

**Roadmap to Implementing High Quality Mathematics Instruction**

**Resource**

The Roadmap to Implementing High Quality Mathematics Instruction seeks to:

* Ground instruction in the *Kentucky Academic Standards (KAS) for Mathematics*, thus reaffirming a commitment to equitable learning

opportunities for all students in Kentucky;

* + Support intentional integration of evidence-based instructional practices; and
	+ Expand educator familiarity with strategies to interweave the development of social emotional competencies with the development of mathematics content.

A completed Roadmap serves as a sample answer to the question, “*How do we decide which roads to take through this world of mathematics?*”, demonstrating how to cultivate vibrant student learning experiences that incorporate evidence-based instructional practices while valuing educator expertise and autonomy.

**Intended Audiences:**

This Roadmap is intended for educators looking for support with:

* *continuous improvement*, as the Roadmap encourages self-reflection around current understanding of the standards and instructional approaches, considering shifts to those approaches when needed/appropriate;
* *collaboration*, as the Roadmap offers educators opportunities to build collective agency around high-quality mathematics instruction and empowers educators to determine how to approach moving forward in a way that is manageable but meaningful.

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| Educators in schools/districts with adopted [High-Quality Instructional Resources (HQIRs)](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/High-Quality_Instructional_Resources.pdf/) should look to that HQIR as a starting point for the Roadmap with an array of pedagogical supports to help meet the needs of all learners. The Roadmap then becomes a place to collect those supports that align with the instructional vision for this specific learning experience. For those using an HQIR, this Roadmap might serve more to support the intellectual prep that goes into a lesson, rather than as a tool for instructional planning.  |

**Intended Purpose:**

Using this Roadmapoffers educators the opportunity to consider:

* What is your *goal for this learning experience*? What are your *success indicators*? In what ways will this learning experience *advance student access to and mastery of the KAS for Mathematics*?
* What evidence-based instructional practices will be prioritized throughout facilitation? What might be some strategies or approaches you are considering? How will you decide which strategies or approaches to take?
* Are there authentic opportunities to interweave social and emotional support for students in service of engaging students with mathematics? If so, what design considerations might you choose? How do those design considerations support the evidence-based practice you chose to prioritize in this learning experience?

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| **A Note About Navigating the Roadmap:** * Map markers are included as virtual flags marking progress through the Roadmap. These markers align with the purpose of the Roadmap as described above.
* As the Standards for Mathematical Practice and Effective Mathematics Teaching Practices are the same K-12, embedded bookmarks within the Roadmap offer ease of navigation without opening additional tabs. Bookmarks support engaging with the practice descriptions while maintaining the ability to quickly return to the section of the Roadmap being considered. As a result of embedding these resources, the Roadmap includes several pages that may or may not be utilized.
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**Grounding instruction in the *KAS for Mathematics*, thus reaffirming a commitment to equitable learning opportunities for all Kentucky students**

 **Supporting Resources:**

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| * [*KAS for Mathematics*](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/Kentucky_Academic_Standards_Mathematics.pdf)
* [*Breaking Down a Mathematics Standard*](https://kystandards.org/standards-resources/mathematics-resources/breaking-down-a-mathematics-standard/)
 | * [*Mathematics Assignment Review Protocol*](https://kystandards.org/standards-resources/mathematics-resources/mathematics-assignment-review-protocol/)
* [*Engaging the Standards for Mathematical Practice: Look fors and Question Stems*](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/SMP_Look_Fors_and_Question_Stems.pdf)
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|  ***KAS for Mathematics*** | **Cluster:** | **Learning Experience:** |
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| **Identify the Target of the Standard:** |
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|  | **Conceptual Understanding** refers to understanding mathematical concepts, operations and relations. Conceptual understanding is more than knowing isolated facts and methods; students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. Conceptual understanding allows students to connect prior knowledge to new ideas and concepts. |
|  | **Procedural Skill/Fluency** is the ability to apply procedures accurately, efficiently, flexibly and appropriately. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students’ ability to solve more complex application and modeling tasks is dependent on procedural skill and fluency |
|  | **Application** provides a valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution(s) makes sense by reasoning and develop critical thinking skills.  |

