Algebraic Thinking:
Snail in the Well
Grades 1 – 4
Formative Assessment Lesson
Problem Solving

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at:
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Revised 2019

A Snail in the Well

Primary/Intermediate Grades

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This problem solving lesson is intended to help you assess how well students are able to use addition and subtraction in a problem solving situation. In particular, this lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns
- Describing and explaining findings clearly and effectively.

Kentucky Academic Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on: **MP1.** Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the content standards*:*

Operations and Algebraic Thinking
Grade 1 Cluster: Represent and solve problems involving addition and subtraction.
Grade 2 Cluster: Represent and solve problems involving addition and subtraction.
Grade 3 Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Grade 4 Cluster: Use the four operations with whole numbers to solve problems.

Introduction

This lesson is structured in the following way:

- Before the lesson, students attempt the task individually. You then review their work and formulate questions for students to answer in order for them to improve their work. • At the start of the lesson, students work individually to answer your questions.
- Next, they work collaboratively, in small groups, to produce a better collective solution than those they produced individually. Throughout their work, they justify and explain their decisions to peers.
- In the same small groups, students critique examples of other students' work.
- In a whole-class discussion, students explain and compare the alternative approaches they have seen and used.
- Finally, students work alone to reflect on their individual & group work.

Materials required

- Each individual student will need one copy of the *A Snail in the Well* sheet and one copy of the *How did you work?* sheet.
- Each small group of students will need d a copy of *Sample Responses to Discuss* and whichever samples of student work chosen.

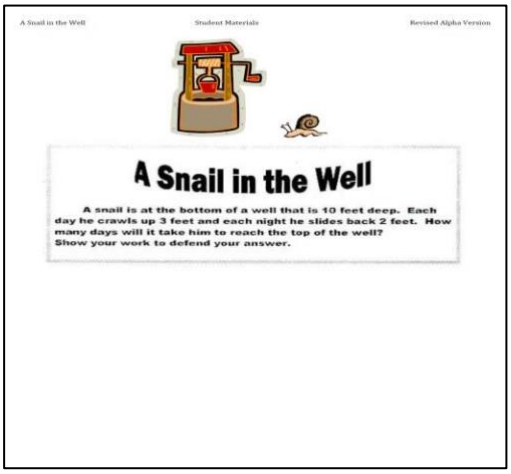
Time needed

Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson. All times are approximate. Exact timings will depend on the needs of the class.

Before the lesson **Assessment task:**

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *A Snail in the Well*. Introduce the task briefly and help the class to understand the problem and its context.



Spend fifteen minutes on your own, answering this question. Show your work.

Don't worry if you are not sure if your solution is correct. There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer this question with confidence by the end of that lesson.

It is important that students answer the question without assistance, as far as possible. Students who sit together often produce similar answers, and then, when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written/displayed on the board at the beginning of the lesson.

Common Issues – Suggested questions and prompts:

Common Issues	**Suggested questions and prompts**
Student forgets to consider what the snail does each day and each night.	• How could you make this task easier? • What sort of picture could you draw that might be helpful? • How can you show the path the snail follows until he gets out of the well?
Student work is unsystematic.	• What pattern do you notice? • What is the same and what is different about how the snail moves during the day and at night? • How can you organize your work?
Student assumes that the initial pattern continues indefinitely and over-generalizes.	• What do you think about how far the snail travels each day? • Does the snail always fall back?
Student writes answer without explanation.	• How could you explain/show how you got your answer so that someone in another class understands? • How can you use numbers, words, or pictures to describe the path of the snail?
Student correctly identifies when the snail gets out of the well.	• Think of another way of solving the problem. Is this method better or worse than your original one? Explain your answer. • Can you make a new problem with a different size well and/or a snail that travels different amounts each day and night?

Suggested lesson outline

Improve individual solutions to the assessment task (10 minutes)

Return your students' work on the *A Snail in the Well* problem. Ask students to re-read both the *A Snail in the Well* problem and their solutions. If you have not added questions to students' work, write a short list of your most common questions on the board. Students can then select a few questions appropriate to their own work and begin answering them.

Recall what we were working on previously. What was the task?

Draw students' attention to the questions you have written.

I have read your solutions and I have some questions about your work.

