Kindergarten Standards

1. Representing and comparing whole numbers, initially with sets of objects
   - Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Describing shapes and space
   - Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Counting and Cardinality

Know number names and the count sequence.
- K.CC.1: Count to 100 by ones and by tens.
- K.CC.2: Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- K.CC.3: Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Counting to tell the number of objects.
- K.CC.4: Understand the relationship between numbers and quantities; connect counting to cardinality.
  - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
  - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  - c. Understand that each successive number name refers to a quantity that is one larger.
- K.CC.5: Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Comparing numbers.
- K.CC.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Note: Include groups with up to ten objects.)
- K.CC.7: Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking

Understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from.
- K.OA.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Note: Drawings need not show details, but should show the mathematics in the problem – this applies wherever drawings are mentioned in the Standards.)
- K.OA.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
- K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- K.OA.5: Fluently add and subtract within 5.

Number and Operations in Base Ten

Working with numbers 11 – 19 to gain foundations for place value.
- K.NBT.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Describe and compare measurable attributes.
- K.MD.1: Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
- K.MD.2: Directly compare two objects with a measurable attribute in common, to see which object has “more of”/”less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Classify objects and count the number of objects in each category.
- K.MD.3: Classify objects or people into given categories; count the numbers in each category and sort the categories by count. (Note: Limit category counts to be less than or equal to 10.)

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
- K.G.1: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
- K.G.2: Correctly name shapes regardless of their orientations or overall size.
- K.G.3: Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
First Grade Standards

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20
   - Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including concrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Developing understanding of whole number relationship and place value, including grouping in tens and ones
   - Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. The compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Developing understanding of linear measurement and measuring lengths as iterating length units
   - Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Note: students should apply the principle of transitivity of measurement to make direct comparisons, but they need not use this technical term.)

4. Reasoning about attributes of, and composing and decomposing geometric shapes
   - Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 1.)

1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 1.)

1.OA.3: Apply properties of operations as strategies to add and subtract. (Note: Students need not use formal terms for these properties.) Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

1.OA.4: Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

1.OA.5: Relate counting to addition and subtraction (e.g., by counting on to add 2).

1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

1.OA.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 7, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

1.OA.8: Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + ? = 11.

Number and Operations in Base Ten

Extend the counting sequence.

1.NBT.1: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

1.NBT.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a “ten.”
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.3: Compare two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

1.NBT.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.5: Give a two-digit number, mentally find 10 more or 10 less than the number, without having to count, explain the reasoning used.

1.NBT.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2: Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Tell and write time.

1.MD.3: Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

1.MD.4: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

Reason with shapes and their attributes.

1.G.1: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1.G.2: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Note: Students do not need to learn formal names such as “right triangular prism.”)

1.G.3: Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of, the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Second Grade Standards

1. Extending understanding of base-ten notation
   - Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Building fluency with addition and subtraction
   - Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Using standard units of measure
   - Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. Describing and analyzing shapes
   - Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding attributes of two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 1.)

Add and subtract within 20.

2.OA.2: Fluently add and subtract within 20 using mental strategies. (Note: See standard 1.OA.6 for a list of mental strategies). By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3: Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

Understand place value.

2.NBT.1: Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2: Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3: Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6: Add up to four two-digit numbers using strategies based on place values and properties of operations.

2.NBT.7: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8: Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9: Explain why addition and subtraction strategies work, using place value and the properties of operations. (Note: Explanations may be supported by drawings or objects.)

Measurement and Data

Measure and estimate lengths in standard units.

2.MD.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3: Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4: Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6: Represent whole-number lengths as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.7: Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

2.MD.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Represent and interpret data.

2.MD.9: Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph. (Note: See Glossary, Table 1.)

Geometry

Reason with shapes and their attributes.

2.G.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Note: Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

2.G.2: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3: Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, fourths, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Third Grade – Standards

1. Developing understanding of multiplication and division and strategies for multiplication and division within 100
   • Students develop an understanding of the meanings of multiplication and division and relate multiplication and division strategies involving equal-sized groups, arrays, and area models; multiplication is found by counting product, and division as finding an unknown factor in these situations. For equal-sized group situations, division can find the unknown number of groups or the unknown group size. Students use properties of operations to calculate products in different ways, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1)
   • Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and use them to represent fractions with larger numerators. Students understand that a fraction 1/b describes the quantity formed by 1 part when a whole is partitioned into b equal parts; and can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect this idea to decomposing fractions as built out of unit fractions.

