Fractions on a Number Line
Grade 3
Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at:
kdemath@education.ky.gov

Revised 2019

Representing Fractions on a Number Line – Grade 3

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This lesson is intended to help you assess how well students are able to:

- Understand a fraction as the quantity formed by 1 part when a whole is partitioned into equal parts.
- Represent a fraction on a number line diagram
- Understand two fractions are equivalent (equal) if they are the same point on a number line.
- Solve fraction word problems using the number line to represent solutions.

Kentucky Academic Standards

This lesson asks students to select and apply mathematical content standards from within the grade, with the emphasis on:

<u>Grade 3 Number and Operations – Fractions</u> (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Cluster: Developing understanding of fractions as numbers.

- KY.3.NF.1 Understand a fraction 1 bb as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction aa bb as the quantity formed by a parts of size $1/b$.
- KY.3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line.
    - Represent a fraction $1/b$ (unit fraction) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.
        - ✓ Recognize each part has size $1/b$.
        - ✓ a unit fraction, $1/b$ is located $1/b$ of a whole unit from 0 on the number line.
    - b. Represent a non-unit fraction a/b on a number line by marking off a lengths of $1/b$ (unit fractions) from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the non-unit fraction a/b on the number line.
- KY.3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.
    - a. Understand two fractions as equivalent (equal) if they are the same size, or same point on a number line.
    - b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent through writing or drawing.
    - c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
    - d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or < and justify the conclusions.

This lesson involves a range of Standards for Mathematical Practice from the standards, with emphasis on:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics
MP6. Attend to precision
MP7. Look for and make use of structure.

Introduction

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work in pairs on a collaborative discussion task using number lines to show evidence of their thinking. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.
- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Big Ideas Addressed in this lesson:

The goal is for students to see unit fractions as the basic building block of fractions, in the same sense that the number 1 is the basic building block of the whole numbers. Just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions.

On the number line, the whole is the *unit interval*, that is, the interval from 0 to 1, measured by length. Iterating this whole to the right, marks off the whole numbers, so that the intervals between consecutive whole numbers, from 0 to 1, 1 to 2, 2 to 3, etc., are all of the same length, as shown. Students might think of the number line as an infinite ruler.

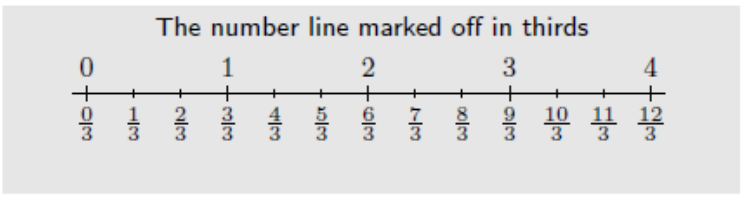
Students sometimes have difficulty perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use as the unit the entire portion of the number line that is shown on the diagram. For example, indicating the number 3 when asked to show 3/4 on a number line diagram marked from 0 to 4.

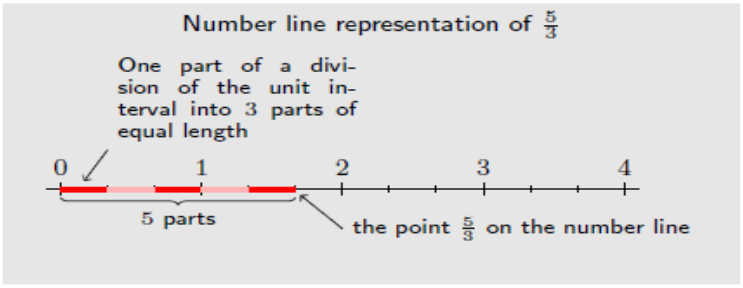
The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so 5/3 is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to 1/3.

Students should have experience with using number lines that go beyond 1 whole and counting the iterations of the fractional parts (see example on page 4).

Linear models like the number line are closely connected to real world-measuring. The number line also emphasizes that a fraction is one number as well as its relative size to other numbers. The number line reinforces that there is always one more fraction to be found between two fractions.

Examples:





Source: Math Progressions from William McCallum

http://ime.math.arizona.edu/progressions/

Materials required

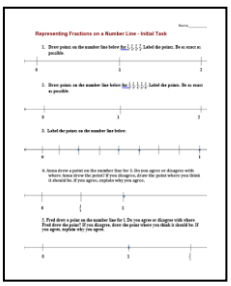
- Each student will need a copy of the initial assessment task, Representing Fractions on the Number Line.
- Each pair of students will need a copy of the Five Friends Swimming task.
- Optional: Whiteboards and markers
- Optional: String or clothesline rope for hands on Fraction Number Line. Number cards with fractions written on the cards.
- Optional: Fraction strips- pre-folded by students, fraction tiles, Cuisenaire rods,

Time needed

Approximately 15 minutes for the assessment task, one-hour or more for the lesson, and 20 minutes for the follow-up lesson where students revisit individual assessment task. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task: Representing Fractions on the Number Line



Ask the students do the initial task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and identify areas of concern/need and target your follow-up instruction effectively.

Give each student a copy of **Representing Fractions on the Number Line – Initial Task**. Introduce the task briefly and help the class to understand the problem and its context. Students should have some prior experience working with number lines and whole

numbers. This is also an opportunity to help students make connections with fractions in the real world. Having students share any experiences with measurement and rulers may also help in giving some context to the initial task.
Note: This formative assessment expects that students have previous experince

Possible instructions for students:

Spend 15 minutes working individually on this task.

Don't worry if you can't understand or do everything or do not finish. There will be a lesson [tomorrow] that will help you improve your work.

Your goal is to be able to answer this question with confidence by the end of that lesson.