I would like you to work on your own to answer my questions for ten minutes.

Collaborative Activity:

Organize the students into small groups of two or three. In trials, teachers found keeping groups small helped more students play an active role.

Put your solutions aside until later in the lesson. I want you to work in groups now.

Your task is to work together to produce a solution that is better than your individual solutions.

As the teacher, you have two tasks during small-group work, to note different student approaches to the task, and to support student problem solving.

Note different student approaches to the task

Notice how students work on finding the path of the snail. Notice what strategies they use and how they organize their data. Note the representations they use, including incorrect versions, for use in whole-class discussion. You can use this information to focus the whole-class plenary discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, you could write a relevant question on the board. You might also ask a student who has performed well on one part of the task to help a student struggling with that part of the task.

The following questions and prompts would be helpful:

What information have you been given?

What do you need to find out?

What changes in the diagram? What stays the same?

How will you write down your pattern?

Why do you think your thinking might be true?

You may find that some students do not work systematically when organizing their data.

What can you do to organize your data?

If students have used equation, focus their attention on improving explanations, or exploring alternative methods.

How can you be sure your explanation works in all cases?

Ask another group if your argument makes sense.

Some stronger explanations are shown in the *Sample Responses to Discuss*.

Make a note of student approaches to the task

Give each small group of students a copy of the *Sample Responses to Discuss*. Choose the samples of student work that match your students' level of understanding. Display the following questions on the board or project the provided sheet: *Analyzing Student responses to discuss*. ***Describe the problem solving approach the student used.***

You might, for example:

- ***Describe the way the student has organized the data.***
- ***Explain what the student could do to make his or her solution correct or clearer if they calculated correctly.***

This analysis task will give students an opportunity to evaluate a variety of alternative approaches to the task, without providing a complete solution strategy.

During small-group work, support student thinking as before. Also, check to see which of the explanations students find more difficult to understand. Identify one or two of these approaches to discuss in the plenary discussion. Note similarities and differences between the sample approaches and those the students took in small-group work.

Individual reflection (10 minutes) – possible homework

Once students have had a chance to discuss the sample responses as a whole class, distribute the questionnaire *How Did You Work?* Ask students to spend a couple of minutes, individually, answering the questions.

Think carefully about your work on this lesson and the different methods you have seen and used.

If you are running out of time, you could schedule this activity for the next lesson or for homework.

Solution

The snail will reach the top of the well on the 8th day. He does gain one foot per day, so that at the end of the 7th day he is 7 feet from the bottom of the well. When he travels up 3 ft on the 7th day he does not slide back down because he reaches the top at 10ft and is out of the well.

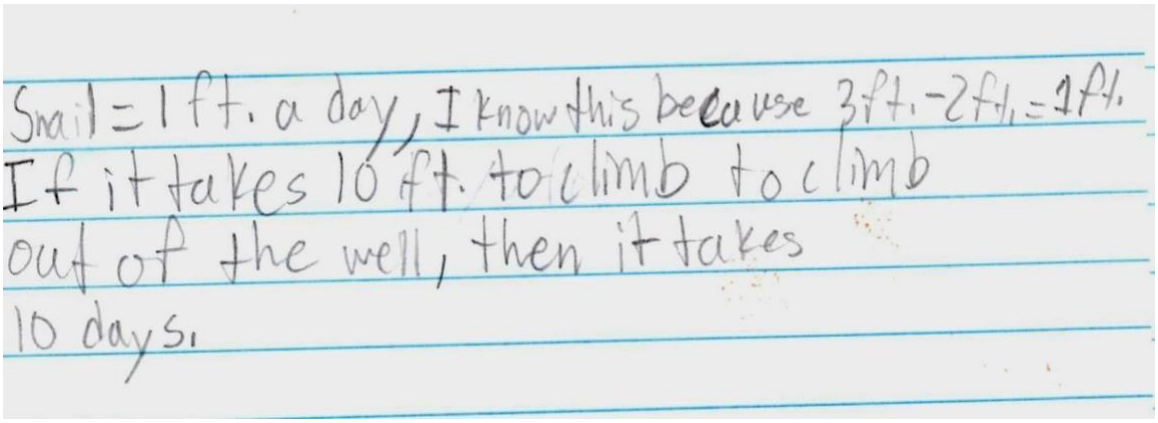
Analysis of *Student Responses to Discuss*

Will's Method

Will noticed that up 3 then down 2 always results in a gain of 1 ft. He over-generalizes this fact and concludes in will take 10 days for the snail to get out of the well.

