

Kentucky Academic Standards for Mathematics: Grade 4 Overview

Operations/Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Number and Operations Fractions (NF)	Measurement and Data (MD)	Geometry (G)
<ul style="list-style-type: none"> • Use the four operations with whole numbers to solve problems. • Gain familiarity with fractions and multiples. • Generate and analyze patterns. 	<ul style="list-style-type: none"> • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. 	<ul style="list-style-type: none"> • Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions and compare decimal fractions. 	<ul style="list-style-type: none"> • Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Understand and apply the statistics process. • Geometric measurement: understand concepts of angle and angle measurements. 	<ul style="list-style-type: none"> • Draw and identify lines and angles and classify shapes by properties of their lines and angles.

In grade 4, instructional time should focus on three critical areas:

1. In the Number and Operations in Base Ten domain, students will:

- generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place;
- apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value and properties of operations, in particular the distributive property, as they develop, discuss and use efficient, accurate and generalizable methods to compute products of multi-digit whole numbers;
- determine and accurately apply appropriate methods to estimate or mentally calculate products;
- develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems;
- apply their understanding of models for division, place value, properties of operations and the relationship of division to multiplication as they develop, discuss and use efficient, accurate and generalizable procedures to find quotients involving multi-digit dividends;
- select and accurately apply appropriate methods to estimate and mentally calculate quotients and interpret remainders based upon the context.

2. In the Numbers and Operations--Fractions domain, students will:

- create an understanding of fraction equivalence and operations with fractions;
- recognize that two different fractions can be equal and they develop methods for generating and recognizing equivalent fractions;
- extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions; decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. In the Geometry domain, students will:

- describe, analyze, compare and classify two-dimensional shapes;
- strengthen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry through building, drawing and analyzing two-dimensional shapes.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

MP.7. Look for and make use of structure.

MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use the four operations with whole numbers to solve problems.

Standards

Clarifications

KY.4.OA.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

MP.2, MP.4

Students interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.

Coherence KY.3.OA.1 → KY.4.OA.1 → KY.5.NF.5

KY.4.OA.2 Multiply or divide to solve word problems involving multiplicative comparisons by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MP.1, MP.2, MP.3

Students solve multiplicative comparison problems using drawings and equations to determine situations like the ones below (**Table 2 in Appendix A**) on which quantity is being multiplied and which factor is telling how many times.

Common Comparison Problems for Multiplication and Division		
Unknown product	Group size unknown	Number of groups unknown
<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p>Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p>Measurement example: A rubber band is stretched to be 18 cm long and is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue?</p> <p>Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Coherence KY.3.OA.3 → KY.4.OA.2 → KY.5.NF.3

KY.4.OA.3 Solve multistep problems.

a. Students use their knowledge of order of operations even when

Standards	Clarifications
<p>a. Perform operations in the conventional order when there are no parentheses to specify a particular order.</p> <p>b. Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computations and estimation strategies including rounding.</p> <p>MP.1, MP.4</p>	<p>there are no parentheses or brackets. $31 + 3 \times 8 - 20 =$</p> <p>b. For example, Mr. May's grade four class is collecting canned goods for a food drive. Their goal is to bring in 50 cans of food by Friday. So far, the students have brought in 10 on Monday and Tuesday, 14 cans on Wednesday and 13 on Thursday. How many more cans will the class need to bring in to reach their goal?</p> $50 = 2 \times 10 + 14 + 13 + c$ $50 = 20 + 14 + 13 + c$ $50 = 47 + c$ $3 = c$ <p>Note: Estimation skills include identifying when estimation is appropriate, determining method of estimation and verifying solutions or determining the reasonableness of situations using various estimation strategies. The skill of estimating within context allows students to further develop their number sense.</p> <p style="text-align: right;">Coherence KY.3.OA.8 → KY.4.OA.3 → KY.7.NS.3</p>

Attending to the Standards for Mathematical Practice
<p>Students recognize a number represents a specific quantity and connects the quantity to written symbols and creates a logical representation of the problem considering both the appropriate units involved and the meaning of quantities (). In an equation such as $35 = 5 \times 7$, students identify and verbalize which quantity is being multiplied and which number tells how many times, saying, "Sally is five years old. Her mom is seven times older. How old is Sally's Mom?"</p> <p>Students discover a pattern or structure (). For example, a student distinguishes an additive comparison by identifying this type of question asks, "How many more?" and a multiplicative comparison focuses on comparing two quantities by asking, "How many times as much?" or "How many times as many?" Students solve contextual problems using models and equations using a symbol to represent the unknown ().</p>

