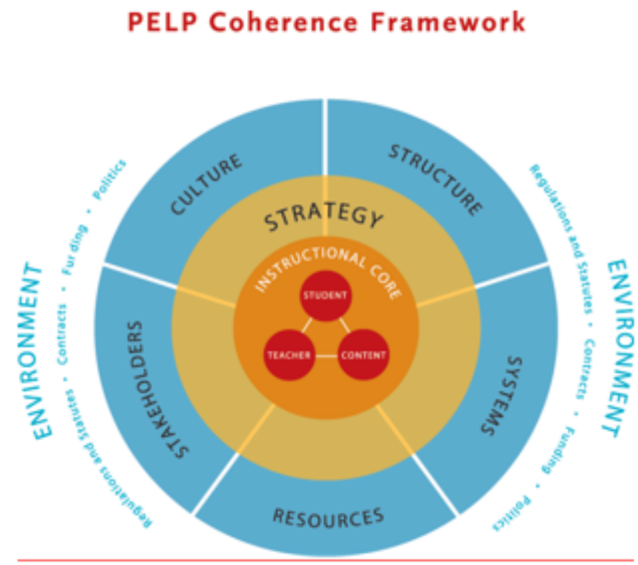
Characteristics of High Quality Mathematics Teaching and Learning in Kentucky Schools



Introduction

This document is an effort to describe the roles of the teacher and student in an exemplary mathematics instructional environment. The focus of the document is on the "instructional core" at the center of the educational process as described in detail in the *Public Education Leadership Program (PELP)*. Future documents will address the "outer ring" factors that are present in mathematics classrooms in high achieving schools and districts – essential resources for mathematics programs, stakeholder involvement, the learning culture, structures and system components, including sustained high quality professional learning opportunities for teachers who are at the core of the instructional process.

Note: The following documents are not cited in the table below as they are the original sources and embody the vision for the characteristics, the overviews of all mathematics standards-based content, instruction, and assessment, and the frameworks that initiated the ideas about which the research was conducted:

- **National Council of Teachers of Mathematics (NCTM). (2000).** ***Principles and Standards for School Mathematics.*** **Reston, VA: NCTM.**
- **National Council of Teachers of Mathematics (NCTM). (1995).** ***Assessment Standards for School Mathematics.*** **Reston, VA: NCTM.**
- **National Council of Teachers of Mathematics (NCTM). (1991).** ***Professional Standards for Teaching Mathematics.*** **Reston, VA: NCTM.**
- **National Council of Teachers of Mathematics (NCTM). (1989).** ***Curriculum and Evaluation Standards for School Mathematics.*** **Reston, VA: NCTM.**

In addition, the following document was developed to serve as a companion to the ***Principles and Standards for School Mathematics*** **(2000). It synthesizes a sizable portion of the literature that provides the foundation for the** ***Standards.***

- **National Council of Teachers of Mathematics (NCTM). (2003).** ***A Research Companion to Principles and Standards for School Mathematics,*** **J. Kilpatrick, G. Martin, and D. Schifter (Eds.). Reston, VA: NCTM.**

1. Knowledge of Content	Research Connections
The teacher: • A. Demonstrates an understanding of all pedagogical mathematics content and an ability to convey this content to students • B. Keeps abreast of current developments in mathematics	**1A, 1B, 1C, 1D, 1E**. National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. * **Chapter 2**, The State of School Mathematics in the United States, pp 31-36: Since the establishment of the NCTM *Curriculum and Evaluation Standards for School Mathematics* in 1989…states have developed content standards or curriculum frameworks describing what students should know and be able to do in mathematics. ***Chapter 7**, Developing Proficiency with Other Numbers, pp 231-254: Many students acquire useful informal knowledge of fractions, decimals, ratios, percents, and integers…but that knowledge needs to be made more explicit and extended through carefully designed instruction…The disconnections that many students exhibit among their conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning pose serious barriers to their progress in learning and using mathematics.

• C. Designs standards-based courses/lessons/units using Kentucky's Program of Studies Revised 2006, Academic Expectations, and Core Content for Assessment Version 4.1 • D. Demonstrates that mathematical understandings are outcomes of solving meaningful problems rather than merely of procedural instruction • E. Encourages students to analyze mathematics by identifying the underlying procedures, applying mathematical knowledge, and making generalizations • F. Uses and promotes the understanding of appropriate mathematics vocabulary • G. Provides essential supports for students in mathematics who are learning English or have limited English proficiency The students: • H. Use and seek to understand appropriate mathematics vocabulary	***Chapter 8**, Developing Mathematical Proficiency Beyond Number, pp 255-280: Students can learn to express the laws (of arithmetic) algebraically and can use them to support their reasoning and to justify their claims about numbers…they become aware of the role played by general statements expressed in algebraic symbols when justifying numerical arguments or discussing classes of situations. * **Chapter 10**, Developing Proficiency in Teaching Mathematics, pp 369-405: Teachers' knowledge is of value only if they can apply it to their teaching…Effective programs of teacher preparation and professional development cannot stop at simply engaging teachers in acquiring knowledge; they must challenge teachers to develop, apply, and analyze that knowledge in the context of their own classrooms so that knowledge and practice are integrated. ***Chapter 11**, Conclusions and Recommendations, pp 420-421: Problem solving should be the site in which all of the strands of mathematics proficiency converge; and pp 428-431: Very few (K-8) teachers currently have the specialized knowledge needed to teach mathematics in the way envisioned in this report…professional development in mathematics needs to be sustained over time that is measured in years, not weeks or months. **1D, 1E, 1I, 1J.** National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on *How Students Learn,* A Targeted Report for Teachers, M.S. Donovan and J. D. Bransford, (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press. ***Chapter 5**, Mathematical Understanding: An Introduction, pp 217-236:…teach mathematics so students come to appreciate that it…is about solving important and relevant quantitative problems…understanding that the rules for computation and solution are a set of clever human inventions that allow us to solve complex problems more easily, and to communicate about those problems with each other effectively and efficiently…how can we link formal mathematics training with students' informal knowledge and problem-solving capacities? ***Chapter 7**, Pipes, Tubes, and Beakers: New Approaches to Teaching the Rational-Number System, pp 309-349: Rational number concepts underpin many topics in advanced mathematics and carry significant academic consequences. Students cannot succeed in algebra if they do not understand rational numbers. But rational numbers also pervade our daily lives… ***Chapter 8**, Teaching and Learning Functions, pp 351-393: …importance of building new knowledge on the foundation of students' existing knowledge and understanding…Instruction should help students develop a conceptual understanding of function, the ability to represent a function in a variety of ways, and fluency in moving among multiple representations of functions… Functions are all around us, though students do not always realize this…Algebraic tools allow us to express these functional relationships very efficiently…and to solve highly complex problems and display in a way that provides a powerful image of change over time. **1A, 1B. 1E, 1F, 1G, 1H.** National Council of Teachers of Mathematics. (2002). *Lessons Learned from Research.* J. Sowder and B Schappelle (Eds.). Reston, VA: NCTM. ***Chapter 10**, Teacher Appropriation and Student Learning of Geometry Through Design, pp 85-91: Achievement was sustained over time by second (2^{nd}) grade students whose teachers were more knowledgeable about students' thinking about space and geometry and who elicited more elaborate

• I. Connects mathematical ideas in different content strands, e.g., number and data, and in different content areas, e.g., science • J. Uses mathematical ideas in realistic problems	patterns of classroom conversations. ***Chapter 11**, A Longitudinal Study of Invention and Understanding: Children's Multidigit Addition and Subtraction, pp 93-100: …the use of invented strategies can help children develop understanding of multidigit addition and subtraction that can enhance their performance even when algorithms are taught. ***Chapter 14**, Developing Concepts of Sampling for Statistical Literacy, pp 117-124: …skills are associated with…understanding of statistical terms and topics… ***Chapter 19**, Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms, pp 155-162: …active teaching that supports children's construction of a web of multiunit conceptions in which number words and written number marks (numerals) are related to ten-structured quantities. **1 (A-J).** Hyde, A., foreword by E. Keene (2006). *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K-6.* Portsmouth, NH: Heinemann *Book provides research and suggestions for implementing it and "braiding" the concepts of thinking, language, and math…providing students more opportunities to make connections, to understand their own thinking, to use language, to create representations, to revise, and to visualize as they approach math problems. Chapters: -Braiding Mathematics, Language, and Thinking -Asking Questions -Making Connections -Visualization -Inferring and Predicting -Determining Importance -Synthesizing -The Power of Braiding (Planning for problem solving and teaching content through problem solving) **1 (A-J).** Mid-continent Research for Education and Learning (McREL). (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning,* J. Sutton and A. Krueger (Eds.). Aurora, CO: McREL. *Summary of educational research and surveys of best classroom practices, with implications for improved teaching and learning. Questions addressed: -How does teacher content knowledge impact instruction? -How does teacher pedagogical knowledge impact instruction? -What is the impact of teacher learning on student learning? -What is the importance of standards-based curricula in mathematics? -How do we determine what students should know and be able to do in mathematics? -What are the characteristics of effective professional development for mathematics? -What instructional methods support mathematical reasoning and problem solving? -How is mathematical thinking addressed in the mathematics classroom? -In what ways can integrating curriculum enhance learning in mathematics?