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| **Identify the Practice Standard:** May reference [*Engaging the SMPs: Look fors & Question stems*](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/SMP_Look_Fors_and_Question_Stems.pdf) |
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|  | [MP.1.](#te9f9mknscqy) Make sense of problems and persevere in solving them. |  | [MP.5.](#qc01vw72nnba) Use appropriate tools strategically. |
|  | [MP.2.](#sv17yzfiiwbn) Reason abstractly and quantitatively. |  | [MP.6.](#gaagaea6nj6l) Attend to precision. |
|  | [MP.3.](#2na6w0cl9seg) Construct viable arguments and critique the reasoning of others. |  | [MP.7.](#fhunhbdofer1) Look for and make use of structure.  |
|  | [MP.4.](#t3nyc4wr4scv) Model with mathematics. |  | [MP.8.](#pwcg3rivsit3) Look for and express regularity in repeated reasoning.  |

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| **Notes on Key Lesson Components (Optional)** |
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**Supporting intentional integration of evidence-based instructional practices**

**Supporting Resource:**

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| * [*NCTM’s Effective Mathematics Teaching Practices*](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/Effective_Mathematics_Teaching_Practices_NCTM.docx)
 | * [KDE’s Evidence-Based Instructional Practice series](https://kystandards.org/standards-resources/pl-mods/evidence-based-instructional-practices-ebips/)
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| **Identify Evidence-based Instructional Practice(s)**May reference [*Effective Mathematics Teaching Practices (NCTM)*](https://education.ky.gov/curriculum/standards/kyacadstand/Documents/Effective_Mathematics_Teaching_Practices_NCTM.docx) |
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|  | [EMTP 1:](#a0u4elfzmyy9) Establish mathematics goals to focus learning. |  | [EMTP 5:](#ldsdxa93iqua) Pose purposeful questions.  |
|  | [EMTP 2:](#p6t8xxbd1ii9) Implement tasks that promote reasoning and problem solving. |  | [EMTP 6:](#e0l9klws59g6) Build procedural fluency from conceptual understanding.  |
|  | [EMTP 3:](#cda7qutiuoqp) Use and connect mathematical representations. |  | [EMTP 7:](#a9jqa3jzg1p5) Support productive struggle in learning mathematics. |
|  | [EMTP 4:](#l3kjr3gihsy0) Facilitate meaningful mathematical discourse.  |  | [EMTP 8:](#pp4zu3u1cszg) Elicit and use evidence of student thinking.  |

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| **Teacher Actions:** | **Student Actions:** |
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**Expanding educator familiarity with strategies to interweave the development of social emotional competencies with development of mathematics content**

**Supporting Resource:** [*Integrating SEAD within the KAS for Mathematics*](https://kystandards.org/standards-resources/mathematics-resources/integrating-sead-mathematics/) resource library

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| **Identify the Competency Intended to Support the Evidence-Based Instructional Practice:**May reference [*Integrating SEAD within the KAS for Mathematics*](https://kystandards.org/standards-resources/mathematics-resources/integrating-sead-mathematics/) resource library |

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|  | SELF-AWARENESS  |

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|  | SELF-MANAGEMENT |

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|  | SOCIAL AWARENESS |

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|  | RELATIONSHIP SKILLS |

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|  | RESPONSIBLE DECISION-MAKING |

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| **Specific Design Considerations from** [***Integrating SEAD within the KAS for Mathematics***](https://kystandards.org/standards-resources/mathematics-resources/integrating-sead-mathematics/) **Grade Level Resource** |
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| **Teacher Reflection Questions from** [***Integrating SEAD within the KAS for Mathematics***](https://kystandards.org/standards-resources/mathematics-resources/integrating-sead-mathematics/) **Grade Level Resource** |
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# **S****tandards for Mathematical Practice** [**Back to Top**](#ppw0k3n42nte)

The Standards for Mathematical Practice (SMPs/MPs) support students’ full engagement in mathematical learning. Examining the SMP text below makes it clear that students cannot fully achieve the competencies described in these standards without demonstrating strength in the Collaborative for Academic, Social, and Emotional Learning (CASEL) social and emotional learning competencies (Charles A. Dana Center, 2016). Mainly, the SMPs reflect the view that learning is a social process, implicitly calling for teaching practices that leverage the power of a positive classroom climate and opportunities for collaborative learning.

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| **MP.1. Make sense of problems and persevere in solving them.** | **Connection to CASEL Competencies** |
| Mathematically proficient students **start by explaining the meaning of a problem and looking for entry points to its solution**. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than **simply jumping into a solution attempt**. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They **monitor and evaluate their progress and change course, if necessary**. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs, or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They **can understand other approaches to solving complex problems** and identify correspondences between different approaches. | **Self-awareness is evident when students:*** Are aware of the personal strengths and knowledge they bring to problem solving.
* Take ownership of where they are in the learning process.
* Embrace opportunities to demonstrate a growth mindset.

**Self-Management is evident when students:*** Resist impulses and regulate their thoughts and behaviors.
* Manage their time and energy toward a goal while appraising their work.