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Operations and Algebraic Thinking

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
 MP.2. Reason abstractly and quantitatively.
 MP.3. Construct viable arguments and critique the reasoning of others.
 MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.
 MP.6. Attend to precision.
 MP.7. Look for and make use of structure.
 MP.8. Look for and express regularity in repeated reasoning.

Cluster: Gain familiarity with factors and multiples.

Standards

KY.4.OA.4 Find factors and multiples of numbers in the range 1-100.

- a. Find all factor pairs for a given whole number.
- b. Recognize that a whole number is a multiple of each of its factors.
- c. Determine whether a given whole number is a multiple of a given one-digit number.
- d. Determine whether a given whole number is prime or composite.

MP.5, MP.7

Clarifications

Students extend their knowledge of multiplication and division facts by exploring patterns they have found by building conceptual understanding of prime numbers (numbers with exactly two factors) and composite numbers (numbers with more than two factors).

Patterns include:

- Numbers that end in 0 have 10 as a factor. These are multiples of 10.
- Numbers that end in 0 or 5 as a factor. These are multiples of 5.
- Even numbers have 2 as a factor. These numbers are multiples of 2.
- Numbers that can be halved twice have 4 as a factor. These numbers are multiples of 4.

Coherence KY.3.OA.7 → KY.4.OA.4 → KY.6.NS.4

Attending to the Standards for Mathematical Practice

Students use the structure and pattern of the counting numbers to find factor pairs, recognizing once they reach a certain point they don't have to keep looking for factors (). Students build arrays with a given area and look for patterns such as numbers of possible arrays to identify if the number is prime or composite. For example, noticing the number 7 has only two possible arrays, 1 x 7 and 7 x 1, therefore, it is prime. The number 4 has more than two rectangular arrays, 1 x 4, 4 x 1 and 2 x 2 and therefore, it is composite.

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Operations and Algebraic Thinking	
Standards for Mathematical Practice	
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Cluster: Generate and analyze patterns.	
Standards	Clarifications
KY.4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern not explicit in the rule itself. MP.2, MP.3	For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. Coherence KY.3.OA.9 → KY.4.OA.5 → KY.5.OA.3
Attending to the Standards for Mathematical Practice	
Students analyze growing patterns and determine rules to describe the pattern (). Students know a pattern is a sequence that repeats the same rule over and over. Students generate their own rules and create an example using that rule, for example, they write 1, 3, 9, 27, 81, 243 for the rule “times 3”. Students describe features of the pattern for example, all numbers are odd, or sums of the digits equal 9 and the rule for generating the next number, for example “times 3”, as well as critique the reasonableness of features and rules they hear from others ().	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations in Base Ten

Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
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Cluster: Generalize place value understanding for multi-digit whole numbers.

Standards	Clarifications
KY.4.NBT.1 Recognize in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. MP.7	Students recognize the relationship of same digits located in different places in a whole number. For example, in the number 435, the digit 5 in the ones place, while the digit 5 in 652 is in the tens place. The five in 652 is ten times greater than the five in 435. Coherence KY.2.NBT.1→KY.4.NBT.1→KY.5.NBT.1
KY.4.NBT.2 Represent and compare multi-digit whole numbers. <ol style="list-style-type: none"> a. Read and write multi-digit whole numbers using base-ten numerals, number names and expanded form. b. Compare two multi-digit numbers based on meanings of the digit in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. MP.2, MP.7	a. Students write numbers in three different forms. For example, 435, four hundred thirty-five, $400 + 30 + 5$. b. Students use different forms of the number to help compare. For example, when students are comparing numbers, they determine that 453 is greater than 435 because the 5 is worth 50 in 453, while the tens place only has 3 worth 30 in 435. So $453 > 435$. Coherence KY.4.NBT.2→KY.5.NBT.3
KY.4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. MP.2, MP.6	Students go beyond the application of a procedure when rounding. Students demonstrate a deeper understanding of number sense and place value when they explain and reason about the answers they get when rounding. KY.4.OA.3 Coherence KY.3.NBT.1→KY.4.NBT.3→KY.5.NBT.4