	-How does integrated instruction in mathematics affect teaching and learning? -How does classroom curriculum connect to the outside world? -What does learning theory show teachers about how students learn mathematics? -What is the role of basic skills in mathematics instruction? -What is the role of algorithms in mathematics instruction? -What factors contribute most strongly to students' success in learning mathematics? **1A.** Hill, H., Rowan, B., and Ball, D. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement, 42(2), pp 371-406. American Educational Research Journal. *Teachers' mathematical knowledge was significantly related to student achievement gains in both first and third grades. **1A.** Fennema, E. & Franke, M. L. (1992). Teachers' knowledge and its impact. D. Grouws (Ed.), *NCTM Handbook of Research on Mathematics Teaching and Learning*, pp 147-164. New York: Macmillan. *An overview of the research on teachers' knowledge of mathematics, mathematical representations, and students' cognitions…effective teachers know more about their subject matter than ineffective teachers. **1A.** Usiskin, Z. (2003). Teachers Need a Special Type of Content Knowledge. *Teacher Support.* ENC. *There is a need for more content courses designed for teachers…courses in what might be called "teachers' mathematics"… (Also see Conference Board of the Mathematical Sciences. (2001). The Mathematical Education of Teachers. Providence, RI: American Mathematical Society.) **1A.** Monk, D.H. (1994). Subject matter preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review,* 12(2) pp 125-145. *…undergraduate mathematics education courses positively predicted student achievement **1A, 1B.** Hill, H. (June, 2007). Mathematical Knowledge of Middle School Teachers: Implications for the No Child Left Behind Policy Initiative. Educational Evaluation and Policy Analysis, 29(2), pp 95-114. American Educational Research Association. *Content course work correlates most substantially with teacher knowledge, but the association between mathematics methods course work and knowledge is strong. **1A, 1B.** National Council of Teachers of Mathematics. (July, 2005). Highly Qualified Teachers (Position Statement). NCTM. *A highly qualified teacher understands how students learn mathematics, expects all students to learn mathematics, employs a wide range of teaching strategies, and is committed to lifelong professional learning.

	1A, 1B, 1C. Meier, S., and Rich, B., Preparing Special-Needs Teachers for Teaching Standards-Based Mathematics: Focusing the Curriculum. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 77-89. National Council of Supervisors of Mathematics (NCSM). *…special needs students often need more time spent on important mathematical ideas in order to understand them and perform related skills…curriculum mapping on a broader scale… **1A, 1B, 1C, 1D, 1E, 1I.** Ma, Liping. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States (Studies in Mathematical Thinking and Learning).* Mahwah, NJ: Lawrence Erlbaum Associates, Inc. *…PUFM, "profound understanding of fundamental mathematics," involves more than subject matter expertise…it also involves how to communicate subject matter to students…A study of U.S. and Chinese elementary teachers who responded to four (4) questions related to two-digit subtraction, 3-digit multiplication, division of fractions, and area vs. perimeter found gaps in conceptual and in-depth understanding of the U.S. teachers…such teaching knowledge is more common in China…Enhance the interaction between teachers' study of school mathematics and how to teach it…Legislators, departments of education, and school boards need to understand the potential value in creating a corps of elementary-grade mathematics specialists who have scheduled time for study and collegial interaction. University educators need to understand teacher training in mathematics as a distinct activity, different from but of comparable value to training scientists, engineers, or generalist teachers… refocus teacher preparation. **1C.** National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.* Reston, VA: NCTM. *Describes an approach to curriculum development that focuses on areas of emphasis within each grade from Prekindergarten through grade 8 based on the foundational mathematical ideas from the *Principles and Standards for School Mathematics* (2000). Focal points are addressed in contexts that promote problem solving , reasoning, communication, making connections, and designing and analyzing representations. **1D, 1E.** Hiebert, J., and Grouws, D. (2007). Effective Teaching for the Development of Skill and Conceptual Understanding of Number: What is Most Effective? National Council of Teachers of Mathematics, Research Brief. Reston, VA: NCTM. *…large set of studies analyzed found two learning goals around which substantial data point to effective features of mathematics instruction: skill efficiency and conceptual understanding....In many studies that focused on conceptual development, students' skills increased at a level equal to or greater than those of students in the control groups.

	1D, 1E, 1F, 1G, 1H, 1I. Chapin, S., O'Connor, C., and Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn (Grades 1-6).* Sausalito, CA: Math Solutions Publications. *Book provides tools based on research (cited in back of the book) for the planned use of classroom discourse to support students' thinking and reasoning in mathematics, including productive talk moves, formats, ground rules, and planning and implementing lessons to incorporate math talk. **1D, 1E, 1F, 1G, 1H, 1J.** Draper, R., (March 2002). School Mathematics Reform, Constructivism and Literacy: A Case for Literacy Instruction in the Reform-oriented Math Classroom. *Journal of Adolescent* & *Adult Literacy,* 45(6). International Reading Association. *Incorporating literacy instruction with mathematics lessons can improve students' ability to learn and understand--as they construct meaning for the mathematics….before, during, and after reading strategies assist students with comprehension. **1F, 1H.** Kenney, J., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C. (2005). *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development. * Blending of current research on selected aspects of language literacy and practical strategies and suggestions for improving students' understanding of mathematical language and their ability to read mathematics text, to write about their mathematical thinking, and to enhance their communication through graphic representation and discourse. **1G.** McCargo, C., (1999). Addressing the Needs of English-Language Learners in Science and Math Classrooms. *The ERIC Review.* Vol 6 Issue 2, 52-54 *To prepare English learners to meet high academic standards at all grade levels, educators have to consider several factors such as social language vs. academic language, cultural backgrounds and previous schooling, effective assessment techniques. **1G.** Coggins, D. (2007). Strategies for Enhancing English Language Learners' Success with Mathematics. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 22-30. National Council of Supervisors of Mathematics (NCSM). *Teachers need preparation to meet the needs of English Language Learners…strategies described and suggestions for helping teachers implement—creating and maintaining access to lessons, using visual tools, and providing language support.

2. Instructional Rigor and Student Engagement	
The teacher: • A. Teaches the complex processes, concepts and principles contained in the Kentucky Core Content for Mathematics Version 4.1 and the Programs of Studies Revised 2006 using differentiated strategies that make them accessible to all students • B. Scaffolds instruction to help students reason and solve cognitively challenging mathematical tasks that provide insights into the structure of mathematics or strategies for solving problems • C. Orchestrates effective classroom discussions, questioning, and learning tasks that promote higher-order thinking skills • D. Challenges students to think deeply about problems and encourages/models a variety of approaches to a solution	**2B, 2C, 2E, 2F, 2K, 2L.** National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. ***Chapter 3**, Number: What is There to Know?, pp 94-110: All mathematical ideas require representations…one must be able to choose and translate among representations. ***Chapter 6**, Developing Proficiency with Whole Numbers--"Multidigit Whole Number Calculations," and "Mental Arithmetic and Estimation," pp 195-216: Such analyses can boost conceptual understanding by revealing much about the structure of the number system itself and can facilitate understanding of place-value representations….Mental arithmetic…and estimation…should integrate the various strands of mathematical proficiency. ***Chapter 8**, Developing Mathematical Proficiency Beyond Numbers, pp 274-276 (Role of technology): In this study…some support was found for the notion that learning how to interpret results of algebraic calculations is not highly dependent on the ability to perform the calculations themselves…At present…the traditional rule-based methods for developing manipulative skills tend to dominate. ***Chapter 9**, Teaching for Mathematical Proficiency-- "Findings from Research on Teaching," pp 333-356:…provide some scaffolding to assist students as they reason through …problems…without reducing the complexity of the task at hand or specifying exactly how to proceed….Teachers have the responsibility for moving the mathematics along while affording students opportunities to offer solutions, make claims, answer questions, and provide explanations to their colleagues….Manipulatives require careful use over sufficient time to allow students to build meaning and make connections….Studies have generally shown that the use of calculators does not threaten the development of basic skills and that it can enhance conceptual understanding, strategic competence, and disposition toward mathematics. **2B, 2C, 2D, 2E, 2F, 2J, 2K, 2L, 2M.** National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on *How Students Learn,* A Targeted Report for Teachers. M.S. Donovan and J. D. Bransford, (Eds). Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press. ***Chapter 5**, Mathematical Understanding: An Introduction, pp 237, 241-242: Teachers can move their students through increasingly productive levels of classroom discourse. * **Chapter 7**, Pipes, Tubes, and Beakers: New Approaches to Teaching the Rational-Number System, pp 309-349:…in this environment, a focus on the interconnections among decimals, fractions, and percents fosters students' ability to make informed decisions on how to operate effectively with rational numbers. * **Chapter 8**, Teaching and Learning Functions, pp 351-393: Instruction should help students develop