**Social Awareness is evident when students:*** Take on others’ perspectives.

**Responsible Decision-Making is evident when students:*** Anticipate and evaluate the consequences of one’s actions.
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| **MP.2. Reason abstractly and quantitatively.** | **Connection to CASEL Competencies** |
| Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize —to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, **to pause as needed during the manipulation process in order to probe into the referents for the symbols involved**. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. | **Self-Management is evident when students:*** Self-regulate and think metacognitively.
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| **MP.3. Construct viable arguments and critique the reasoning of others.**  | **Connection to CASEL Competencies** |
| Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They **justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose**. Mathematically proficient students also are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can **listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.**  | **Social Awareness is evident when students:*** Anticipate how their own arguments may be interpreted/received..
* Understand others’ perspectives to effectively interpret their arguments.
* Recognize strengths in others.

**Self-Management is evident when students:*** Think metacognitively and organize their own thoughts with given information.

**Relationship Skills is evident when students:*** Listen actively to further explore the arguments of others, providing feedback to help move thinking forward.

**Responsible Decision-Making is evident when students:*** Make a reasoned judgment after analyzing information, data, facts.
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| **MP.4. Model with mathematics.** | **Connection to CASEL Competencies** |
| Mathematically proficient students can apply the mathematics they know to solve problems that arise in everyday life. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are **comfortable making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They **routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.** | **Self-awareness is evident when students:*** Are aware of their own strengths and limitations.

**Self-management is evident when students:*** Self-reflect and self-motivate by recognizing the need to improve and work toward goals.

**Responsible Decision-Making is evident when students:*** Identify solutions for personal and social problems.
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| **MP.5. Use appropriate tools strategically.** | **Connection to CASEL Competencies** |
| Mathematically proficient students **consider the available tools** when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with appropriate tools to make sound decisions about when each of these tools might be helpful, recognizing both the potential for insight and limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to **use technological tools to explore and deepen their understanding of concepts**. | **Self-management is evident when students:*** Think metacognitively to identify when to use what tool.
* Motivate themselves to deepen their current understanding.

**Responsible Decision-Making is evident when students:*** Make a reasoned judgment after analyzing information, data and facts.
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| **MP.6. Attend to precision.** | **Connection to CASEL Competencies** |
| Mathematically proficient students try to **communicate precisely to others**. They try to **use clear definitions in discussions with others** and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign **consistently** and **appropriately**. They are **careful** about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students **provide carefully formulated explanations to each other**. By the time they reach high school, they can examine claims and make explicit use of definitions. | **Social Awareness and Relationship Skills is evident when students:*** Take on the perspective of others and are aware of other’s thoughts and feelings in order to strengthen the effectiveness of communication.

**Self-management is evident when students:*** Self-regulate thoughts and behaviors.
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| **MP.7. Look for and make use of structure.** | **Connection to CASEL Competencies** |
| Mathematically proficient students **look closely to discern a pattern or structure.** Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. **They also are able to shift perspectives.** They can **see complicated things**, such as some algebraic expressions, **as single objects or as being composed of several objects**. For example, they can see 5 - 3(x - y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. | **Self-awareness is evident when students:*** Understand their strengths and possess confidence/optimism about their ability to look for and make use of structure.

**Self-management is evident when students:*** Motivate themselves, persist and regulate against impulses to give up when a pattern or structure is not immediately apparent.
* Manage their own progress.
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| **MP.8. Look for and express regularity in repeated reasoning.** | **Connection to CASEL Competencies** |
| Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1) (x + 1), (x - 1) (x2 + x + 1) and (x - 1) (x3 + x2 + x + 1) might lead to awareness of the general formula for the sum of a geometric series. As they work to solve a problem, **mathematically proficient students maintain oversight of the process, while attending to the details**. They **continually evaluate the reasonableness of their intermediate results**. | **Self-awareness is evident when students:*** Accurately appraise their own abilities and work.

**Self-management is evident when students:*** Regulate their thoughts to know when organizational strategies are needed (e.g., writing key facts or organizing information on paper).
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**Effective Mathematics Teaching Practices** [**Back to Top**](#2fswzle5rkxa)

The Effective Mathematics Teaching Practices (EMTPs), laid out within [Principles to Actions](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) from the National Council of Teachers of Mathematics (NCTM), provide a research-based framework for guiding and strengthening the teaching and learning of mathematics. When implemented effectively, they are high-leverage, essential, equitable practices; and taken together, they support and promote a deep learning of mathematics.