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Attending to the Standards for Mathematical Practice

Students use precise language, such as “ten times as much as” rather than “ten times more than” as they describe place value relationships (**MP.6**). Students make the conceptual connection between place value and multiplying and dividing by 10, noticing when any digit is multiplied by 10, the place of the digit moves one place to the left and when a digit is divided by 10, it moves one place to the right. Beyond noticing this pattern, students understand this pattern exists because place value is structured this way (). For example, in solving $35 \times 10 = \underline{\quad}$, students might place 35 in a place value chart and explain 5 tens is 50, therefore, moving the 5 to the tens place and 30 tens equals 3 hundreds, therefore, moving the 3 to the hundreds place.

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Number and Operations in Base Ten

Note: grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Standards for Mathematical Practice

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 MP.8. Look for and express regularity in repeated reasoning.

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standards

KY.4.NBT.4 Fluently add and subtract multi-digit whole numbers using an algorithm.

MP.2, MP.8

Clarifications

Students make connections from previous work with addition and subtraction, using models/representations to develop an efficient algorithm to add and subtract multi-digit numbers. These are types of algorithms/strategies one could possibly use (but not limited to) to solve adding and subtracting multi-digit whole numbers.

Standard Algorithm	Expanded Form	Models
$\begin{array}{r} 1 \\ 542 \\ + 63 \\ \hline 605 \end{array}$	$542 + 63 = \underline{\quad}$ $500 + 40 + 2$ $+ 60 + 3$ $500 + 100 + 5 = 605$	

Coherence KY.3.NBT.2 → KY.4.NBT.4 → KY.5.NBT.5

KY.4.NBT.5 Multiply whole numbers

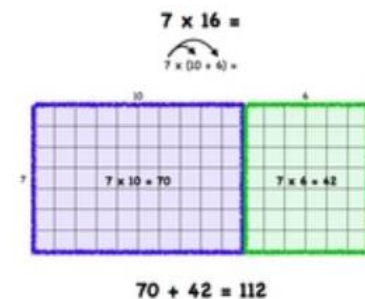
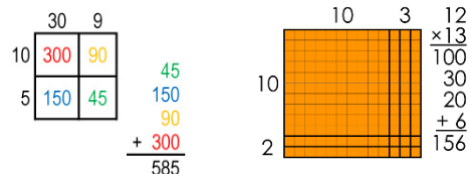
- Up to four digit number by a one-digit number
- Two-digit number by two-digit number

Multiply using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

MP.3, MP.4, MP.8

Students use a variety of models (rectangular arrays and area models) and strategies to represent multi-digit factors times a one-digit factor and a two-digit number by a two-digit number. Students also connect their reasoning to a written equation.

Some examples include:



Standards	Clarifications																																				
<p>KY.4.NBT.6 Divide up to four-digit dividends by one-digit divisors. Find whole number quotients and remainders using</p> <ul style="list-style-type: none"> strategies based on place value the properties of operations the relationship between multiplication and division <p>Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.</p> <p>MP.3, MP.7, MP.8</p>	<p style="text-align: center;">KY.3.OA.5 Coherence KY.3.NBT.3→ KY.4.NBT.5→KY.5.NBT.5 KY.3.MD.7</p> <p>Student use a variety of models (rectangular arrays and area models) and strategies to divide up to four-digit dividends by one-digit divisors.</p> <div style="text-align: center;"> <table border="1" style="margin: auto;"> <tr> <td></td> <td style="text-align: center;">1,000</td> <td style="text-align: center;">300</td> <td style="text-align: center;">70</td> <td style="text-align: center;">5</td> <td></td> </tr> <tr> <td style="text-align: center;">4</td> <td style="text-align: center;">1,000 x 4</td> <td style="text-align: center;">300 x 4</td> <td style="text-align: center;">70 x 4</td> <td style="text-align: center;">5 x 4</td> <td style="text-align: center;">1,000</td> </tr> <tr> <td></td> <td style="text-align: center;">4,000</td> <td style="text-align: center;">1,200</td> <td style="text-align: center;">280</td> <td style="text-align: center;">20</td> <td style="text-align: center;">300</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td style="text-align: center;">70</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td style="text-align: center;">+ 5</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td style="text-align: center;">1,375</td> </tr> </table> </div> <p>5,500 ÷ 4 = ?</p> <p>Note: By the end of grade 4 students should be able to model, write and explain division by a one-digit divisor.</p> <p style="text-align: center;">KY.3.OA.5 Coherence KY.3.OA.6→ KY.4.NBT.6→KY.5.NBT.6 KY.3.MD.7</p>		1,000	300	70	5		4	1,000 x 4	300 x 4	70 x 4	5 x 4	1,000		4,000	1,200	280	20	300						70						+ 5						1,375
	1,000	300	70	5																																	
4	1,000 x 4	300 x 4	70 x 4	5 x 4	1,000																																
	4,000	1,200	280	20	300																																
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Attending to the Standards for Mathematical Practice

