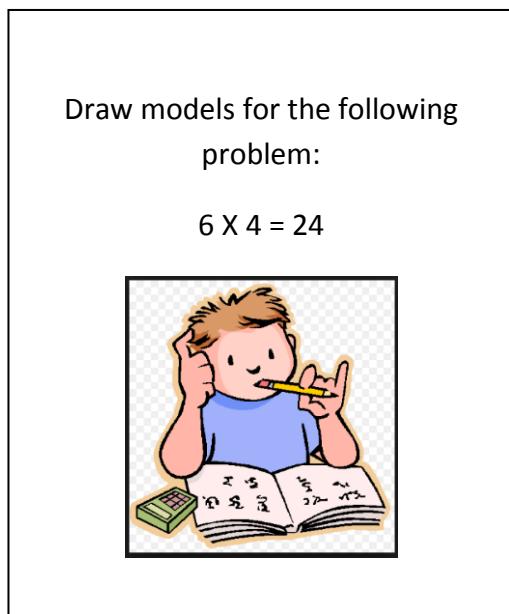
Instructions:

Today we are continuing our work with multiplication.

I want you to use the next three minutes to brainstorm models for this multiplication problem.

Draw your models on your whiteboard.

Display the problem $6 \times 4 = 24$. Set the timer.



Ready-Set-Think Start the timer.

Walk around and monitor students as they are working.
When the timer sounds, ask students to hold their boards up.
Ask students to share their models with a shoulder partner.
Point out student models and call on a few students to explain their model. Briefly compare different and similar models.

Collaborative Activity (30 minutes)

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the preassessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

Introduce the lesson carefully:

I want you to work together with your partner. You are going to work on matching cards that represent the same math idea. There may be extra cards that are not needed and all sets may not have a match.

This lesson is strategically designed for cards to not match. As students finish, they may use the blank cards to complete sets.

Have cards ready to show the students a model of two cards that would match and two cards that would not match. Have a student tell why the cards do/do not match.

Every pair of students may not work through all card sets. Groups should work at their own pace.

During the partner work, the teachers' tasks are to:

- question but not interfere with student work;
- make notes of student approaches to the task;
- support student problem solving through guided questions.

Make notes of student approaches to the task

You use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may not consider or understand the concept of multiplication. Students may be able to recall the facts, but have difficulty attaching the meaning to a model. This lack of understanding prevents students from applying multiplication in real life situations.

Support student problem solving

Try not to make suggestions that move students toward a particular approach to the task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

When a student creates a match, challenge their partner to provide an explanation.

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a brief whole class discussion.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations.

Card Set A and Card Set B

*Note: There will be several expression that do not have a matches and there may also be extra cards. However, there are **blank cards** provided for the students to draw their own representation to complete the set.

Card Set C

As students finish with Card Sets A & B and are able to explain their reasoning give them Card Set C. Do not take up the previous sets of cards. Students may use these for guidance in making further decisions. Card Set C will consist of area model representations. There may be extra cards and cards with no matches. This is designed to allow students to be more intentional with their thinking.

Card Set D

As students finish with Activity C and are able to explain their reasoning give them Card Set D. Card Set D will consist of repeated addition representations. *Note: There will be one blank card for students to fill in to complete this level. Do not tell students; allow them to develop this idea on their own. Again there may be extra cards and cards with no matches. This is designed to allow students to be more intentional with their thinking.

Card Set E

As students finish with Activity D and are able to explain their reasoning give them Card Set E.

Card Set E is the “sets of” cards. *Note: There will be one blank card in this set in which the students will need to fill in to complete the set. Do not tell students; allow them to develop this idea on their own. There may be extra cards and cards with no matches. This is designed to allow students to be more intentional with their thinking.

Card Set F

As students finish with Card Set E and are able to explain their reasoning give them Card Set F. Card Set F will consist of interpreting word problems. There will be extra cards and cards with

no matches. This is designed to allow students to be more intentional with their thinking.

Card Set G

As students finish with Card Set F and are able to explain their reasoning give them Card Set G. This is an opportunity for students to stretch their thinking of how the multiplication problem can also be represented through the distributive property of multiplication. There may be extra cards and cards with no matches. This is designed to allow students to be more intentional with their thinking.

Card Set H

As students finish with Card Set G and are able to explain their reasoning give them Card Set H. Card Set H will consist of finding the product. There may be extra cards and cards with no matches. This is designed to allow students to be more intentional with their thinking.

Sharing Work Optional (10 minutes)

When students get as far as they can with the task, ask one student from each pair to visit another pair's work. Students remaining at their desk should explain their reasoning for the way they worked the problem at their own desk.

If you are staying at your desk, be ready to explain the reasons for your pair's work.

If you are visiting another pair, check to see which answers or explanations are different from your own.

If their matches are different than yours, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider, as a pair, whether to make any changes to your work.

Provide time for pairs to discuss and make changes to their original work.