3. Developing understanding of the structure of rectangular arrays and of area
   • Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. A two-dimensional figure can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect this idea to decomposing fractions as built out of unit fractions.

4. Describing and analyzing two-dimensional shapes
   • Students describe, analyze, and compare two-dimensional shapes. They classify shapes by their sides and angles, and connect these with definitions of shapes. Students connect this work with their work in area by decomposing rectangles into rectangular arrays of squares, expressing the area of a part of a shape as a fraction of the whole.

Operations and Algebraic Thinking

3.OA.1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

3.OA.2: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into 8 equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and areas, and measure distances or areas in real-world problems by using strategies based on multiplication and division. Include problems in which remainders must be interpreted. (Note: See Glossary, Table 2.)

3.OA.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, ? ÷ 3 = 8, ? + 8 = 20, and 5 × ? = 35.

Understanding properties of multiplication and the relationship between multiplication and division

3.OA.5: Apply properties of operations as strategies to multiply and divide. (Note: Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2, or by 3 × 2 = 6, then 6 × 5 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

3.OA.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 multiplied by 8.

3.OA.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.NF.2: Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; and can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect this idea to decomposing fractions as built out of unit fractions.

Solve problems involving the four operations, and identify and explain patterns in arithmetic

3.OA.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Note: This standard is limited to problems posed with whole numbers and having-whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order—Order of Operations.)

3.OA.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

3.MD.1: Tell and write time to the nearest 5 minutes, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Note: Excludes compound units like cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve problems involving measurements.

3.MD.3: Relate area to the operations of multiplication and addition. Find areas of rectilinear figures by tiling with unit squares as examples of qu

3.MD.5: Recognize area as additive. Find areas of rectilinear figures by tiling with unit squares of the same size, or by decomposing the figure into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

3.MD.6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7: Relate area to the operations of multiplication and addition.

Geometric measurement: recognize perimeter as an attribute of plane figures and understand concepts of area measurement.

3.MD.5: Recognize perime

3.MD.6: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Reason with shapes and their attributes.

3.G.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2: Partition shapes into parts with equal areas. Express the area of each part as a fraction of the whole.

Geometric measurement: understand concepts of area and relate area to multiplication and addition.

3.MD.5: Recognize an area as an attribute of plane figures and understand concepts of area measurement.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Fourth Grade – Standards

1. Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends
   - Students generalize their understanding of place value to 1,000,000, understanding the size of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, area models, place value, properties of operations, and the relationship of division to multiplication) as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers.

2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, generation and recognition of equivalent fractions
   - Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 1/3 = 3/9) and, in whole number contexts, develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions by applying their knowledge of fractions to the situation of dividing objects or sets of objects.

3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry
   - Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students develop an understanding of the attributes of two-dimensional objects and the use of them to solve problems involving symmetry.

Operations and Algebraic Thinking

4. OA.3: Solve multiplicative word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4. OA.4: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4. OA.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate the terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate. Generalize number patterns.

4. OA.6: Analyze patterns using two variables.

Number and Operations in Base Ten

4. NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4. NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and properties of operations. Illustrate and explain how to multiply decimals, using examples such as 0.9 × 0.9 and 0.99 × 0.99.

4. NBT.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain calculations by using equations, rectangular arrays, and/or area models.

Number and Operations – Fractions

4. NF.1: Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4. NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4. NF.3: Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and like denominators, e.g., by using visual fraction models and equations to represent the problem.

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4. NF.4: Multiply a fraction by a whole number. a. Understand a fraction ab as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). b. Understand a multiple of a unit fraction as a multiplicative comparison of one. Use visual fraction models to represent a multiple of a fraction as a multiple of a multiple. c. Solve word problems involving multiplication of a fraction by a whole number, using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pizza, and there are 5 people at the party, how many pizzas will be eaten? d. Extend understanding of multiplication to include multiplying a fraction by a whole number. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4. MD.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

4. MD.4: Represent and interpret data.

4. MD.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

(a) An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

(b) An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4. MD.6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4. MD.7: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, using an equation with a symbol for the unknown angle measure.