Students select from their repertoire of strategies to solve multi-digit whole number addition or subtraction problems. For example, for the problem $345,402 - 67,087 = \square$, a student might choose to stack it and subtract using an algorithm. The same student seeing $56,708 - 9,998 = \underline{\quad}$, might notice how close the subtrahend (second value) is to 10,000 and decide to subtract 10,000 and add 2 onto the answer (). In general, students determine their approach based on the numbers in the problem seeking an efficient strategy. For multiplication and division, students recognize the relationship between area and multiplication and take advantage of rectangular arrays to model multiplication problems (). In creating such models and recording them as equations, students notice repetitive actions in computation and make generalizations to solve other similar problems (). Students explain how and why their selected models and/or algorithms work ().

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations – Fractions

Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
 MP.2. Reason abstractly and quantitatively.
 MP.3. Construct viable arguments and critique the reasoning of others.
 MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.
 MP.6. Attend to precision.
 MP.7. Look for and make use of structure.
 MP.8. Look for and express regularity in repeated reasoning.

Cluster: Extend understanding of fraction equivalence and ordering.

Standards	Clarifications
<p>KY.4.NF.1 Understand and generate equivalent fractions.</p> <p>a. Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size.</p> <p>b. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$.</p> <p>MP.4, MP.7, MP.8</p>	<p>Students draw fractions and see equivalent fractions.</p> <div style="text-align: center;"> </div> <p style="text-align: center;"> $\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$ </p> <p style="text-align: right; color: red;">Coherence KY.3.NF.3 → KY.4.NF.1 → KY.5.NF.1</p>
<p>KY.4.NF.2 Compare two fractions with different numerators and different denominators using the symbols $<$, $=$, or $>$. Recognize comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions.</p> <p>MP.2, MP.3</p>	<p>Students use a variety of representations to compare fractions including concrete models, benchmarks, common denominators and common numerators.</p> <p>Note: Students determine which strategy makes the most sense to them, realizing they use different strategies for different situations.</p> <p style="text-align: right; color: red;">Coherence KY.3.NF.3d → KY.4.NF.2 → KY.5.NF.2</p>

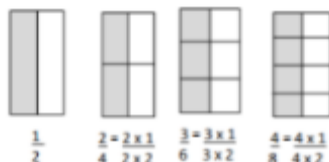
Attending to the Standards for Mathematical Practice

Work in this standard extends the work in grade 3 by using additional denominators (5, 10, 12 and 100). Students use visual models such as area models, number lines, or sets of objects to illustrate how two fractions are equivalent ()