• E. Creates multiple opportunities for students to communicate and connect mathematical ideas through appropriate representations such as diagrams or pictures, examples, demonstrations, manipulative models, writing, symbols, and logical arguments
 	
• F. Frequently and consistently demonstrates proficiency with the use of appropriate tools and technology to solve problems
    o Provides to students appropriate tools, such as pattern blocks, algebra tiles, calculators, rulers, reference materials, and computers, so that they can make sense of tasks
    o Appropriately integrates technology as a tool (e.g., graphing calculators, CBLs and probes for data collection, spreadsheets, problem-solving software)
	a conceptual understanding of function, the ability to represent a function in a variety of ways, and fluency in moving among multiple representations of functions.
	
	2A, 2B, 2C, 2D, 2E, 2F, 2J, 2K, 2L, 2M. National Council of Teachers of Mathematics. (2002). *Lessons Learned from Research.* J. Sowder and B Schappelle (Eds.). Reston, VA: NCTM.
	***Chapter 3**, Mathematics Learning in Multiple Environments, pp 23-26: Students…make mathematical environments (a configuration of tools such as manipulatives, written number table, or computer software) into "lived-in spaces" for themselves and connect environments…across their experiences.
	***Chapter 4**, Supporting Students' High-level Thinking, Reasoning, and Communication in Mathematics, pp 27-35: The teacher must not only select and appropriately set up worthwhile mathematical tasks but also proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task.
	***Chapter 5**, Advancing Children's Mathematical Thinking, pp 37-39: Teachers can and should intervene to advance children's thinking…through eliciting, supporting, and extending…
	***Chapter 9**, Learning in an Inquiry-Based Classroom: Fifth Graders' Enumeration of Cubes in 3D Arrays, pp 75-83: This study illustrates how powerful mathematics learning can occur in problem-centered inquiry-based teaching.
	***Chapter 10**, Teacher Appropriation and Student Learning of Geometry Through Design, pp 85-91: Achievement was sustained over time by second (2nd) grade students whose teachers were more knowledgeable about students' thinking about space and geometry and who elicited more elaborate patterns of classroom conversations.
	***Chapter 15**, When a Student Perpetually Struggles, pp 125-128:…demonstrated the vital role language processes play in developing the concept flexibility necessary for success in mathematics…has implications for pedagogy in classrooms that include mainstreamed students with learning disabilities.
	***Chapter 16**, Open and Closed Experiences and Understandings, pp 135-142: Students who followed a traditional approach developed procedural knowledge that was of limited use in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed conceptual understanding that yielded advantages in a range of assessments and situations.
	***Chapter 19**, Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms, pp 155-162: The results reported here clearly indicate that all U.S. children can do enormously better than they ordinarily do in primary school mathematics….Doing so requires…active teaching that supports children's construction of a web of multiunit conceptions in which number words and written number marks (numerals) are related to ten-structured quantities.
	***Chapter 21**, Good Intentions Were Not Enough: Lower SES Students' Struggles to Learn Mathematics Through Problem Solving, pp 171-177….lowering expectations is not the solution for lower SES students…but teachers need to support lower SES students' efforts to understand and adapt to new roles…these students' learning must be carefully monitored. Through careful analyses of students' writing and talking, teachers must continually assess whether their students are learning mathematics in such a way that they can transfer their understanding from one situation to another.
	***Chapter 23**, An Investigation of African American Students' Mathematical Problem Solving, pp

• G. Integrates a variety of learning resources with classroom instruction to increase learning options for all students; these should include guest presenters, field experiences, and career exploration • H. Clarify and share with students learning intentions/targets and criteria for success The students: • I. Articulate learning intentions/targets and criteria for success • J. Justify solutions to problems by communicating mathematically using written, hands-on, spoken and symbolic representations • K. Use mathematics and technology appropriately in problem solving situations (e.g. spreadsheets, symbolic manipulation software, graphing technology, geometry software, simulations, formulas, etc.)	191-195: Successful African-American students' problem-solving actions matched previously reported actions of good mathematical problem solvers: successful use of strategies, flexibility in approach, use of verification-of-solution actions, and success in dealing with irrelevant detail. ***Chapter 24**, Grade 6 Students' Preinstructional Use of Equations to Describe and Represent Problem Situations, pp 197-201: …the emphasis in the curriculum should be on developing and linking multiple representations to generalize problem situations instead of on merely constructing representations that students do not link with problem situations. **2A, 2B, 2C, 2D, 2E, 2F, 2G, 2H, 2I, 2J, 2L, 2M.** Hyde, A., foreword by E. Keene (2006). *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K-6.* Portsmouth, NH: Heinemann *Book provides research and suggestions for implementing it and "braiding" the concepts of thinking, language, and math…providing students more opportunities to make connections, to understand their own thinking, to use language, to create representations, to revise, and to visualize as they approach math problems. Chapters: -Braiding Mathematics, Language, and Thinking -Asking Questions -Making Connections -Visualization -Inferring and Predicting -Determining Importance -Synthesizing -The Power of Braiding (Planning for problem solving and teaching content through problem solving) **2 (A-M).** Mid-continent Research for Education and Learning (McREL). (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning,* J. Sutton and A. Krueger (Eds.). Aurora, CO: McREL. *Summary of educational research and surveys of best classroom practices, with implications for improved teaching and learning. Questions addressed: -What is equity and how is it evident in mathematics classrooms? -What can schools do to facilitate students' opportunity to learn mathematics? -How can different learning styles be addressed with consistent expectations? -What instructional methods support mathematical reasoning and problem solving? -How is mathematical thinking addressed in the mathematics classroom? -What role does teacher questioning play in learning mathematics? -How can using instructional technology affect mathematics reasoning and problem solving? -What effect do calculators have on student learning? -How can technology make mathematics teaching more learner-centered? -What role does active hands-on learning play in mathematics instruction? -How does using contextual or applied activities improve student learning in mathematics?

• L. Engage in active, hands-on, open-ended, problem-based learning experiences using meaningful mathematics that also reveal the structure of the mathematics • M. Solve realistic problems using a variety of strategies	**2A.** Meier, S., and Rich, B., Preparing Special-Needs Teachers for Teaching Standards-Based Mathematics: Focusing the Curriculum. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 77-89. National Council of Supervisors of Mathematics (NCSM). *…special needs students often need more time spent on important mathematical ideas in order to understand them and perform related skills…curriculum mapping on a broader scale… **2A.** Strong, R., Thomas, E., Perini, M., and Silver, H. (2004). Creating a Differentiated Mathematics Classroom. *Educational Leadership,* 61(5), pp 73-78. Association for Supervision and Curriculum Development. *To construct a differentiated mathematics classroom…include computation, explanation, application, and problem solving in every unit; help students recognize their own mathematical learning styles; use a variety of teaching strategies; create assessments to reflect all four dimensions of mathematical learning and all four learning styles. **2A, 2B, 2C, 2D, 2E, 2F, 2G, 2J, 2K, 2L.** Connell, M., Klein, R., Harnisch, D. (2007). Technology Uses in Special Education Mathematics Classrooms. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 54-63. National Council of Supervisors of Mathematics (NCSM). *…use of technology (Smart Board) to support an instructional model…to provide “mindful engagement” that enhances students’ abilities to build conceptual knowledge. **2A, 2B, 2C, 2D, 2E, 2F, 2H, 2I, 2J, 2L.** Chapin, S., O’Connor, C., and Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn (Grades 1-6).* Sausalito, CA: Math Solutions Publications. *Book provides tools based on research (cited in back of the book) for the planned use of classroom discourse to support students’ thinking and reasoning in mathematics, including productive talk moves, formats, ground rules, and planning and implementing lessons to incorporate math talk. **2A, 2B, 2E, 2J.** Draper, R., (March 2002). School Mathematics Reform, Constructivism and Literacy: A Case for Literacy Instruction in the Reform-oriented Math Classroom. *Journal of Adolescent & Adult Literacy,* 45(6). International Reading Association. *Incorporating literacy instruction with mathematics lessons can improve students’ ability to learn and understand--as they construct meaning for the mathematics….before, during, and after reading strategies assist students with comprehension. **2A, 2B, 2E, 2F, 2H, 2I, 2J, 2K, 2L, 2M.** Olson, M., Olson, J., Swarthout, M., and Hartweg, K. (2007). Meeting Special Needs in Mathematics Through Learning Stations. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 31-41. National Council of Supervisors of Mathematics (NCSM). *Multiple-day learning stations for teaching mathematics can be used to accommodate students of special needs, and can benefit all students, by removing barriers, structuring the environment, and providing more time and practice… focused tasks with clear expectations…