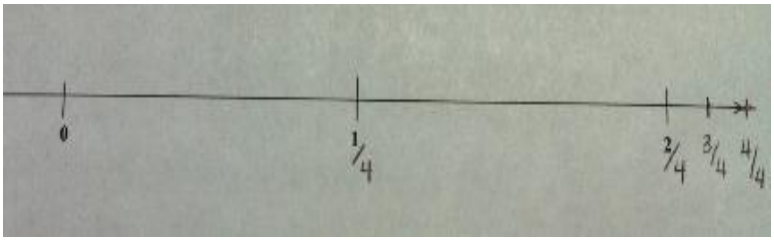
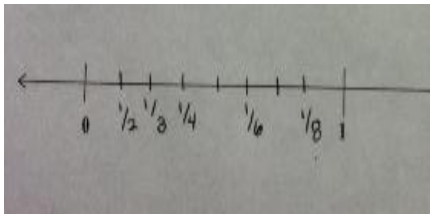
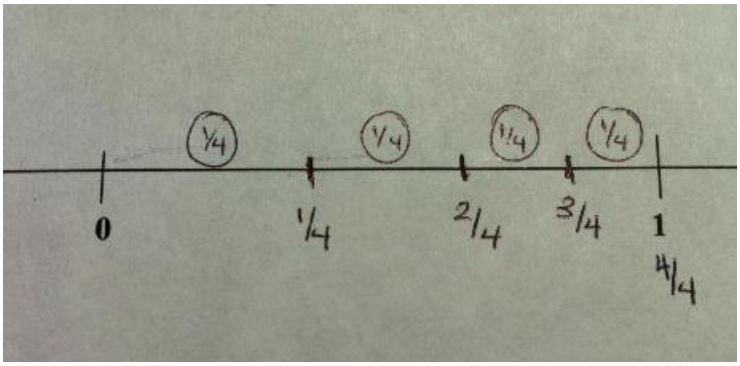
It is important that students are allowed to answer the questions without assistance. For struggling students, direct by paraphrasing or questioning, but do not complete the task for them.

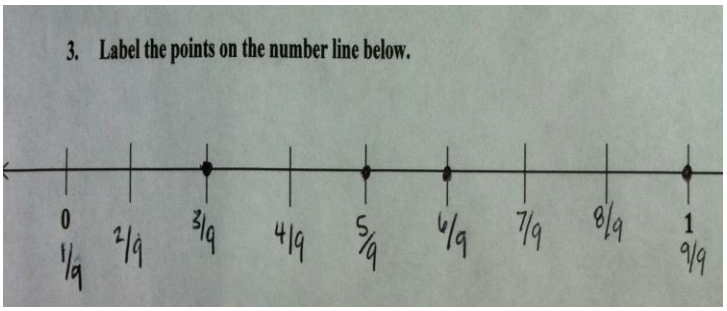
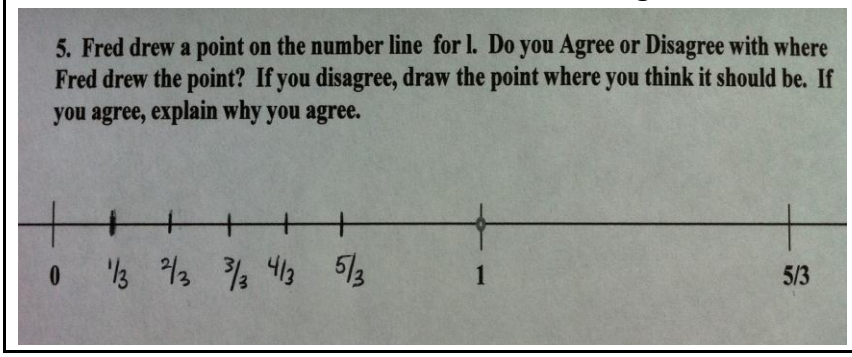
Assessing students' responses

Collect students' responses to the task. Make some notes about what their work reveals concerning their current levels of understanding and their different problem solving approaches. This will help you prepare for the lesson and anticipate issues that may arise. If time allows you may write questions on each student's work. If there are time constraints, select a few questions that will help the majority of students. These can be written/displayed on the board at the end of the lesson.

It is suggested that you do not score students' work. Instead, help students progress by asking questions that focus attention on aspects of their work. Anticipating the different ways the task can be solved will help you in developing questions. Consider how your students mathematically interpret the task, use of correct and incorrect strategies to solve it, and how those strategies and interpretations relate to the mathematical ideas embedded in the task.

It is also suggested that you plan student pairings based on their work on this initial task - pairing students homogeneously (common understandings and misconceptions).

Common Misconceptions	Suggested Feedback Questions
Student plots points based on understanding fractions as whole numbers instead of fractional parts. For example: Students order fractions using the numerator:  **Students order unit fractions by the denominator:**  	Do ½ and 1 mean the same thing? Tell me the difference between the size of ¼ and 1 whole? Can you draw a picture of ½? Can you draw ¼? Which is closer to 1 whole?
Student sees the numbers in fractions as two unrelated whole numbers separated by a line.	When I show this fraction 2/3, what does it mean? Does it mean 2 and 3 separated by a line?
Students do not understand that when partitioning a whole or a fraction into unit fractions, the intervals must be equal.  	What can you tell me about all the [fourths] of a whole?
Student does not understand the importance of the whole of a fraction and identifying it. For example, students may use a fixed size of ¼ based on the manipulatives used or previous experience with a ruler.	Is ¼ inch the same as ¼ of [this whole]?

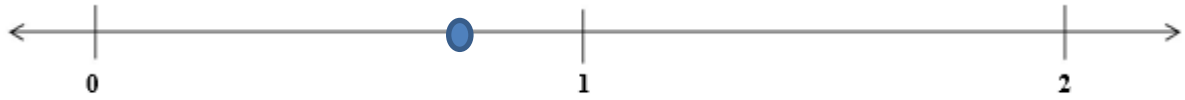
Student does not count correctly on the number line. For example, students may count the hash mark at zero as the first number in the sequence:  	Where are the parts of your whole? How many parts are there?
Student does not understand there are many fractions less than 1.	Could you place one half on the number line? Where would you place one fourth? What do you notice when you do this? Can you do it for other fractions?
Student does not understand fractions can be greater than 1.  	Can you count using fractions like you do with whole numbers? How would you count if you were counting by 1's, 2's, and ½'s? Can I have 3/2 cookies, hours, days, inches? Would that be less than or more than 1 [cookie]?