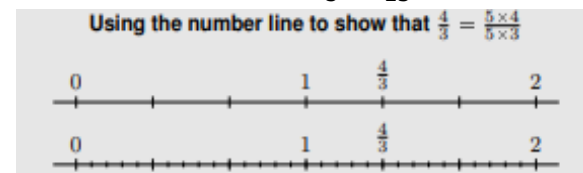
set model $\frac{6}{8} = \frac{3}{4}$



area model $\frac{1}{2} = \frac{4}{8}$



number line $\frac{4}{3} = \frac{20}{15}$



When students are asked to compare two fractions, they do not use a strategy they don't understand, such as the butterfly method, but rather employ reasoning strategies. They first consider whether they can decide which fraction is greater by observation (for example, the fractions have the same numerator or denominator or one fraction is greater than a benchmark and the other is less). If the fractions cannot be compared in this way, students decide whether to find a common denominator or a common numerator and then find the necessary fraction **equivalencies** to compare. For example, to compare $\frac{3}{8}$ and $\frac{5}{12}$, one can see $\frac{5}{12}$ is closer to $\frac{1}{2}$ (only $\frac{1}{12}$ away, while $\frac{3}{8}$ is $\frac{1}{8}$ away) and therefore know that $\frac{5}{12}$ is greater. Another student might not see this relationship, but decide that finding a common numerator is easier (being a basic fact) and multiply $\frac{3}{8}$ by $\frac{5}{5}$ to get $\frac{15}{40}$ and $\frac{5}{12}$ by $\frac{3}{3}$ to get $\frac{15}{36}$. Then recognize and explain that $\frac{15}{36}$ is greater (the pieces are larger) (,).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Operations Fractions

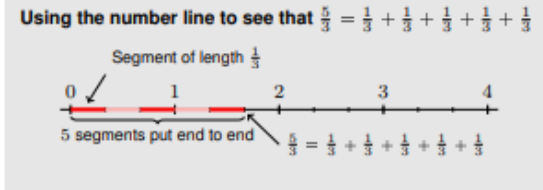
Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
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MP.5. Use appropriate tools strategically.
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Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards	Clarifications
<p>KY.4.NF.3 Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.</p> <ol style="list-style-type: none"> Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. Decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions. Add and subtract mixed numbers with like denominators. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. <p>MP.1, MP.5, MP.7</p>	<p>b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ OR $\frac{3}{5} = \frac{2}{5} + \frac{1}{5}$ $3\frac{1}{4} = 1 + 1 + 1 + \frac{1}{4}$ OR $3\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$</p> <p>c/d. Adding and subtracting using visual fraction models and/or equations to represent the problem.</p> <div style="text-align: center;">  <p>Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$</p> </div> <p style="text-align: right; color: red;">KY.5.NF.1 Coherence KY.3.NF.1 → KY.4.NF.3 → KY.5.NF.2</p>
<p>KY.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <ol style="list-style-type: none"> Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a fraction by a whole number. Solve word problems involving multiplication of a fraction by a whole number. <p>MP.5, MP.8</p>	<p>Students refer this standard to n groups of a fraction (where n is a whole number) for example 3 groups of $\frac{1}{4}$, which can be seen as repeated addition. In grade 5 students will multiply a fraction by a whole number.</p> <ol style="list-style-type: none"> Students use visual fraction models to represent $\frac{7}{5} = 7 \times \frac{1}{5}$ Students use the same thinking to see $3 \times \frac{2}{5}$ as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5}$ <p style="text-align: right; color: red;">KY.4.OA.2 Coherence KY.3.NF.1 → KY.4.NF.4 → KY.5.NF.4</p>

Attending to the Standards for Mathematical Practice

As students begin to work with fractions greater than unit fractions such as $\frac{2}{3} + \frac{2}{3} = \underline{\quad}$, they recognize, like whole numbers, they can decompose the non-unit fraction solve problems (Example: $\frac{2}{3} + \frac{2}{3} = \frac{2}{3} + \frac{1}{3} + \frac{1}{3} = 1\frac{1}{3}$) (). Students apply this knowledge make sense of word problems and persevere in solving them (). By using tools and situations, students notice a pattern and generalize how to multiply a fraction by a whole number (for example, problems in the form $n \times \frac{a}{b}$). For example, they use pattern blocks or Cuisenaire Rods to determine the answer to a set of tasks: $4 \times \frac{1}{2}$, $5 \times \frac{1}{3}$, $6 \times \frac{1}{3}$, $5 \times \frac{2}{3}$, $6 \times \frac{2}{3}$ and notice they multiply to find how many parts and thereby multiplying the whole number by the numerator (,). Note: Following a rote process of “putting a one under the whole number” or other rules not understood work against building understanding of 4.NF.4 and the development of mathematical practices.