Geometry

4. G.1: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4. G.2: Classify two-dimensional shapes based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4. G.3: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Number and Operations: Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. For example, by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 3/10 = 3/10 because 3/10 is a third.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, if a rectangular park has an area of 36 square meters and width 4 meters, then multiply side length by width as 36 ÷ 4 as the result of dividing 36 by 4, noting that 36/4 multiplied by 4 is 3, and that when three whole squares are shared equally among four people each person has a share of 3/4 square meter. If we want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4: Apply and extend previous understandings of multiplication to multiply a fraction by a fraction.

Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (3/4) × (5/6). (In general, a/b × c/d = (ac)/(bd).

a. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show how the area is computed by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

b. Interpret multiplication as scaling (resizing), by:

i. Comparing the size of a product to the size of one factor on the basis of sizes of the unit squares used to tile both, and showing that the area of a rectangle need not be smaller than either factor.

ii. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying ab by the reciprocal of b.

5.NF.6: Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

b. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to represent the question. Use the relationship between multiplication and division to explain that (10 ÷ 4) = 10/4 because (10/4) × 4 = 10.

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. For example, by using visual fraction models and equations to express the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are there in 2 cups of raisins?

Measurement and Data

Convert like measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Represent and interpret data.

Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same standard units.

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2: Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpreting coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4: Classify two-dimensional figures in a hierarchy based on their properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geo metric measurement: understand concepts of volume and relate volume to multiplication and addition.

5.MD.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5: Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as (8 + 7) × 2. Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Analyze patterns and relationships.

5.OA.3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form and evaluate expressions for these patterns.

5.OA.4: Generate patterns using the rule “Add 3” and the starting number “2.” Compare the resulting sequence to the sequence resulting from the rule “Add 6” and the starting number “0.” Notice that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Fifth Grade – Standards

1. Developing fluency with addition and subtraction of fractions, developing understanding of the multiplication of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

a. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent operations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, in particular the meaning of a unit fraction, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: This is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.

a. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number). They understand that the place value system allows for the volume to multiplication and to addition.

b. Students understand why division procedures work. They understand that, to understand and explain why the procedures for multiplying and dividing decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Developing understanding of volume.

a. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be quantified by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1 × 1 × 1 cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems involving volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into arrays of cubes and measure necessary attributes of shapes in order to solve real world and mathematical problems.
Sixth Grade Standards

1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems
   - Students use reasoning about multiplication and division to solve ratio problems. By viewing equivalent ratio as a way to connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve ratio problems, and connections.
   - Students solve a wide variety of problems involving ratios and rates.

2. Completing understandings of multiplication and division and extending the notion of number to the system of rational numbers, which includes negative numbers
   - Students use the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. Students use these operations to solve problems.
   - Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers. A rational number is a point on the number line. Extend and use arithmetic operations, for all rational numbers.

3. Writing, interpreting, and using expressions and equations
   - Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations; evaluate expressions, and use expressions and equations to solve problems.
   - Students understand that expressions are a way to record numbers and that equivalent expressions represent the same number regardless of which values are substituted into them.

4. Developing understanding of statistical thinking
   - Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution defines a center and spread that different ways to measure center yield different values. The median center is the number that is in the middle of the data. The mean measures center by finding the balance point of the data and recognizing that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median but very different measures of variability.

5. Applying and extending understandings of multiplication and division to divide fractions by fractions.

6. Expressions and Equations
   - Apply and extend previous understandings of arithmetic to algebraic expressions.

7. Reasoning with linear equations and inequalities.
   - Use properties of operations to generate equivalent expressions.
   - Understand solving an equation or inequality as a process of answering a question: which values from a specified set make the equation or inequality true? The process depends on an initial understanding of variables as quantities that can represent arbitrary numbers. 
   - Use variables to represent two quantities in a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

8. Analyze proportional relationships and use them to solve real-world and mathematical problems
   - Apply the concepts of proportionality to the situation in the context of solving real-world and mathematical problems.
   - Draw proportions in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Analyze these techniques in the context of solving real-world and mathematical problems.