Suggested lesson outline

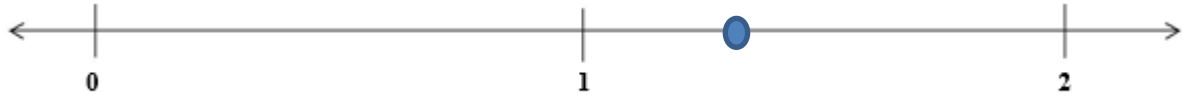
Whole Class Introduction (10 minutes)

Display this image on the board:

Number Line 1



Number Line 2



Ask students to identify the fraction located on each number line and explain their reasoning.

For example, if a student labels the fraction on Number Line 1 as ¾ ask, "How might you partition the number line into equal parts to justify your answer?"

Note: If a student labeled Number Line 1 as ¾ (or 6/8, 2/3, or 4/6) then the number line should be partitioned into four equal parts from 0 to 1. If the student labels Number Line 1 as 2/3, the explanation should include partitioning into three equal parts from 0 to 1. The same applies to 6/8 and 4/6 respectively. The student explanation should match the identified fraction.

Do the same questions for Number Line 2.

Optional: Each student have their own copy of the number lines to manipulate.

Collaborative Activity: Five Friends Swimming Task

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the preassessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

There are two versions of the Five Friends Swimming Task. Version B is slightly more challenging. Based on your assessment of the initial student task, you may want some pairs to start with Version A, while other pairs start with Version B. You may want all pairs to start with Version A, then move to Version B. Students may need to see an image of swimming lanes in a pool for background information.

> *I want you to work with your partner. Today you and your partner are going to use number lines to record where five swimmers are in a race at the lake.*

Each time you do this, explain your thinking clearly to your partner. If your partner disagrees with your placement then challenge him or her to explain why. It is important that you both understand why each marker is placed where it is on the number line. There is a lot of work to do today and you may not all finish. The important thing is to learn something new, so take your time.

Have students work in pairs to solve the Five Friends Swimming task.

Monitoring Student Groups

Your task during the partner work is to make notes of student approaches to the task, and to support student problem solving. As you monitor the work, listen to the discussion and ask questions to help students understand concepts and clarify misconceptions. Misconceptions to look for:

Students not equally partitioning the number line.
Students ordering fractions using the numerator.
Students ordering fractions using the denominator.
Students not seeing the relationship between the numerator and the denominator.

Whole class discussion: comparing different approaches (10 minutes)

Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Strategically select students or partners who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each pair shares, highlight the connections between strategies.

For pairs to share their solutions you might use a smart board or document camera, a clothesline number line, or floor number line. The whole group discussion is a great time for students to use the mathematical practice, *Construct viable arguments and critique the reasoning of others.*

The discussion offers students the opportunity to learn from each other and for you to address some of the common misconceptions observed in the initial task and Five Friends Swimming.

Students should be expected to use this time to compare their solutions, discuss misconceptions and eventually evaluating their own responses based on correct answers. As part of the culture in your mathematics classroom students need to feel safe to share their solution strategies and ask questions of the teacher and each other.

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions the students come up with.

- *Were there certain problems that were more difficult?*
- *Did you use the number line, or did you use a different method to solve the problems?*
- *Can different fractions represent the same place on the number line?*

- *Could you use what you know about whole numbers to help you solve the problems?*

Try to avoid making evaluative comments yourself. Instead, encourage students to respond to other students' explanations.

Note: You could introduce the fraction strips as another way to represent fractions used in the number line. *How could you use the fraction strips to help you solve the Five Friends Swimming problems? How are the fraction strips similar/different than the number line?*

Improving individual solutions to the assessment task

Return the student's original assessment, Representing Fractions on a Number Line. You will also want to provide them with a copy of Representing Fractions on a Number Line – Revisit Task.

Possible instructions for students:

Look at your original responses and think about what you have learned during this lesson. Using what you have learned, is there anything you want to change?

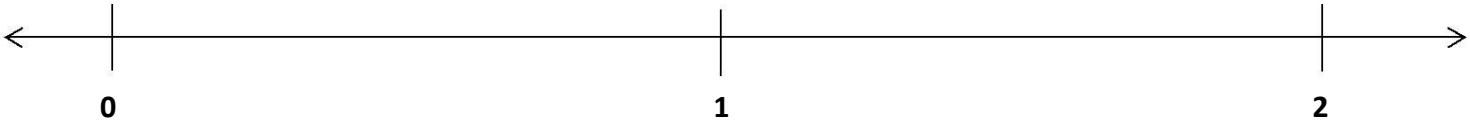
If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

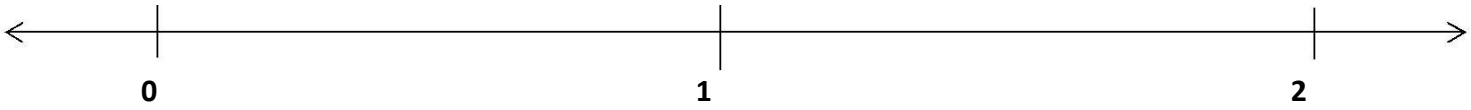
Name_____

Representing Fractions on a Number Line - Initial Task

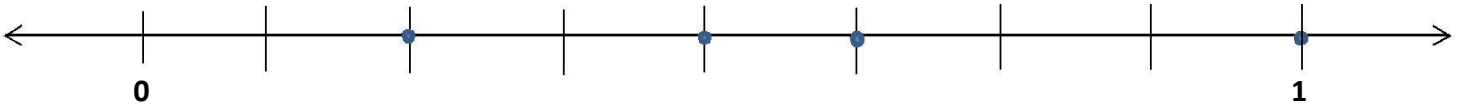
1. **Draw points on the number line below for $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{6}{4}, \frac{8}{4}$. Label the points. Be as exact as possible.**