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Number and Operations Fractions

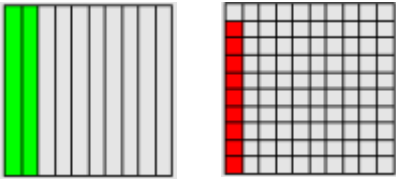
Note: grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
 MP.2. Reason abstractly and quantitatively.
 MP.3. Construct viable arguments and critique the reasoning of others.
 MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.
 MP.6. Attend to precision.
 MP.7. Look for and make use of structure.
 MP.8. Look for and express regularity in repeated reasoning.

Cluster: Understand decimal notation for fractions and compare decimal fractions.

Standards	Clarifications
KY.4.NF.5 Convert and add fractions with denominators of 10 and 100. <ul style="list-style-type: none"> a. Convert a fraction with a denominator of 10 to an equivalent fraction with a denominator of 100. b. Add two fractions with respective denominators 10 and 100. MP.5, MP.7	For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$ Note: Students who generate equivalent fractions develop strategies for adding fractions with unlike denominators in general. Addition and subtraction with unlike denominators in general is not a requirement at grade 4. <p style="text-align: right; color: red;">Coherence KY.3.NF.3→KY.4.NF.5→KY.5.NBT.7</p>
KY.4.NF.6 Use decimal notation for fractions with denominators 10 or 100. MP.4, MP.7	For example, students rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line. <p style="text-align: right; color: red;">Coherence KY.4.NF.6→KY.5.NBT.3</p>
KY.4.NF.7 Compare two decimals to hundredths. <ul style="list-style-type: none"> a. Compare two decimals to hundredths by reasoning about their size. b. Recognize that comparisons are valid only when the two decimals refer to the same whole. c. Record the results of comparisons with the symbols >, =, or < and justify the conclusions. MP.2, MP.3, MP.5	Students recognize comparisons are valid only when the two decimals refer to the same whole. For example, students use a visual model: seeing $0.2 > 0.09$  <p style="text-align: right; color: red;">Coherence KY.4.NF.7→KY.5.NBT.3</p>

Attending to the Standards for Mathematical Practice

Students consider available tools and choose to use base ten blocks, graph paper, place value charts, number lines and other place value models to explore the relationships between fractions with denominators of 10 and denominators of 100 (). By using these tools, students begin to make abstract and quantitative connections to the relationship between fractions with denominators of 10 and 100 (). Through these experiences and work with fraction models, they build the understanding comparisons between fractions and decimals are only valid when the

whole is the same for both cases (hundredths or tenths) (). Students use base ten blocks, 10 by 10 geoboards and 10 by 10 grids to illustrate and compare decimal fractions and justify their conclusions (,).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Measurement and Data

Standards for Mathematical Practice

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Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Standards

KY.4.MD.1 Know relative size of measurement units (mass, weight, liquid volume, length, time) within one system of units (metric system, U.S. standard system and time).

- a. Understand the relationship of measurement units within any given measurement system.
- b. Within any given measurement system, express measurements in a larger unit in terms of a smaller unit.
- c. Record measurement equivalents in a two-column table.

MP.5, MP.6

KY.4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money.

- a. Solve measurement problems involving whole number, simple fractions or decimals.
- b. Solve problems that require converting a given measurement from a larger unit to a smaller unit within a common measurement system, such as 2 km = 2,000 m.
- c. Visually display measurement quantities using representations such as number lines that feature a measurement scale.

MP.1, MP.4

KY.4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

MP.1, MP.3

Clarifications

c. Two- column tables may include:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

Coherence KY.4.MD.1→KY.5.MD.1

Note: grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Coherence KY.3.MD.2→KY.4.MD.2

Students apply the area and perimeter formulas to real world problems with an unknown factor:
 Area = length × width ($A = l \times w$)

Standards	Clarifications
	perimeter = length + width + length + width ($p = l + w + l + w$ OR $p = 2l + 2w$) KY.3.MD.8 Coherence KY.3.MD.7→KY.4.MD.3→KY.5.MD.5

Attending to the Standards for Mathematical Practice

Students know relative sizes of measurement units by actually measuring with the units and establishing a reference to an object. For example, recognizing a centimeter is about the width of their finger (). Students also measure objects using different units within the same system, such as meters and in centimeters (using a meter stick). Record the measurements in a table and notice relationships (). They explain why this pattern is true, arguing each meter has 100 centimeters, so 3 meters will have 300 centimeters and more generally explaining the smaller the unit the more units there will be when measuring the same object (). As students solve problems, they attend to and explain the attribute being measured (length or area), the unit being used to measure and make sense of the problem using drawings, tools, or strategies that make sense to them (,).