Will's strategy of noticing a gain of 1 foot per day is correct but he did not systematically organize the data to show the complete path of the snail.

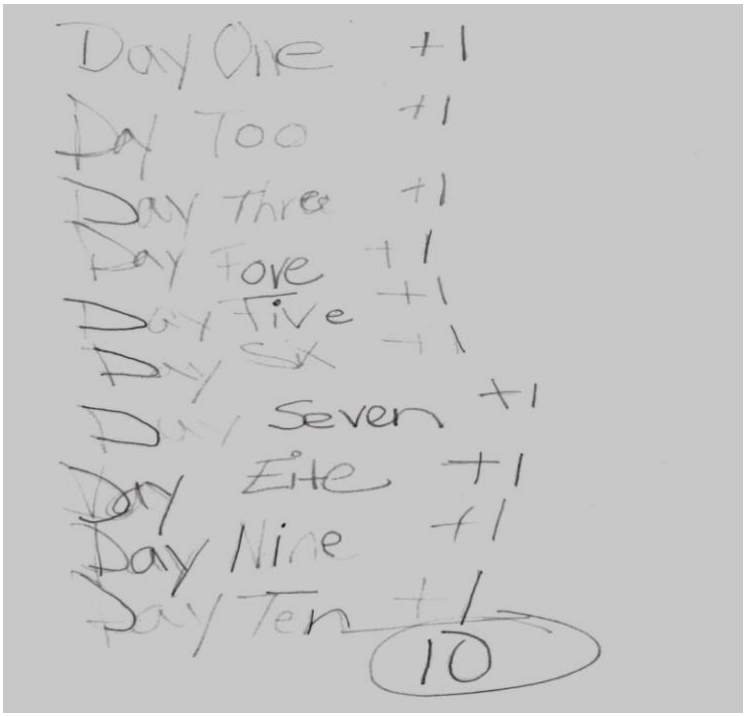
Will's Solution



Whitney's Method

Whitney made a list of each day and the amount the snail gained. She over-generalizes this fact and concludes it will take 10 days for the snail to get out of the well. Whitney's strategy of noticing a gain of 1 foot per day is correct but she did not show the complete path of the snail.

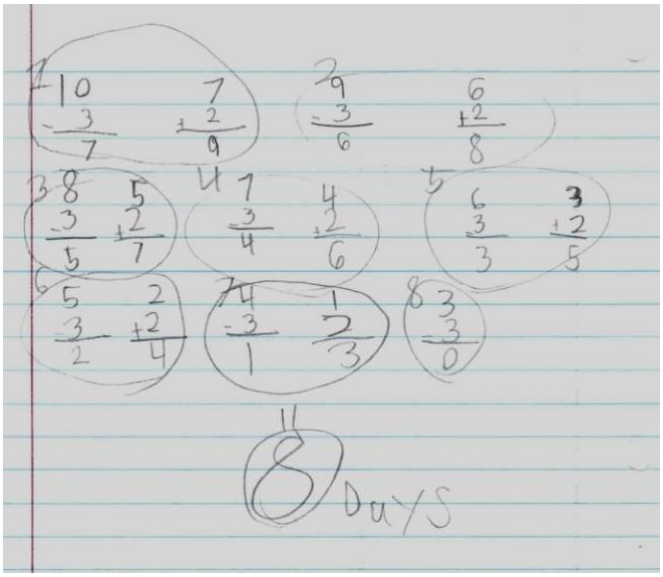
Whitney's Solution



Chuck's Method

Chuck correctly shows subtraction and addition calculations organized by day so that he arrives at the correct solution to the problem. He could enhance his solution by labeling each day and night clearly and/or showing an additional representation (picture, words, chart, etc.) to validate the solution.