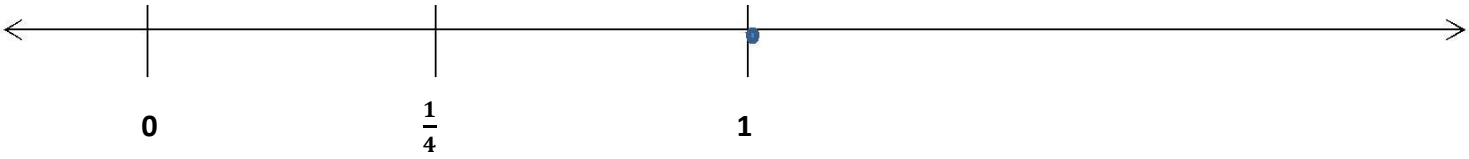
2. **Draw points on the number line below for $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$. Label the points. Be as exact as possible.**



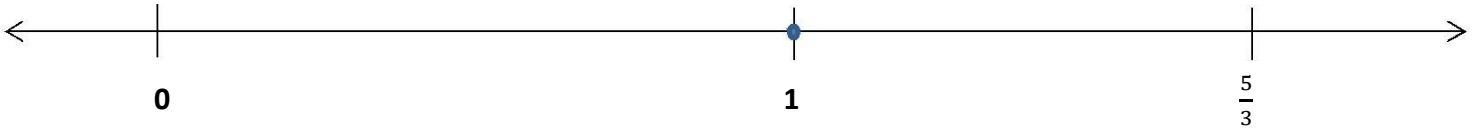
3. **Label the points on the number line below.**



4. **Anna drew a point on the number line for 1. Do you agree or disagree with where Anna drew the point? If you disagree, make changes to the number line for a correct representation. If you agree, explain why you agree.**



5. **Fred drew a point on the number line for l. Do you agree or disagree with where Fred drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.**



Five Friends Swimming-Version A

Name_____

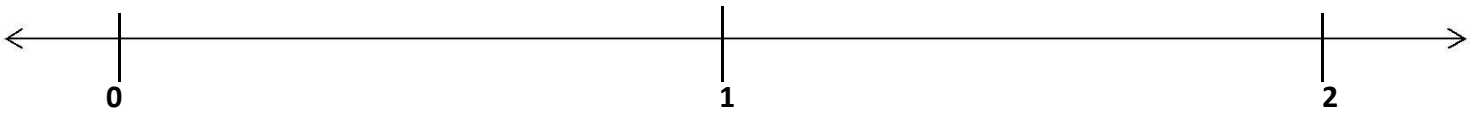
Five friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they swam.

Addison – $\frac{1}{3}$ Caleb – $\frac{1}{2}$ Benjamin – $\frac{1}{4}$

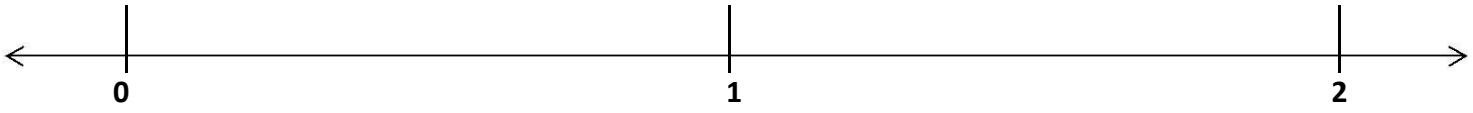
Mary – $\frac{1}{6}$ Ellen – $\frac{1}{8}$