	2A, 2B, 2F, 2G, 2H, 2I. Bradsby, L. (2007). Methods and Strategies to Help At-Risk Students Be Successful in Secondary Mathematics. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 42-53. National Council of Supervisors of Mathematics (NCSM). *A systematic approach consisting of six (6) hierarchical teaching processes incorporates many of the aspects of mastery learning as analyzed by Guskey and Gates (1986) in 27 studies… **2A, 2C, 2E, 2J.** Kenney, J., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C. (2005). *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development. *Blending of current research on selected aspects of language literacy and practical strategies and suggestions for improving students' understanding of mathematical language and their ability to read mathematics text, to write about their mathematical thinking, and to enhance their communication through graphic representation and discourse. **2A, 2E, 2F.** Tomlinson, C. (Aug 2000). Differentiation of Instruction in the Elementary Grades. *Teacher Support.* ENC. (Also see Tomlinson, C. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development. ED 429 944) *Teachers can differentiate at least four classroom elements based on student readiness, interest, or learning profile: content, process, products, and learning environment. **2A, 2E, 2F, 2J.** Gersten, R., and Clarke, B. (2007). Effective Strategies for Teaching Students with Difficulties in Mathematics. National Council of Teachers of Mathematics, Research Brief. Reston, VA: NCTM. *Research points to several strategies consistently effective in teaching students who experience difficulties in mathematics: structured peer-assisted learning activities, systemic and explicit instruction, student think-alouds, formative assessment data provided to teachers and students, extensive use of visual representations. **2C, 2D, 2E, 2H, 2I, 2J, 2L.** Black, P., and Wiliam, D. (October, 1998). Inside the Black Box: Raising Standards Through Classroom Assessment. *Phi Delta Kappan,* 80(2), 139-148. *Research summary of more than 250 articles/chapters…found that formative assessment can contribute more to improving outcomes than any other school-based factor, especially benefiting low achievers….essential component of classroom work. **2C, 2E, 2H, 2I, 2J.** Wiliam, D. (2007). Five "Key Strategies" for Effective Formative Assessment. Research Brief. National Council of Teachers of Mathematics (NCTM). *Formative assessment can be increased by using five (5) key strategies: Clarifying, sharing, and understanding goals for learning and criteria for success with learners; engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning; providing feedback that moves learning forward; activating students as owners of their own learning; and activating students as learning resources for one another.

	2E, 2J. Allman, J. with McCoy, L. (Ed). (December, 2005). Mathematical Reasoning in Multiple Representations: Connections and Confidence. Studies in Teaching 2005 Research Digest. Winston-Salem, NC: Wake Forest University. *Ten (10) high school students were observed and interviewed about their performance on a task with regard to intertwining strands of mathematical proficiency described by the National Research Council in 2001 for grades K-8: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The researcher determined that students should investigate with multiple representations of the content to construct their web ofunderstanding. **2F, 2K.** National Council of Teachers of Mathematics. (October, 2003), The Use of Technology in the Learning and Teaching of Mathematics (Position Statement). NCTM *Technology increases both the scope of the mathematical content and the range of the problem situations that are within students' reach. **2F, 2K.** National Council of Teachers of Mathematics. (May, 2005), Computation, Calculators, and Common Sense (Position Statement). NCTM. *Students need an understanding of number and operations, including the use of computational procedures, estimation, mental mathematics, and the appropriate use of the calculator (to expand students' mathematical understanding, not to replace it). **2F, 2K.** Dessart, D., DeRidder, C, and Ellington, A., (May/June 1999). The Research Backs Calculators. *Mathematics Education Dialogues*, 2, 8. *The authors cite meta-analyses (112 studies and one large longer-term study) of the effects of calculator use…significantly improved students' attitudes and self-concepts, had a positive effect on increasing conceptual knowledge and in problem solving and computation, and did not hinder the development of pencil-and-paper skills. **2F, 2K.** McCauliff, E. (2004). The Calculator in the Elementary Classroom: Making a Useful Tool out of an Ineffective Crutch. *Concept: An Interdisciplinary Journal of Graduate Studies*. Villanova University. *…most research supports the use of calculators but cautions that responsibility lies with the teacher….It is imperative that teachers be educated in the use of calculators so that they are able to teach students to use calculators effectively to learn mathematics. **2F. 2K.** Heller, J., Curtis, D., Jaffe, R., and Verboncoeur, C. (January, 2006). Impact of Handheld Graphing Calculator Use on Student Achievement in Beginning Algebra. Heller Research Associates. *When teachers incorporated graphing calculators into their curriculum more frequently and with greater intensity…student achievement was higher (even when students did not use graphing calculators during testing).

	2F. 2K. Using Graphing Calculators in Secondary Mathematics: What Scientifically-Based Research Has to Say. (May 2003). Interactive Educational Systems Design (Prepared for Texas Instruments). *Five (5) independent studies by different researchers from 1990-2001 found that the use of graphing calculators in general algebra improved student knowledge of functions and improved student performance (one study included low-performing students). Researchers: Ruthven, 1990; Schwarz and Hershkowitz, 1999; Hollar and Norwood, 1999; Harskamp, Suhre and Van Streun, 2000; Thompson and Senk, 2001. **2F, 2K.** U.S. Department of Education. Office of Educational Research and Improvement. (2001). *The Nation's Report Card: Mathematics 2000.* J. Braswell, A. Lutkus, W. Grigg, S. Santapau, B. Tay-Lim, and M. Johnson. Washington, DC: National Center for Education Statistics. *Eighth graders with unrestricted use of calculators had higher average scores than the students whose teachers restricted calculator use. Eighth graders who used calculators on class tests had higher average NAEP scores than students whose teachers did not permit calculator use on tests. Frequent usage of calculators by fourth graders was associated with lower average mathematics scores than less frequent usage, but for eighth and twelfth graders, more frequent calculator usage was associated with higher scores. **2H, 2I.** Stiggins, R., Arter, J., Chappuis, J., Chappuis, S. (2006). *Classroom Assessment for Student Learning: Doing It Right—Using It Well.* Educational Testing Service. *Book (can be used for professional development) offers research-based tools for quality classroom assessment that produces accurate information (from defining and assessing clear learning targets, understanding the purpose for assessing, and designing and using a variety of methods well) used effectively to maximize student learning (to plan instruction and to involve students in their own assessment, and communicating results clearly to meet the needs of the user—including the student).