Extension activities

Students may create their own set of cards with different multiplication facts or add cards to sets with missing matches.

If time allows, they may challenge each other to sort their sets.

Whole-class discussion (10 minutes)

Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Select students or pairs who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each pair shares, highlight the connections between strategies.

Choose students to share how they figured out which model matched the expressions. Discuss which model they liked best and why.

Possible questions to ask:

- Which cards were easiest/hardest to match? Why?*
- What might be a different way to explain?*
- Did anyone do the same or something different?*
- How would you explain in words your model?*

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves extending what has been learned to new examples.

Improving individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Multiplication*, as well as a second blank copy of the task.

*Look at your original responses and think about what you have learned during this lesson.
Using what you have learned, try to improve your work.*

If you (teacher) have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

*Card sorting tasks adapted from www.makingmathmagic.com

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the [Math Assessment Project](#).

Multiplication

Using the multiplication problem at the top of each chart, create a representation of the problem that matches the labels in each of the four boxes.

$$4 \times 7 = 28$$

Area Model

Equal Groups

Repeated Addition

Word Problem

$$7 \times 8 = 56$$

Area Model

Equal Groups

Repeated Addition

Word Problem

Card Set A

$$3 \times 4$$

$$4 \times 8$$

$$6 \times 3$$

$$4 \times 9$$

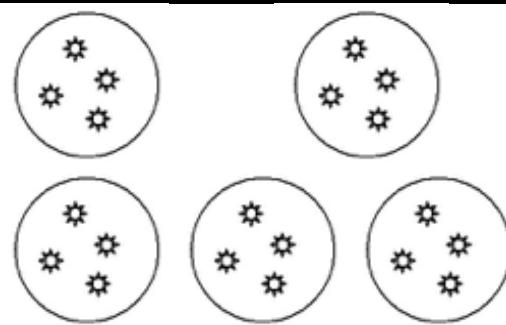
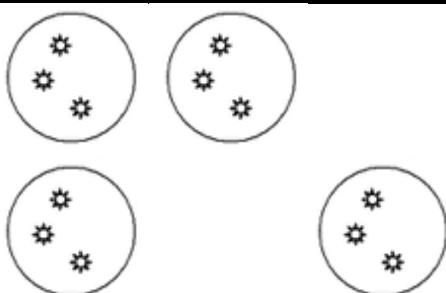
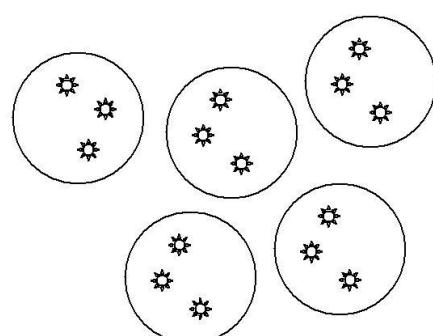
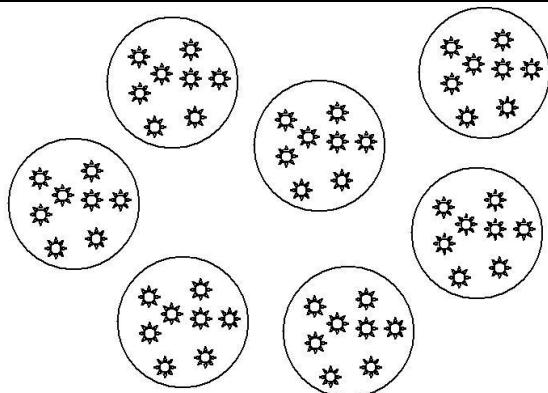
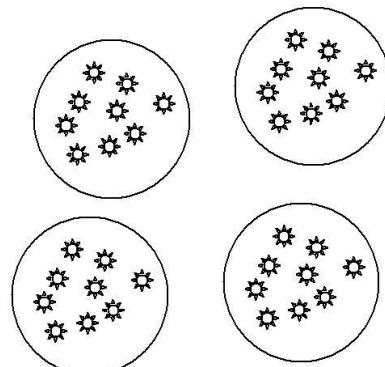
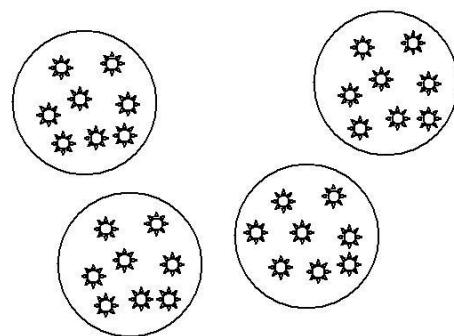
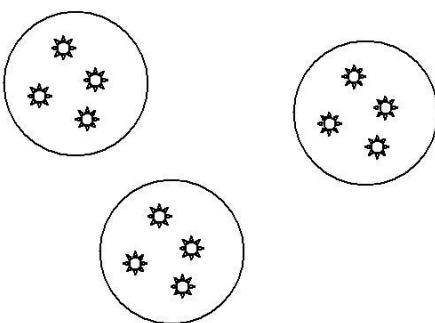
$$7 \times 8$$