9. Develop understanding of statistical variability.
   - Summarize numerical data sets in relation to their context, such as by:
     a. Reporting the number of observations.
     b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
     c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern, with reference to the context in which the data were gathered.
   - Using the context of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seventh Grade Standards

1. Developing understanding of and applying proportional relationships
   ▪ Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use reasoning about ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale dilations by relating corresponding lengths between the objects or by using the fact that lengths of corresponding sides are preserved in dilations. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Developing understanding of operations with rational numbers and working with expressions and linear equations
   ▪ Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers. Developing the properties of operations for operations with rational numbers and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume
   ▪ Students continue their work with area from Grade 6, solving problems involving the area, circumference, and volume of two- and three-dimensional objects. For example, students calculate unknown dimensions in problems involving area in the context of tiling and perimeter in the context of designing a storage container. To work with the volume of right rectangular prisms, students apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with integer edge lengths in the context of solving real-world and mathematical problems. In particular, they find surface areas of prisms and lateral and surface areas of cylinders.

4. Drawing inferences about populations based on samples
   ▪ Students build on their prior work with single and multiple data distributions to compare two data distributions and to draw inferences about two populations. They use measures of center and variability for numerical data from samples to compare two populations. For example, they estimate the mean word length in the grade science book are generally longer than the words in the basketball team's yearbook. They do this by using representative samples for drawing inferences and comparing two data distributions and address questions about the people or places the data represents. (In Grade 8 they reason about relationships among two populations using samples with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is easily visible.)

### Seventh Grade Standards

#### 7.RP.1: Develop understanding of and applying proportional relationships

1. **a.** Example, if a person walks 1/2 mile in each 1/4 hour, compute the walking speed in miles per hour.
2. **b.** Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
3. **c.** Explain what a proportionality constant means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

#### 7.RP.2: Recognize and represent proportional relationships between quantities.

1. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equal ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

#### 7.EE.3: Solve real-world and mathematical problems involving the area and circumference of a circle and surface area of right rectangular prisms and right rectangular pyramids.

1. **a.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. **b.** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. For example, given a triangle with a specific area and a specific height, generate a number of different triangles that can be drawn.
3. **c.** Describe the two-dimensional figures that result from slicing threedimensional objects with planes. For example, given a rectangular prism, describe the sections of right rectangular prisms and right triangular prisms.

### Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**
Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems.

- Students recognize the meaning of the x- and y-intercepts and the vertical and horizontal asymptotes of a graph of a function in terms of the situation.
- Students use linear equations, systems of linear equations, and linear functions to represent, analyze, and explain situations where one quantity determines another.
- Students reason abstractly and quantitatively.
- Students make sense of problems and persevere in solving them.

The Number System
Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1: Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat. Know that other numbers are called irrational.

8.NS.2: Use rational approximations of irrational numbers to compare the size of irrational numbers (e.g., by writing the number as a decimal approximation), place them on a number line diagram, and estimate the value of expressions (e.g., p2).

8.NS.3: Use properties of integer exponents to generate equivalent numerical expressions. For example, use the properties of exponents to rewrite expressions for exponential functions.

8.EE.5: Use proportional relationships to describe dilations, both as a single transformation and as a sequence of transformations, and explain why the x- and y-coordinates change in specific ways.

8.EE.6: Explain why the slope m of a non-vertical line is the same from any two distinct points on the line. Use similar right triangles to explain why the slope is the same from any two distinct points on the line.

8.EE.7: Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8: Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot be simultaneously equal to 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates of two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions
Define, evaluate, and compare functions.

8.F.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Note: Function notation is not required in Grade 8.)

8.F.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or verbally). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.3: Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

8.F.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5: Describe qualitatively the functional relationship between two quantities by analyzing a (graph, where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the relationship between the two quantities given by the statements of the features of a function that has been described verbally.

Geometry
Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1: Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. The angle of rotation is the same.

c. Parallel lines are taken to parallel lines.

8.G.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations: given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of dilations, translations, reflections, and rotations: given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Practices
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