3. Instructional Relevance	
The teacher: • A. Designs lessons that allow students to participate in empowering activities in which they understand that learning is a process and mistakes are a natural part of the learning • B. Incorporates student experiences, interests, and real-life situations in instruction • C. Links mathematics concepts and key ideas to students' prior learning experiences and understandings, using multiple representations, examples and explanations • D. Teaches students to express their understanding of how big ideas in mathematics are connected (e.g., through use of benchmark problems such as area and multiplication, data and numbers) • E. Works with other teachers to make connections between and among disciplines to show how mathematics is a part of other major subjects	**3A, 3C, 3D, 3F, 3J,** National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. ***Chapter 3**, Number: What is There to Know?, pp 71-114: Numbers are ideas—abstractions that apply to a broad range of real and imagined situations…related to measurement, algebra, geometry, probability, and statistics….represented by symbols, words, pictures, objects and actions…The number line provides a link between arithmetic and geometry…helps students develop a sense of magnitudes and relationships of integers and rational numbers…lets students interpret whole numbers, negative numbers, and fractions all as part of one overall system. ***Chapter 8**, Developing Mathematical Proficiency Beyond Number, pp 255-312: The representational activities of algebra involve translating verbal information into symbolic expressions and equations…Measurement of length, area, and volume are the basis for the connection between geometry and number…technology offers promise for helping to support and link students' developing conceptions of data and chance, measurement and geometry. ***Chapter 9**, Teaching for Mathematical Proficiency, pp 333-349: …students need to believe that what they are learning is worth learning…Classrooms that function as a community of learners value mistakes as sites of learning for everyone…Help maintain student engagement at a high level by…choosing tasks that build on students' prior knowledge. **3A, 3C.** National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on *How Students Learn,* A Targeted Report for Teachers. M.S. Donovan and J. D. Bransford, (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press. ***Chapter 5**, Mathematical Understanding: An Introduction- (see 1 and 2 below) (1) Principle #3: A metacognitive approach enables student self-monitoring, pp 236-242: Technological advances mean that more adults will need to do more complex problem solving and error identification throughout their lives, so debugging—locating the source of an error—is a good general skill that can be learned in the math classroom. (2) Engaging Students' Preconceptions and Building on Existing Knowledge, pp 223-231: Certain features support engaging and building on student preconceptions…allow students to use their own informal problem-solving strategies…encourage math talk…design instructional activities to bridge commonly held conceptions and targeted mathematical understandings. (See examples of such bridging contexts in **Chapter 6**, Fostering the Development of Whole-Number Sense: Teaching Mathematics in the Primary Grades, pp 257-308; **Chapter 7**, Pipes, Tubes, and Beakers: New Approaches to Teaching the Rational-Number System, pp 309-349; and **Chapter 8**, Teaching and Learning Functions, pp 351-393). ***Chapter 16**, Open and Closed Experiences and Understandings, pp 135-142: Students who learned mathematics in an open, project-based environment …developed a predisposition to think about and use mathematics in novel situations…believed that mathematics involves active and flexible thought and developed the ability to adapt and change methods to fit new situations.

• F. Effectively incorporates technology that prepares students to meet future challenges, as articulated in the **Partnership for 21st Century Skills.** The students: • G. Respond to and pose non-trivial questions • H. Express their understanding of mathematics and how to apply it to problem-solving activities by creating responses to a variety of classroom activities and compiling their work in a form that they can access and use, e.g., mathematics journal, open-response item portfolio, entry and exit slips folder, 3-ring binder of problem solving experiences • I. Use appropriate tools and techniques to gather, analyze, and interpret data • J. Use multiple representations (e.g., words, numbers, charts, models, graphs, symbols, tables, diagrams, and manipulatives) to communicate mathematically and to uncover different aspects of the problem	**3B, 3C, 3I, 3K.** National Council of Teachers of Mathematics. (2002). *Lessons Learned from Research.* J. Sowder and B Schappelle (Eds.). Reston, VA: NCTM. ***Chapter 14**, Developing Concepts of Sampling for Statistical Literacy, pp 117-124: As well as teaching appropriate methods for selecting samples, teachers must help students develop appreciation for situations in which bias can occur. **3A, 3B, 3C, 3D, 3E, 3G, 3H, 3I, 3J, 3K.** Hyde, A., foreword by E. Keene (2006). *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K-6.* Portsmouth, NH: Heinemann *Book provides research and suggestions for implementing it and "braiding" the concepts of thinking, language, and math…providing students more opportunities to make connections, to understand their own thinking, to use language, to create representations, to revise, and to visualize as they approach math problems. Chapters: -Braiding Mathematics, Language, and Thinking -Asking Questions -Making Connections -Visualization -Inferring and Predicting -Determining Importance -Synthesizing -The Power of Braiding (Planning for problem solving and teaching content through problem solving) **3 (A- K).** Mid-continent Research for Education and Learning (McREL). (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning,* J. Sutton and A. Krueger (Eds.). Aurora, CO: McREL. *Summary of educational research and surveys of best classroom practices, with implications for improved teaching and learning. Questions addressed: -What instructional strategies make mathematics teaching more learner-centered? -What do we know about how students learn mathematics? -What does learning theory show teachers about how students learn mathematics? -How can teachers help students reflect on and communicate their own learning? -How does using contextual or applied activities improve student learning in mathematics? -How is mathematical thinking addressed in the mathematics classroom? -In what ways can integrating curriculum enhance learning in mathematics? -How does integrated instruction in mathematics affect teaching and learning? -How does classroom curriculum connect to the outside world? -How can using instructional technology affect mathematics reasoning and problem solving? -How can technology make mathematics teaching more learner-centered? -How can students best use information and data from the Internet? -How has technology changed the mathematics that is important for students to learn?

• K. Work on mathematics that is connected to other content areas and to realistic problems	**3A.** Deana, M. (May 2007). The Effects of Cooperative Learning on Student Motivation (Master's Project). State University College at Cortland. *Findings from 15 studies…suggest that cooperative learning may increase motivation by creating a more enjoyable learning environment, by increasing student self-efficacy in the content area, and by holding students accountable to their peers…students became more engaged and had a more positive attitude toward learning. **3A.** Lannin, J., Barker, D., Townsend, B. (September, 2007). How Students View the General Nature of Their Errors. *Educational Studies in Mathematics.* Vol 66 No 1, pp 43-59. *Case study of two students and how their perception of errors shape their understanding of proportional reasoning and the role errors play in restructuring a student's conceptual schema. **3A, 3B, 3C, 3G, 3H, 3I, 3J, 3K.** Olson, M., Olson, J., Swarthout, M., and Hartweg, K. (2007). Meeting Special Needs in Mathematics Through Learning Stations. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 31-41. National Council of Supervisors of Mathematics (NCSM). *Multiple-day learning stations for teaching mathematics can be used to accommodate students of special needs, and can benefit all students, by removing barriers, structuring the environment, and providing more time and practice. The learning stations approach capitalizes on the strengths of all students, and attends to the diversity of ability levels and learning styles. **3A, 3B, 3C, 3F, 3G, 3J.** Connell, M., Klein, R., Harnisch, D. (2007). Technology Uses in Special Education Mathematics Classrooms. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 54-63. National Council of Supervisors of Mathematics (NCSM). *…use of technology (Smart Board) to support an instructional model…to provide "mindful engagement" that enhances students' abilities to build conceptual knowledge. **3A, 3C, 3D, 3G, 3J.** Wiliam, D. (2007). Five "Key Strategies" for Effective Formative Assessment. Research Brief. National Council of Teachers of Mathematics (NCTM). *Formative assessment can be increased by using five (5) key strategies: Clarifying, sharing, and understanding goals for learning and criteria for success with learners; engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning; providing feedback that moves learning forward; activating students as owners of their own learning; and activating students as learning resources for one another. **3A, 3C, 3D, 3G, 3J.** Chapin, S., O'Connor, C., and Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn (Grades 1-6).* Sausalito, CA: Math Solutions Publications. *Book provides tools based on research (cited in back of the book) for the planned use of classroom discourse to support students' thinking and reasoning in mathematics, including productive talk moves, formats, ground rules, and planning and implementing lessons to incorporate math talk.

	3A, 3C, 3G, 3I, 3J. Black, P., and Wiliam, D. (October, 1998). Inside the Black Box: Raising Standards Through Classroom Assessment. *Phi Delta Kappan,* 80(2), 139-148. *Research summary of more than 250 articles/chapters…found that formative assessment can contribute more to improving outcomes than any other school-based factor, especially benefiting low achievers….essential component of classroom work. **3B, 3H, 3K.** Tomlinson, C. (Aug 2000). Differentiation of Instruction in the Elementary Grades. *Teacher Support.* ENC. (Also see Tomlinson, C. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development. ED 429 944) *Teachers can differentiate at least four classroom elements based on student readiness, interest, or learning profile: content, process, products, and learning environment, e.g., giving students options of how to express required learning, providing interest centers. **3C, 3D, 3E.** Meier, S., and Rich, B., Preparing Special-Needs Teachers for Teaching Standards-Based Mathematics: Focusing the Curriculum. *Leadership to Math Success for All*. E. Bazik (Ed). Monograph Series, Vol 5, pp 77-89. National Council of Supervisors of Mathematics (NCSM). *…special needs students often need more time spent on important mathematical ideas in order to understand them and perform related skills…developing “big ideas” and making connections between them… **3C, 3J.** Gersten, R., and Clarke, B. (2007). Effective Strategies for Teaching Students with Difficulties in Mathematics. National Council of Teachers of Mathematics, Research Brief. Reston, VA: NCTM. *Research points to several strategies consistently effective in teaching students who experience difficulties in mathematics: structured peer-assisted learning activities, systemic and explicit instruction, student think-alouds, formative assessment data provided to teachers and students, extensive use of visual representations. **3C, 3J, 3K.** Allman, J. with McCoy, L. (Ed). (December, 2005). Mathematical Reasoning in Multiple Representations: Connections and Confidence. Studies in Teaching 2005 Research Digest. Winston-Salem, NC: Wake Forest University. *Ten (10) high school students were observed and interviewed about their performance on a task with regard to intertwining strands of mathematical proficiency described by the National Research Council in 2001 for grades K-8: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The researcher determined that students should investigate with multiple representations of the content to construct their web ofunderstanding.