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Measurement and Data

Standards for Mathematical Practice

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Cluster: Understand and apply the statistics process.

Standards

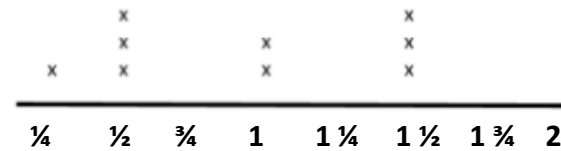
KY.4.MD.4 Use dot plots to analyze data to a statistical question.

- a. Identify a statistical question focused on numerical data.
- b. Make a dot plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$).
- c. Solve problems involving addition and subtraction of fractions by using information presented in dot plots.

MP.1, MP.6

Clarifications

Students create dot plots to show a data set of objects with fractional measurements.



Coherence KY.3.MD.4→KY.4.MD.4→KY.5.MD.2

Attending to the Standards for Mathematical Practice

Students recognize a statistical question is one that has variability in the answer and create such a question of interest to them and for which there are numerical responses (). After gathering data on a question of interest, students recognize they have many data points and therefore creating a graph helps to analyze the data. In creating the dot plot, students create a scale from 0 to 1 and label the scale to include intervals of $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ (). As they solve problems related to the graph, they stay focused on the reason they created the graph - to provide insights into the question they first posed, so responses focus on the statistical question posed ().

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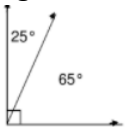
Measurement and Data

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
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Cluster: Geometric measurement: understand concepts of angle and measure angles.

Standards	Clarifications
KY.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement. MP.7	An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. Angles are measured in reference to a circle with the center at that common point. 
KY.4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. MP.5, MP.6	KY.4.MD.6 Coherence KY.4.MD.5→KY.4.MD.7
KY.4.MD.7 Recognize angle measure as additive. When an angle is into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. MP.1, MP.4	For example, students use an equation with a symbol for the unknown angle measure.  $25^\circ + \square = 90^\circ$

Attending to the Standards for Mathematical Practice

Students explore angle measures using tools (). For example, the white rhombus in a pattern block set or a cardboard cut-out is used as a ‘unit’ angle (a non-standard unit). Students use this tool to measure the size of other angles, noticing that angle measures are additive (). Building on concrete experiences, students explain $\frac{1}{360}$ of a circle, called a “one-degree angle,” is the unit for measuring angles (). Students connect their concrete measuring experiences with a new tool, the protractor and use it to more precisely determine angle measures (,). When solving word problems involving angle measures, students use drawings and tools to make sense of the problem, recognizing non-overlapping angles can be added or subtracted to find missing angles ().

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Geometry

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
 MP.2. Reason abstractly and quantitatively.
 MP.3. Construct viable arguments and critique the reasoning of others.
 MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.
 MP.6. Attend to precision.
 MP.7. Look for and make use of structure.
 MP.8. Look for and express regularity in repeated reasoning.

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standards	Clarifications
KY.4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures. MP.5, MP.6	Coherence KY.3.G.1→KY.4.G.1
KY.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence of absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. MP.7	Coherence KY.3.G.1→KY.4.G.2→KY.5.G.3
KY.4.G.3 Identify lines of symmetry. <ol style="list-style-type: none"> a. Recognize a line of symmetry for a two-dimensional figure. b. Identify line-symmetric figures and draw lines of symmetry. MP. 5, MP.7	

Attending to the Standards for Mathematical Practice

Using technology, using straightedges and/or protractors, students draw points, lines, line segments, rays, angles and perpendicular and parallel lines (). Students reason about the possible relationship of two lines or line segments. For example, students might use technology, uncooked spaghetti, or lines drawn on two transparency strips, to arrange two lines in different ways to determine possible events (the two lines might intersect, might intersect and be perpendicular, or may be parallel) (). Students analyze, compare and sort polygons based on their sides, angles and symmetry, explaining whether an attribute is a defining characteristic of that shape ().

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.