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

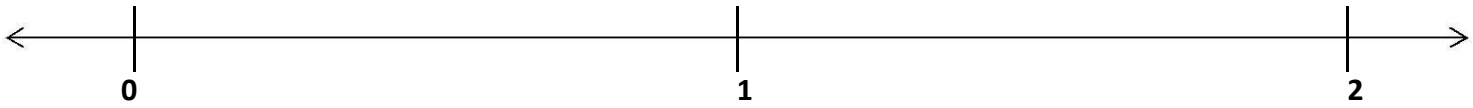
Addison



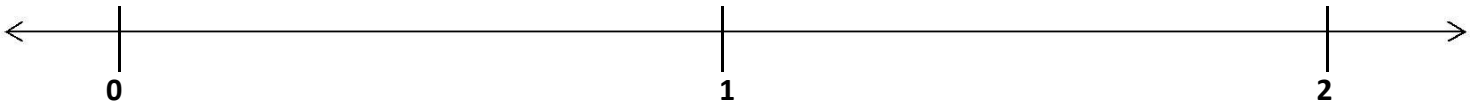
Benjamin



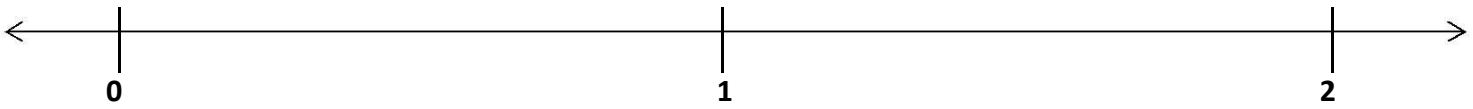
Caleb



Mary



Ellen

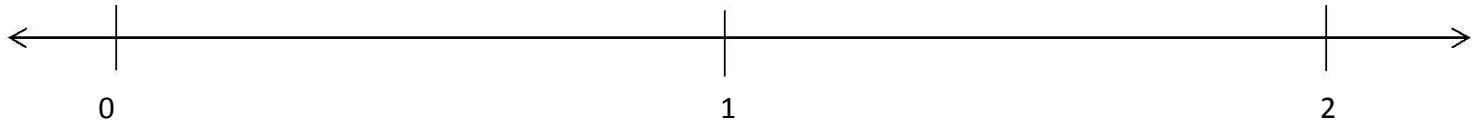


Five Friends Swimming-Version A

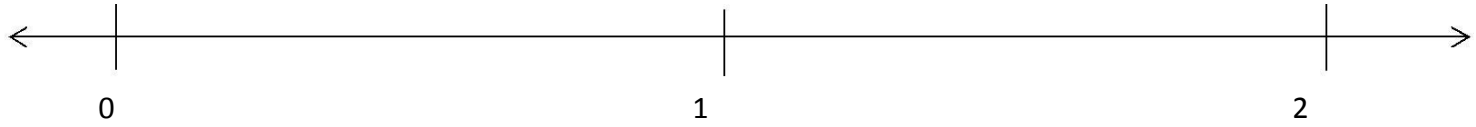
Name_____

Use the (number lines) on page 12 to solve the problems with a partner.

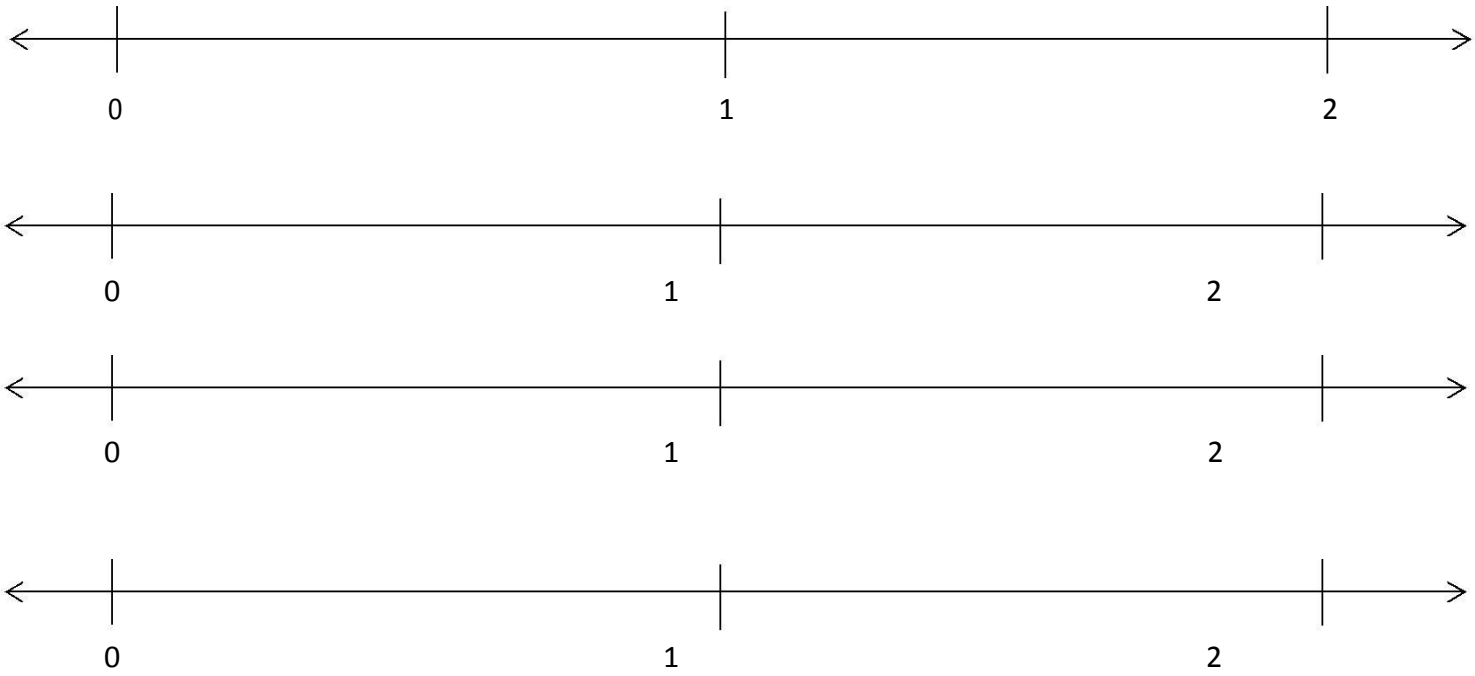
1. Benjamin is training for the mile swim. He swims $\frac{1}{4}$ mile the first day. If he swims $\frac{1}{4}$ mile a day, how many days will take him to swim 1 mile?



2. Ellen is training for the mile swim. She swims $\frac{1}{8}$ mile each day. On the first day, she swims $\frac{1}{8}$ mile. How far did Ellen swim in 2 days? How far will Ellen swim in 4 days?

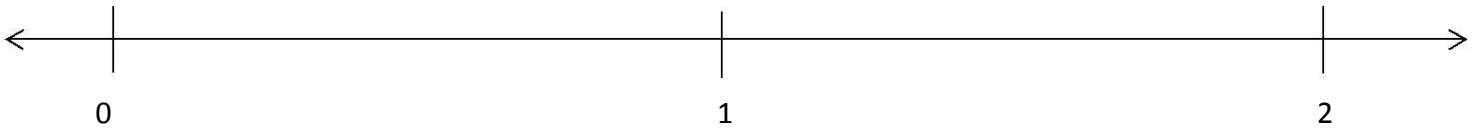


3. On Wednesday, Caleb said he swam $\frac{1}{2}$ mile. Mary swam $\frac{3}{6}$ miles. Ellen swam $\frac{4}{8}$ miles. Addison swam $\frac{2}{4}$ miles. Caleb said, "We all swam the same distance and we are tied." Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.