Chuck's Solution

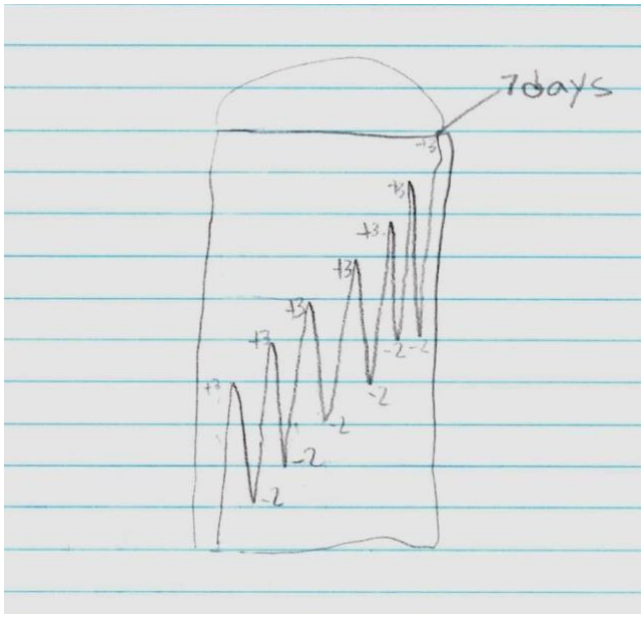


Tim's Method

Tim draws a diagram and attempts to show the path of the snail including how the snail moves up the well during each day and slides back each night. However, his diagram is not clear and consistent so he cannot accurately tell how many total feet the snail has traveled.

Tim's diagram could be enhanced with additional labels showing feet, day, night, etc.

Tim's Solution

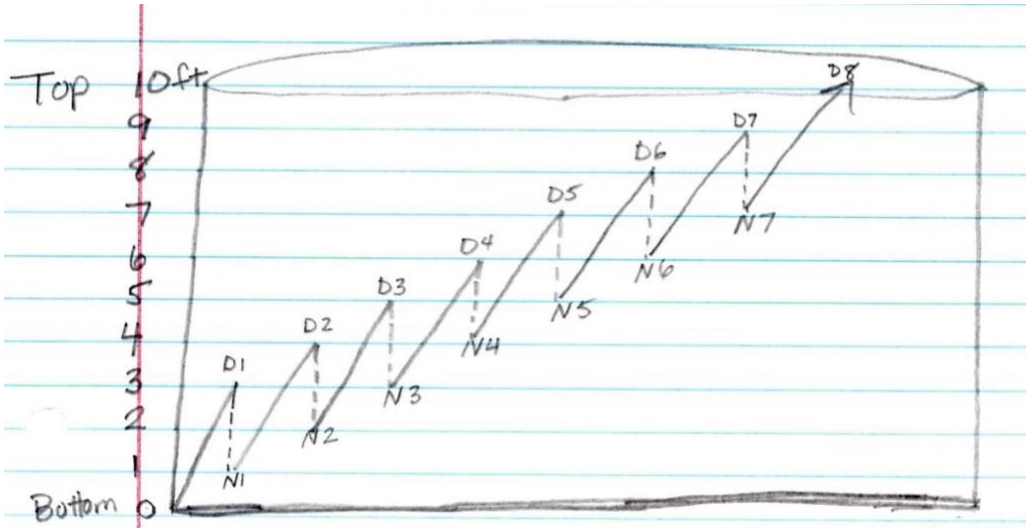


Denise's Method

Denise clearly and accurately shows the path of the snail, labeling feet, day and night. Her distinction in bold versus slashed lines makes the distinction between night and day easy to see. To enhance her

solution, Denise could verify her solution with another representation (chart, words, etc.) to show how they are related.