	3C, 3G, 3H, 3J. Kenney, J., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C. (2005). *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development. *Blending of current research on selected aspects of language literacy and practical strategies and suggestions for improving students' understanding of mathematical language and their ability to read mathematics text, to write about their mathematical thinking, and to enhance their communication through graphic representation and discourse. **3F, 3G, 3I.** Dougherty, B., Akana, K., Cho, C., and Fernandez, J. (November 2005). TI-Navigator Technology and Algebra I. University of Hawaii Curriculum Research and Development Group. * Eighth grade students using the TI-Navigator showed improvement in conceptual understanding classroom interactions, quantity and quality of responses, time on task, and time to start tasks. **3F, 3G, 3I.** Stroup, W., Carmona, L., and Davis, S. (October 2005). Improving on Expectations: Preliminary Results From Using Network-Supported Function-Based Algebra. The University of Texas at Austin. *Scores of students in the reduced lunch category, female students, and Hispanic students improved on test scores in algebra when using the TI-Navigator system. **3H, 3I.** Stiggins, R., Arter, J., Chappuis, J., Chappuis, S. (2006). *Classroom Assessment for Student Learning: Doing It Right—Using It Well.* Educational Testing Service. *Book (can be used for professional development) offers research-based tools for quality classroom assessment that produces accurate information (from defining and assessing clear learning targets, understanding the purpose for assessing, and designing and using a variety of methods well) used effectively to maximize student learning (to plan instruction and to involve students in their own assessment, and communicating results clearly to meet the needs of the user—including the student). The difference between communicating about assessment *of* learning and *for* learning is discussed, e.g., report cards, standardized tests, portfolios, conferences.

4. Learning Climate	
The teacher: • A. Creates learning environments where students are active participants in creating, questioning, sharing, discussing, and analyzing mathematical problems/ tasks. • B. Motivates students to achieve, and nurtures their desire to learn in an environment that promotes empathy, compassion, and mutual respect among students and between students and the teacher. • C. Provides learning experiences that actively engage students as individuals and as members of collaborative groups • D. Encourages students to accept responsibility for their own learning and respects the right of each student to ask questions and to request resources in order to more fully understand, enhance, or add clarity to the learning	**4 (A-K).** National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. ***Chapter 4**, (section) "Proficiency Develops Over Time," p 135: Students need enough time to engage in activities around a specific mathematical topic if they are to become proficient with it. ***Chapter 9**, Teaching for Mathematical Proficiency, pp 333-359: Teachers can motivate students to strive for mathematical proficiency both by supporting their expectations for achieving success through a reasonable investment of effort and by helping them appreciate the value of what they are learning….Classroom discourse provides opportunities to emphasize and model mathematical reasoning and problem solving and to enhance students' disposition toward mathematics….Cooperative group methods are likely to have positive effects on achievement and on other social and psychological characteristics….The teacher's role is to establish a classroom culture that supports learning with understanding…a community of learners…The circumstances that allow students to engage in and spend **time** on academic tasks…have been labeled as "opportunity to learn"…considered the single most important predictor of student achievement. **4A, 4B, 4D, 4G, 4H, 4J, 4K.** National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on *How Students Learn,* A Targeted Report for Teachers. M.S. Donovan and J. D. Bransford (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press. ***Chapter 5**, Mathematical Understanding: An Introduction, pp 228-231, pp 236-242: One important way to make students' thinking visible is through "math talk"…helping students experience their own abilities to find patterns and problems, invent solutions…and contribute to and learn from discussions with others provides the kinds of experiences that can help them learn with understanding… **4A, 4B, 4C, 4D, 4G, 4H, 4J, 4K. *Chapter 16**, Open and Closed Experiences and Understandings, pp 135-142:…Success of students (who learned mathematics in an open, project-based environment)…was enhanced by their desire and abilities to think about unfamiliar situations and determine what was required…able to interpret and develop meaning. **4B, 4C, 4D, 4G, 4H, 4I, 4J, 4K.** National Council of Teachers of Mathematics. (2002). *Lessons Learned from Research.* J. Sowder and B Schappelle (Eds.). Reston, VA: NCTM. ***Chapter 2**, Motivating Students by Teaching for Understanding, pp 17-22: Researchers…found that positive student motivation was associated with increased skills related to fractions…characterized by an increase in learning and understanding, self-confidence, willingness to take risks, enjoyment, and positive feelings. ***Chapters 7 and 8**, Tell Me With Whom You're Learning, and I'll Tell You How Much You've Learned: Mixed-ability Versus Same-ability Grouping in Mathematics, pp 47-61 and pp 63-70: …the

• E. Displays effective and efficient classroom management (e.g., in facilitating cooperative groups, in use of equipment or hands-on materials) • F. Provides sufficient time in mathematics class for students to engage in hands-on experiences, discussions of the content, applications of the mathematics, etc. The students: • G. Accept responsibility for their own learning • H. Actively participate (regardless of gender, race, ability or disability) • I. Collaborate/team with other students • J. Exhibit a sense of accomplishment and confidence • K. Take educational risks in class (e.g., to refute, defend, etc.)	achievements of our average and less able students proved to be significantly higher when compared to their peers in the same-ability classes, whereas highly able students performed about the same. ***Chapter 11**, A Longitudinal Study of Invention and Understanding: Children's Multidigit Addition and Subtraction, pp 93-100: Results indicate that almost all children can and do invent strategies and that this process of invention (especially when it comes before learning standard algorithms) may have multiple advantages. **4A, 4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K.** Hyde, A., foreword by E. Keene (2006). *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K-6.* Portsmouth, NH: Heinemann *Book provides research and suggestions for implementing it and "braiding" the concepts of thinking, language, and math…providing students more opportunities to make connections, to understand their own thinking, to use language, to create representations, to revise, and to visualize as they approach math problems. Chapters: -Braiding Mathematics, Language, and Thinking -Asking Questions-Making Connections -Visualization -Inferring and Predicting -Determining Importance -Synthesizing -The Power of Braiding (Planning for problem solving and teaching content through problem solving) **4 (A-K).** Mid-continent Research for Education and Learning (McREL). (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning,* J. Sutton and A. Krueger (Eds.). Aurora, CO: McREL. *Summary of educational research and surveys of best classroom practices, with implications for improved teaching and learning. Questions addressed: -How can teachers motivate students to enjoy and want to learn mathematics? -What are the impacts of ability grouping and tracking on student learning? -What is equity and how is it evident in mathematics classrooms? -What can schools do to facilitate students' opportunity to learn mathematics? -How can different learning styles be addressed with consistent expectations? -What instructional strategies make mathematics teaching more learner-centered? -How do teacher attitudes about mathematics learning impact student achievement? -What factors contribute most strongly to students' success in learning mathematics? -How can teachers help students reflect on and communicate their own learning? -How do students' attitudes affect their performance and future opportunities?