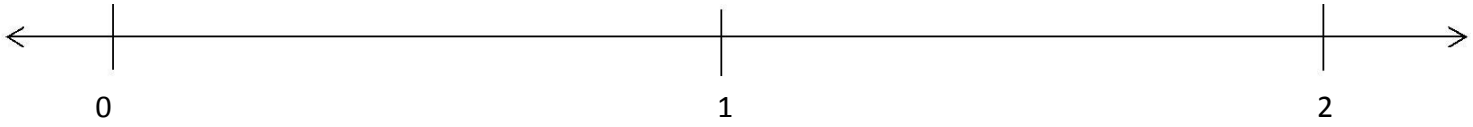


Five Friends Swimming-Version A

4. Caleb swam $\frac{1}{2}$ mile a day for 3 days. At the end of 3 days Caleb told his friends he swam $\frac{3}{2}$ miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as $\frac{3}{2}$?



5. Extension: Write your own story problem using the number line, fractions, and the Five Friends Swimming characters.



Name_____

Five Friends Swimming-Version B

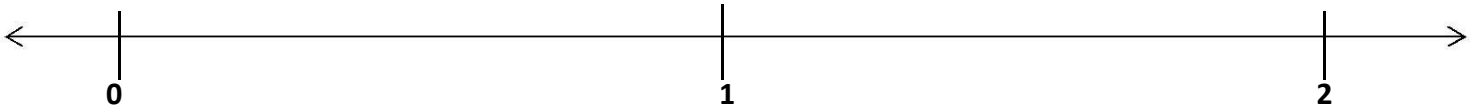
Five friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they have swum.

Addison – $\frac{3}{4}$ Caleb – $\frac{1}{2}$ Benjamin – $\frac{5}{6}$

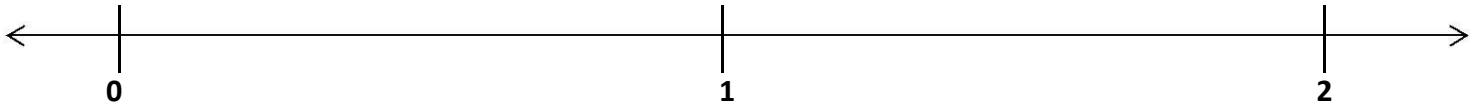
Mary – $\frac{5}{8}$ Ellen – $\frac{2}{3}$

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

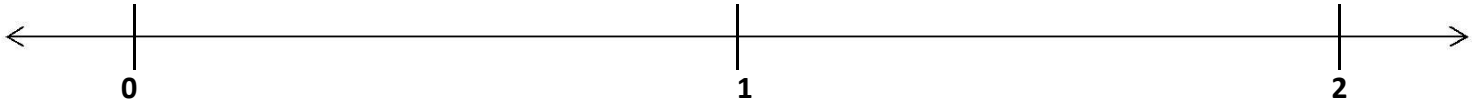
Addison



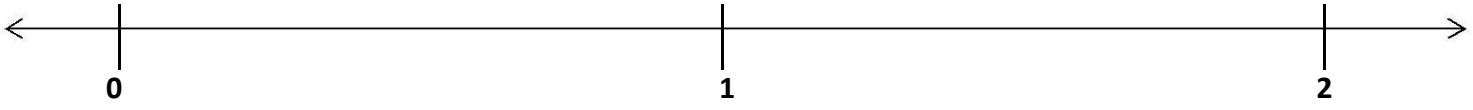
Benjamin



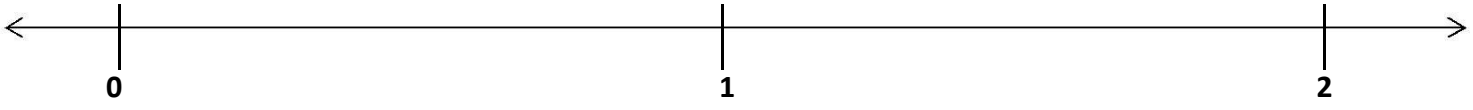
Caleb



Mary



Ellen

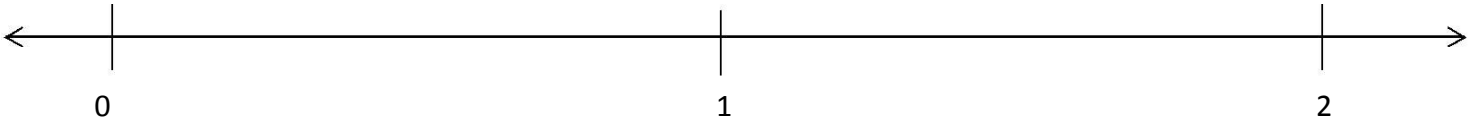


Five Friends Swimming-Version B

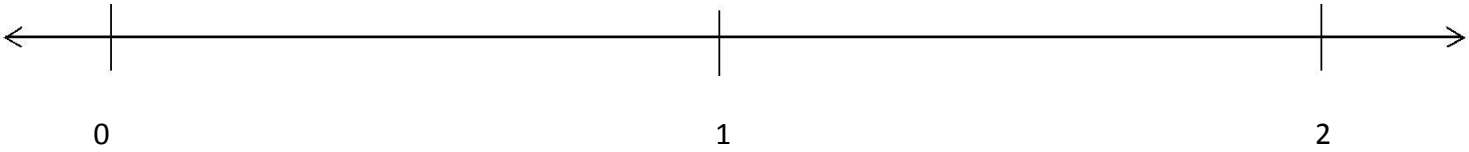
Name_____

Use the (number lines) to solve the problems with a partner.