$$5 \times 3$$

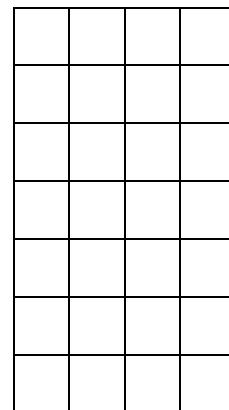
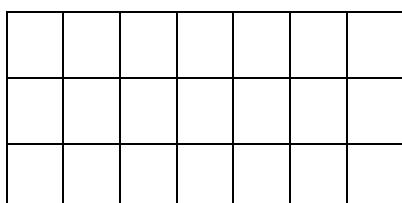
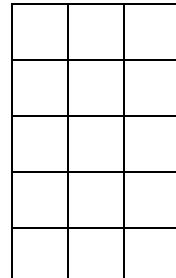
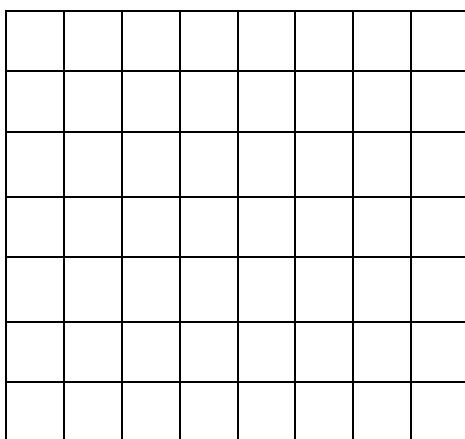
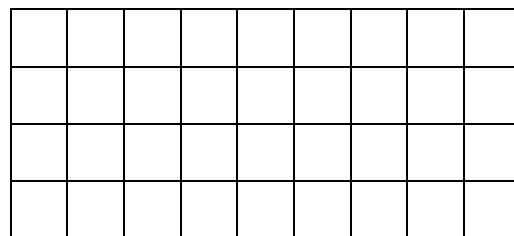
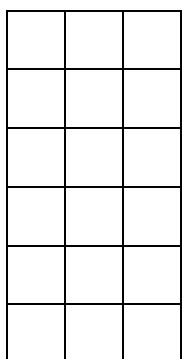
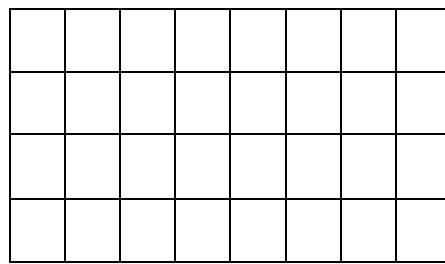
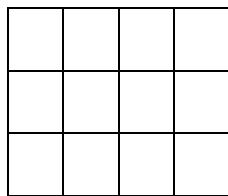
$$5 \times 9$$

$$6 \times 4$$

Card Set B



Card Set C



Card Set D

$$4 + 4 + 4$$

$$8 + 8 + 8 + 8$$

$$3 + 3 + 3 + 3 + 3 + 3$$

$$8 + 8 + 8 + 8 + 8 + 8 + 8$$

$$3 + 3 + 3 + 3 + 3$$

$$9 + 9 + 9 + 9 + 9$$

$$4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$$

Card Set E

3 sets of 4

6 sets of 3

4 sets of 9

7 sets of 8

5 sets of 3

3 sets of 5

8 sets of 4

Card Set F

Connlee walked for 3 hours. He walked at the rate of 4 miles per hour.
How many miles did Connlee walk?

Sarah has 8 bracelets. Madison has 4 times as many bracelets as Sarah.
How many bracelets does Madison have?

Noah made 6 shots in his basketball game. Each shot was worth 3 points.
How many points did Noah score?

Joanna planted 7 rows of trees. She planted 8 trees in each row. How many trees did Joanna plant?

Katrina has 3 oranges. Renee' has five times as many oranges as Katrina.
How many oranges does Renee' have?

Each touchdown scored is 6 points. Dylan scored 3 touchdowns. How many points did he score?

Ms. Smith gives each of her 9 students 4 markers.
How many markers did Ms. Smith give?

Card Set G

| | |
|-------------------------------|-------------------------------|
| | $(2 \times 8) + (2 \times 8)$ |
| $(2 \times 3) + (4 \times 3)$ | $(2 \times 9) + (2 \times 9)$ |
| $(5 \times 8) + (2 \times 8)$ | $(2 \times 3) + (3 \times 3)$ |
| $(2 \times 4) + (3 \times 4)$ | $(3 \times 4) + (3 \times 5)$ |

Card Set H

12

32

18

36

56

15

48

21