	4(A-K). Chapin, S., O'Connor, C., and Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn (Grades 1-6).* Sausalito, CA: Math Solutions Publications. *Book provides tools based on research (cited in back of the book) for the planned use of classroom discourse to support students' thinking and reasoning in mathematics, including productive talk moves, formats, ground rules, and planning and implementing lessons to incorporate math talk. **4 (A-K).** Wiliam, D. (2007). Five "Key Strategies" for Effective Formative Assessment. Research Brief. National Council of Teachers of Mathematics (NCTM). *Formative assessment can be increased by using five (5) key strategies: Clarifying, sharing, and understanding goals for learning and criteria for success with learners; engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning; providing feedback that moves learning forward; activating students as owners of their own learning; and activating students as learning resources for one another. **4 (A-K).** Kenney, J., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C. (2005). *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development. *Blending of current research on selected aspects of language literacy and practical strategies and suggestions for improving students' understanding of mathematical language and their ability to read mathematics text, to write about their mathematical thinking, and to enhance their communication through graphic representation and discourse. **4 (A-K).** Tomlinson, C. (Aug 2000). Differentiation of Instruction in the Elementary Grades. *Teacher Support.* ENC. (Also see Tomlinson, C. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development. ED 429 944) *Teachers can differentiate at least four classroom elements based on student readiness, interest, or learning profile: content, process, products, and learning environment, e.g., varying the length of time a student may take to complete a task, allowing students to work alone or in small groups, use effective management routines. **4(A-K).** Olson, M., Olson, J., Swarthout, M., and Hartweg, K. (2007). Meeting Special Needs in Mathematics Through Learning Stations. *Leadership to Math Success for All*. E. Bazik (Ed). Monograph Series, Vol 5, pp 31-41. National Council of Supervisors of Mathematics (NCSM). *Multiple-day learning stations for teaching mathematics can be used to accommodate students of special needs, and can benefit all students, by removing barriers, structuring the environment, and providing more time and practice…opportunities to work in small groups…build and expand levels of conceptual knowledge…become self-directed…take pride in their accomplishments…All students achieve some level of success…

	4 (A-K). Connell, M., Klein, R., Harnisch, D. (2007). Technology Uses in Special Education Mathematics Classrooms. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 54-63. National Council of Supervisors of Mathematics (NCSM). *…use of technology (Smart Board) to support an instructional model…to provide “mindful engagement” that enhances students’ abilities to build conceptual knowledge. **4A, 4B, 4C, 4D, 4E, 4G, 4H, 4J, 4K.** Deana, M. (May 2007). The Effects of Cooperative Learning on Student Motivation (Master’s Project). State University College at Cortland. *Findings from 15 studies…suggest that cooperative learning may increase motivation by creating a more enjoyable learning environment, by increasing student self-efficacy in the content area, and by holding students accountable to their peers…students became more engaged and had a more positive attitude toward learning. **4A, 4B, 4C, 4F, 4H, 4I.** Gersten, R., and Clarke, B. (2007). Effective Strategies for Teaching Students with Difficulties in Mathematics. National Council of Teachers of Mathematics, Research Brief. Reston, VA: NCTM. *Research points to several strategies consistently effective in teaching students who experience difficulties in mathematics: structured peer-assisted learning activities, systemic and explicit instruction, student think-alouds, formative assessment data provided to teachers and students, extensive use of visual representations (some variation for special needs students). **4A, 4C, 4D, 4G, 4H, 4I, 4J, 4K.** Black, P., and Wiliam, D. (October, 1998). Inside the Black Box: Raising Standards Through Classroom Assessment. *Phi Delta Kappan,* 80(2), 139-148. *Research summary of more than 250 articles/chapters…found that formative assessment can contribute more to improving outcomes than any other school-based factor, especially benefiting low achievers….essential component of classroom work. **4E, 4F.** Wells, C., with Milner, J. (December 2005). Making Lasting Impressions: Teachers’ Use of the First and Last Five Minutes of Class Time. Studies in Teaching 2005 Research Digest. Winston-Salem, NC: Wake Forest University. *Recent brain research suggests that the brain operates using a primacy/recency effect, and therefore the first and last minutes of class should be spent in learning new material. **4F.** National Council of Teachers of Mathematics. (August, 2006). Math Takes Time (Position Statement). NCTM. *All students need to be engaged in learning challenging mathematics for at least one hour a day at the elementary, middle school, and high school levels.

5. Informative Assessment and Reflection	
The teacher: • A. Uses multiple methods within the classroom and systematically gathers data about student understanding and ability (formative and summative assessments) • B. Uses student work/data, observations of instruction, assignments and interactions with colleagues to reflect on and improve teaching practice consistently • C. Revises instructional strategies based upon analysis of student achievement data (short term and long term) • D. Uncovers students' prior knowledge about the concepts to be addressed and addresses misconceptions/incomplete conceptions	**5A, 5B, 5C, 5D, 5G.** National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. ***Chapter 2**, The State of School Mathematics in the United States, "Assessments," pp 39-44: This linking of assessment to instructional efforts is consistent with the recent NRC report *Testing, Teaching, and Learning* (Elmore and Rothman, 1999)…Teachers should administer assessments frequently and regularly in classrooms for the purpose of monitoring individual students' performance and adapting instruction to improve their performance. ***Chapter 9**, Teaching For Mathematical Proficiency, pp 338-350: Successful teachers not only expect their students to succeed but also see themselves as capable of motivating and instructing students effectively. ***Chapter 11**, Conclusions and Recommendations, pp 424-425:…teachers should plan for instruction…focus on learning goals…provide ways of ascertaining what students know …anticipating the ways students will respond and how those responses can be used to further the lesson goals. **5D.** National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on *How Students Learn,* A Targeted Report for Teachers, M.S. Donovan and J. D. Bransford, (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press. ***Chapter 6**, Fostering the Development of Whole-Number Sense: Teaching Mathematics in the Primary Grades, pp 257-308; **Chapter 7**, Pipes, Tubes, and Beakers: New Approaches to Teaching the Rational-Number System, pp 309-349; and **Chapter 8**, Teaching and Learning Functions, pp 351-393)…engaging students' preconceptions and building on existing knowledge…using elementary, middle, and high school models **5A, 5D.** National Council of Teachers of Mathematics. (2002). *Lessons Learned from Research.* J. Sowder and B Schappelle (Eds.). Reston, VA: NCTM. ***Chapter 25**, Using Concept Maps to Assess Conceptual Knowledge, pp 203-205: Concept maps can help us determine if students have made connections among mathematical concepts. **5 (A-M).** Mid-continent Research for Education and Learning (McREL). (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning,* J. Sutton and A. Krueger (Eds.). Aurora, CO: McREL. *Summary of educational research and surveys of best classroom practices, with implications for improved teaching and learning. Questions addressed: -What roles can assessment play in mathematics teaching and learning? -How can the use of varied assessments provide important evidence of learning?

• E. Co-develops scoring guides/rubrics with students and provides adequate modeling to make clear the expectations for quality performance • F. Guides students to apply rubrics to assess their performance and to identify improvement strategies • G. Provides feedback (focused, descriptive, and qualitative) that moves learners forward. • H. Allows students to use feedback to improve their work before a grade is assigned. • I. Facilitates students in self- and peer-assessment • J. Provides qualitative and quantitative feedback to students and parents on a regular and timely basis The students: • K. Recognize what proficient work looks like and determine steps necessary for improving their work, e.g., explaining, verifying, justifying.	-How can mathematical thinking be assessed in the classroom? -What do national/international assessments tell us about teaching and learning mathematics? -What role does teacher questioning play in learning mathematics? -How does linking instruction and classroom assessment impact student learning? -What are characteristics of effective homework in mathematics? **5 (A-M) and 2H, 2I.** Stiggins, R., Arter, J., Chappuis, J., Chappuis, S. (2006). *Classroom Assessment for Student Learning: Doing It Right—Using It Well.* Educational Testing Service. *Book (can be used for professional development) offers research-based tools for quality classroom assessment that produces accurate information (from defining and assessing clear learning targets, understanding the purpose for assessing, and designing and using a variety of methods well) used effectively to maximize student learning (to plan instruction and to involve students in their own assessment, and communicating results clearly to meet the needs of the user—including the student). **5(A-M) and 2H, 2I.** Wiliam, D. (2007). Five "Key Strategies" for Effective Formative Assessment. Research Brief. National Council of Teachers of Mathematics (NCTM). *Formative assessment can be increased by using five (5) key strategies: Clarifying, sharing, and understanding goals for learning and criteria for success with learners; engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning; providing feedback that moves learning forward; activating students as owners of their own learning; and activating students as learning resources for one another. **5A.** Strong, R., Thomas, E., Perini, M., and Silver, H. (2004). Creating a Differentiated Mathematics Classroom. *Educational Leadership,* 61(5), pp 73-78. Association for Supervision and Curriculum Development. *To construct a differentiated mathematics classroom…include computation, explanation, application, and problem solving in every unit; help students recognize their own mathematical learning styles; use a variety of teaching strategies; create assessments to reflect all four dimensions of mathematical learning and all four learning styles. **5A, 5B, 5C.** Popham, W.James. (2003, February). The Seductive Allure of Data. *Educational Leadership,* 60(3), 48-51. Association for Supervision and Curriculum Development. *…the right kinds of assessment data…from properly designed classroom assessments, can help teachers improve student achievement. Five attributes of instructionally useful tests: significance, teachability, describability, reportability, and nonobtrusiveness… **5A, 5B, 5D, 5G, 5I, 5J, 5M.** Chapin, S., O'Connor, C., and Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn (Grades 1-6).* Sausalito, CA: Math Solutions Publications. *Book provides tools based on research (cited in back of the book) for the planned use of classroom discourse to support students' thinking and reasoning in mathematics, including productive talk moves, formats, ground rules, and planning and