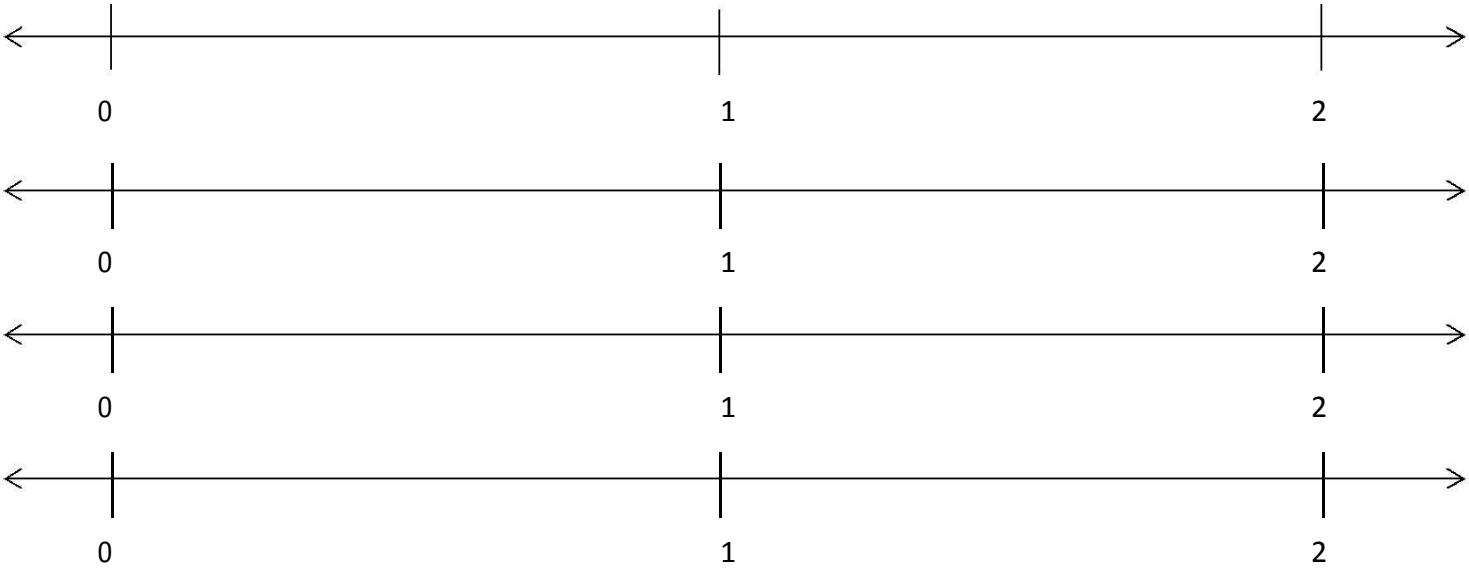
1. Addison is training for a swimming race. If he swims $\frac{1}{4}$ mile a day, how many days will it take him to swim $\frac{1}{2}$ mile?



2. Mary swims $\frac{1}{8}$ mile each day. How far will Mary swim in 12 days?

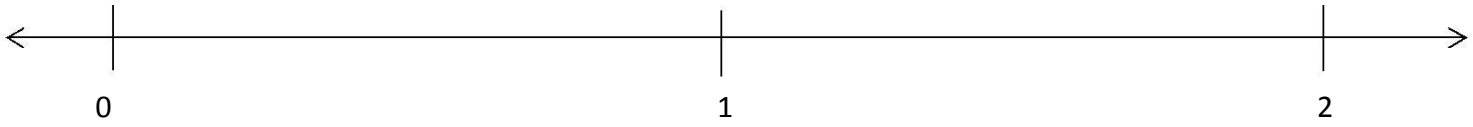


2. On Wednesday, Caleb said he swam 2 miles. Benjamin swam $\frac{6}{3}$ miles. Mary swam $\frac{4}{2}$ miles. Addison swam $\frac{6}{4}$ miles. Caleb said, "We all swam the same distance and we're tired." Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.