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| **EMTP.1. Establish mathematical goals to focus learning.** *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons or throughout a unit.  |
|  | Identifying how the goals fit within a mathematics learning progression. |
|  | Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.  |
|  | Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. |

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|  | Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) |
|  | Using learning goals to stay focused on their progress in improving their understanding of mathematics content and practices.  |
|  | Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. |
|  | Assessing and monitoring their own understanding and progress toward the mathematics learning goals. |

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| **EMTP.2. Implement tasks that promote reasoning and problem solving.** *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Motivating students’ learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. |
|  | Selecting tasks that provide multiple entry points through the use of varied tools and representations. |
|  | Posing tasks on a regular basis that require a high level of cognitive demand. |
|  | Supporting students in exploring tasks without taking over student thinking. |
|  | Encouraging students to use varied approaches and strategies to make sense of and solve tasks.  |

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|  | Persevering in exploring and reasoning through tasks. |
|  | Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. |
|  | Using tools and representations as needed to support their thinking and problem solving. |
|  | Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another. |

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| **EMTP.3. Use and connect mathematical representations.** *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Selecting tasks that allow students to decide which representations to use in making sense of the problems. |
|  | Allocating substantial instructional time for students to use, discuss and make connections among representations. |
|  | Introducing forms of representations that can be useful to students. |
|  | Asking students to make math drawings or use other visual supports to explain and justify their reasoning.  |
|  | Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. |
|  | Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. |

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|  | Using multiple forms of representations to make sense of and understand mathematics.  |
|  | Describing and justifying their mathematical understanding and reasoning with drawings, diagrams and other representations.  |
|  | Making choices about which forms of representations to use as tools for solving problems. |
|  | Sketching diagrams to make sense of problem situations.  |
|  | Contextualizing mathematical ideas by connecting them to real-world situations.  |
|  | Considering the advantages or suitability of using various representations when solving problems. |

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| **EMTP.4. Facilitate meaningful mathematical discourse.** *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. |
|  | Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. |
|  | Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. |
|  | Ensuring progress toward mathematical goals by making explicit connections among student approaches and reasoning. |

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|  | Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.  |
|  | Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.  |
|  | Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others. |
|  | Identifying how different approaches to solving a task are the same and how they are different. |

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| **EMTP.5. Pose purposeful questions.** *Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.* |

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|  | Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. |
|  | Making certain to ask questions that go beyond gathering information to probing thinking and require explanation and justification. |
|  | Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. |
|  | Allowing sufficient wait time so that students can formulate and offer responses. |

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|  | Expecting to be asked to explain, clarify, and elaborate on their thinking.  |
|  | Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. |
|  | Reflecting on and justifying their reasoning, not simply providing answers.  |
|  | Listening to, commenting on, and questioning the contributions of their classmates. |

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| **EMTP.6. Build procedural fluency from conceptual understanding.** *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Providing students with opportunities to use their own reasoning strategies and methods for solving problems. |
|  | Asking students to discuss and explain why the procedures they are using work to solve particular problems. |
|  | Connecting student-generated strategies and methods to more efficient procedures as appropriate.  |
|  | Using visual models to support students’ understanding of general methods.  |
|  | Providing students with opportunities for distributed practice of procedures. |

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|  | Making sure that they understand and can explain the mathematical basis for the procedures that they are using.  |
|  | Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.  |
|  | Determining whether specific approaches generalize to a broad class of problems. |
|  | Striving to use procedures appropriately and efficiently. |

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| **EMTP.7. Support productive struggle in learning mathematics.** *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.  |
|  | Giving students time to struggle with tasks, and asking questions that scaffold students’ thinking without stepping in to do the work for them. |
|  | Helping students realize that confusion and errors are a natural part of learning, but facilitating discussions on mistakes, misconceptions and struggles. |
|  | Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.  |

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|  | Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. |
|  | Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. |
|  | Persevering in solving problems and realizing that it is acceptable to say, “I don’t know how to proceed here,” but it is not acceptable to give up. |
|  | Helping one another without telling their classmates what the answer is or how to solve the problem. |

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| **EMTP.8. Elicit and use evidence of student thinking.** *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.* |

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| **What are teachers doing?** | **What are students doing?** |
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|  | Identifying what counts as evidence of student progress toward mathematics learning goals. |
|  | Eliciting and gathering evidence of student understanding at strategic points during instruction.  |
|  | Interpreting student thinking to assess mathematical understanding, reasoning, and methods.  |
|  | Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.  |
|  | Reflecting on evidence of student learning to inform the planning of next instructional steps.  |

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|  | Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. |
|  | Reflecting on mistakes and misconceptions to improve their mathematical understanding. |
|  | Asking questions, responding to and giving suggestions to support the learning of classmates. |
|  | Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve. |

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