• L. Develop and/or use scoring guides periodically to assess their own work or that of their peers • M. Use teacher feedback to improve their work	implementing lessons to incorporate math talk. **5A, 5B, 5D, 5G, 5H, 5I, 5J.** Research Relating to TI-Navigator. (2004). SRI International (Prepared for Texas Instruments) *Students using the TI-Navigator system in class were able to gauge their own level of understanding, able to understand complex subjects, and more willing to take part in discussion…Teachers were able to immediately assess, provide feedback, and guide student performance… **5A, 5B, 5C, 5D, 5G, 5J, 5M.** Bradsby, L. (2007). Methods and Strategies to Help At-Risk Students Be Successful in Secondary Mathematics. *Leadership to Math Success for All*. E. Bazik (Ed). Monograph Series, Vol 5, pp 42-53. National Council of Supervisors of Mathematics (NCSM). *A systematic approach consisting of six (6) hierarchical teaching processes incorporates many of the aspects of mastery learning as analyzed by Guskey and Gates (1986) in 27 studies… **5A, 5B, 5C, 5D, 5I, 5K.** Buschman, L. (December 2001). Using Student Interviews to Guide Classroom Instruction: An Action Research Project. *Teaching Children Mathematics,* Vol 8, Issue 4, pp 222-227. National Council of Teachers of Mathematics. * Teachers who assessed students through interviews experienced a shift in their beliefs about how children learn to think mathematically, how they reason, why they make errors, and what kind of feedback will help them become better problem solvers. **5A, 5B, 5D, 5E, 5F, 5G.** Tomlinson, C. (Aug 2000). Differentiation of Instruction in the Elementary Grades. *Teacher Support.* ENC. (Also see Tomlinson, C. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development. ED 429 944) *Teachers can differentiate at least four classroom elements based on student readiness, interest, or learning profile: content, process, products, and learning environment. Make assessment ongoing and tightly linked to instruction, build a support system of other educators, reflect constantly on the quality of what is being differentiated, use rubrics that match and extend students' varied skills levels. **5A, 5B, 5C, 5D, 5H, 5I, 5K, 5M.** Kenney, J., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C. (2005). *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development. *Blending of current research on selected aspects of language literacy and practical strategies and suggestions for improving students' understanding of mathematical language and their ability to read mathematics text, to write about their mathematical thinking, and to enhance their communication through graphic representation and discourse. **5A, 5C, 5D, 5J.** Clarke, B., Baker, S., and Chard, D. (2007). Measuring Number Sense Development in Young Children: A Summary of Early Research. *Leadership to Math Success for All*. E. Bazik (Ed). Monograph Series, Vol 5, pp 1-11. National Council of Supervisors of Mathematics (NCSM). *…systems for preventing academic difficulties, e.g., the three-tier model approach to providing

	literacy instruction, are more effective for improving outcomes for struggling students than are remedial efforts after difficulties are discovered. **5A, 5B, 5C, 5D, 5E, 5F, 5I, 5L.** Olson, M., Olson, J., Swarthout, M., and Hartweg, K. (2007). Meeting Special Needs in Mathematics Through Learning Stations. *Leadership to Math Success for All*. E. Bazik (Ed). Monograph Series, Vol 5, pp 31-41. National Council of Supervisors of Mathematics (NCSM). *Multiple-day learning stations for teaching mathematics can be used to accommodate students of special needs, and can benefit all students, by removing barriers, structuring the environment, and providing more time and practice… multiple assessment opportunities give students a variety of ways to represent and demonstrate their learning…All students achieve some level of success… **5A, 5C, 5D.** Niemi, D., Vallone, J., and Vendlinski, T. (August, 2006). The Power of Big Ideas in Mathematics Education: Development and Pilot Testing of POWERSOURCE Assessments. CSE Report 697. National Center for Research on Evaluation, Standards, and Student Testing (CRESST) and Center for the Study of Evaluation (CSE). Los Angeles, CA: Graduate School of Education and Information Studies-UCLA. *…identify and assess understanding of big ideas…different assessments can reveal different information about student understanding…a strategy has the potential to help teachers remediate those deficiencies. **5A, 5B, 5C, 5D, 5G, 5H, 5I, 5J, 5K, 5M.** Black, P., and Wiliam, D. (October, 1998). Inside the Black Box: Raising Standards Through Classroom Assessment. *Phi Delta Kappan,* 80(2), 139-148. *Research summary of more than 250 articles/chapters…found that formative assessment can contribute more to improving outcomes than any other school-based factor, especially benefiting low achievers….essential component of classroom work. **5B, 5C, 5G, 5I, 5J, 5M.** Baker, S., Gersten, R., Lee, D-S. A Synthesis of Empirical Research on Teaching Mathematics to Low-Achieving Students. *The Elementary School Journal*, (103)1, 51-73. Results from 15 studies indicated that different types of interventions led to improvements in the mathematics achievement of students considered low achieving or at risk for failure… include… providing teachers and students with specific information on how each student is performing (and instructional recommendations); using peers as tutors or instructional guides to provide feedback or support; providing clear, specific feedback to parents on their children's successes in mathematics…. On the issue of explicit instruction vs. contextualized instruction, seven (7) studies included three (3) that investigated each of these and one (1) that used both approaches…suggested that principles of explicit instruction can be useful in teaching mathematics concepts and procedures—both for developing problem-solving strategies and more classic direct instruction….less clarity about the benefits of contextualized approaches—students in the four (4) studies had some success working complex, real-world problems after they had been explicitly taught the underlying foundational mathematical concepts….For lower-achieving students, this small number of instructional studies seems to support…a mix of explicit instruction in procedures and ample opportunity to apply

	procedures to open-ended problems with real-world relevance. **5B, 5C, 5G, 5I, 5J, 5M.** Gersten, R., and Clarke, B. (2007). Effective Strategies for Teaching Students with Difficulties in Mathematics. National Council of Teachers of Mathematics, Research Brief. Reston, VA: NCTM. *Research points to several strategies consistently effective in teaching students who experience difficulties in mathematics: structured peer-assisted learning activities, systemic and explicit instruction, student think-alouds, formative assessment data provided to teachers and students, extensive use of visual representations (some variation for special needs students). **5B, 5C, 5G, 5H, 5M.** Guskey, T. (February 2003). How Classroom Assessments Improve Learning. *Educational Leadership.* 60(3). Using Data to Improve Student Achievement, pp 6-11. Association for Supervision and Curriculum Development. *To use assessments to improve instruction and student learning teachers need to make assessments useful, follow assessments with corrective instruction, and give second chances to demonstrate success. **5G, 5H, 5I.** Connell, M., Klein, R., Harnisch, D. (2007). Technology Uses in Special Education Mathematics Classrooms. *Leadership to Math Success for All*, E. Bazik (Ed). Monograph Series, Vol 5, pp 54-63. National Council of Supervisors of Mathematics (NCSM). *…use of technology (Smart Board) to support an instructional model…to provide “mindful engagement” that enhances students’ abilities to build conceptual knowledge.

Additional Resources

Kentucky Department of Education *Program of Studies, Revised 2006*
Kentucky Department of Education Academic Expectations
Kentucky Department of Education *Core Content for Assessment, Version 4.1*
Kentucky Department of Education *Standards and Indicators for School Improvement*
Kentucky Department of Education *Guide for Reflective Classroom Practices: A Self-Assessment Tool for Teachers* (draft)
Kentucky Department of Education Mathematics *PERKS*
The Mathematics Program Improvement Review: A Comprehensive Evaluation for K-12 Schools-ARSI-Ron Pelfrey
Standards-Based School Mathematics Review-ARSI/AMSP-Sheila Vice