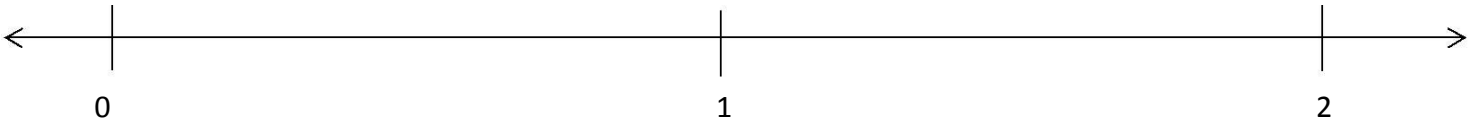


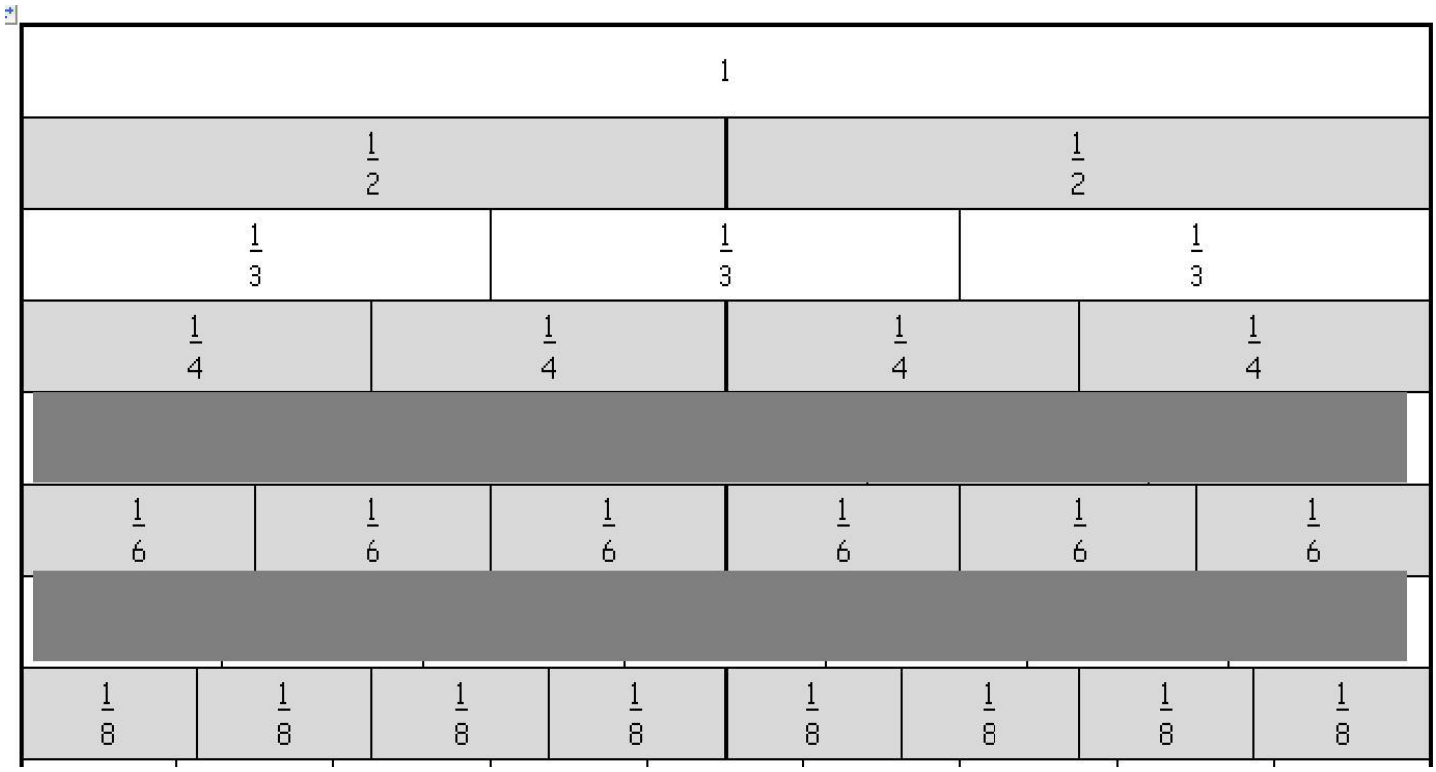
Five Friends Swimming-Version B

4. Caleb swam $\frac{1}{6}$ mile a day for 8 days. At the end of 8 days Caleb told his friends he swam $\frac{4}{3}$ miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as $\frac{4}{3}$?



5. Extension: Write your own story problem using the number line, fractions, and the Six Friends Swimming characters.

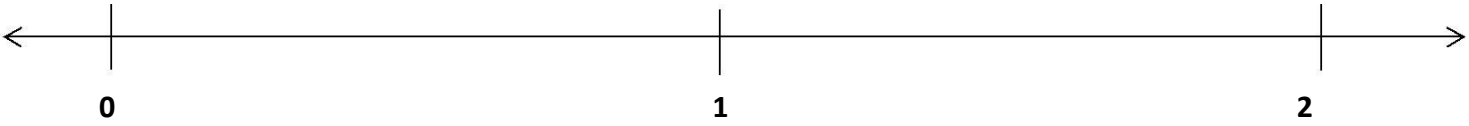



1
1/2
1/2
1/3
1/3
1/3
1/4
1/4
1/4
1/4
1/6
1/6
1/6
1/6
1/6
1/6
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8

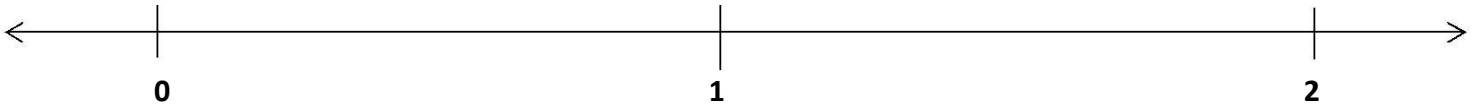
Representing Fractions on a Number Line - Revisit Task

Name_____

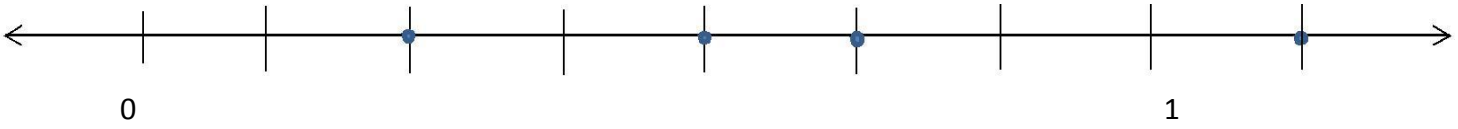
1. **Draw points on the number line below for $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{6}{4}, \frac{8}{4}$. Label the points. Be as exact as possible.**



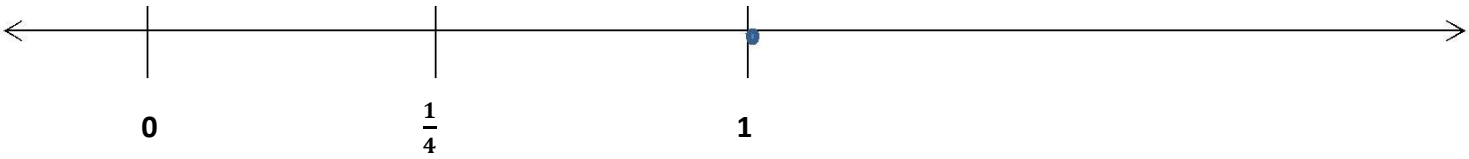
2. **Draw points on the number line below for $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$. Label the points. Be as exact as possible.**



3. **Label the points on the number line below.**



4. **Anna drew a point on the number line for 1. Do you agree or disagree with where Anna drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.**



5. **Fred drew a point on the number line for l. Do you agree or disagree with where Fred drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.**