Denise's Solution

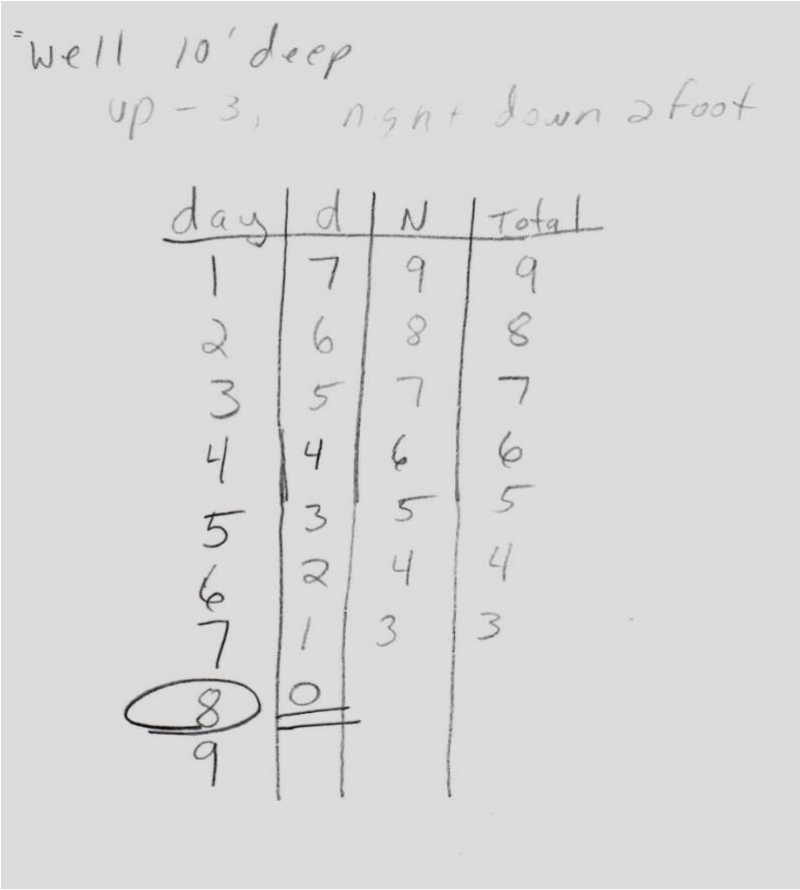


Bill's Method

Bill made a chart to organize the data in this problem. He makes columns to identify the overall day in the well, the distance the snail is from the top of the well each day and each night and the total feet the snail is from the top at the end of the day. On the 8th day Bill shows there are 0 feet left for the snail to travel. His work is organized and labeled so that he correctly arrives at an accurate solution.

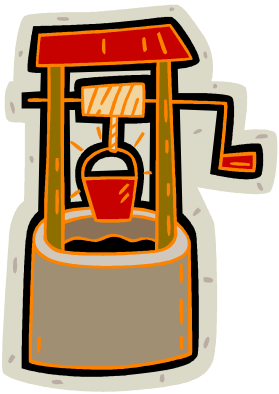
Bill's labels may be a bit confusing for someone else reading his work. He could enhance his solution by clearly labeling the columns so it easy to tell the difference between day and d, and N and total. He could also further verify the solution with alternative representation.

Bill's Solution



This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

Name_____



A Snail in the Well

A snail is at the bottom of a well that is 10 feet deep. Each day he crawls up 3 feet and each night he slides back 2 feet. How many days will it take him to reach the top of the well? Show your work to defend your answer.

Sample Responses to Discuss

Here is some work on ***A Snail in the Well*** from students in another class.

For each piece of work:

1. Write the name of the student whose solution you are analyzing.
2. Describe the problem solving approach the student used.

   For example, you might:

   - Describe the way the student has organized the data.
   - Describe what the student did to calculate the day the snail reaches the top of the well.

3. Explain what the student needs to do to complete or correct his or her solution.

_____'s_ Solution

_____’s_ Solution

_____’s_ Solution

How Did You Work?

Post-Task Reflection: A *Snail in a Well*

Check the boxes and complete the sentences that apply to your work.

1.) Check one, then complete the sentence below:

_____Our group work was better than my own individual work.

OR

_____My own individual work is better than our group work.

I prefer (*circle one)* **our method** / **my method** because:

_____

_____

2.) Check one, then complete the sentence below:

_____ Our method is similar to: _____

(add name of sample response)

OR

_____Our method is different from **all** of the sample responses.

I prefer (*circle one)* **our method** / **the sample response** method because:

_____

_____

3.) Check one and complete the sentence:

_____We checked our method by: _____

OR

_____We checked our method by: _____

Collaborative Work:

(1) Share your method with your partner(s) and your ideas for improving your individual solution.

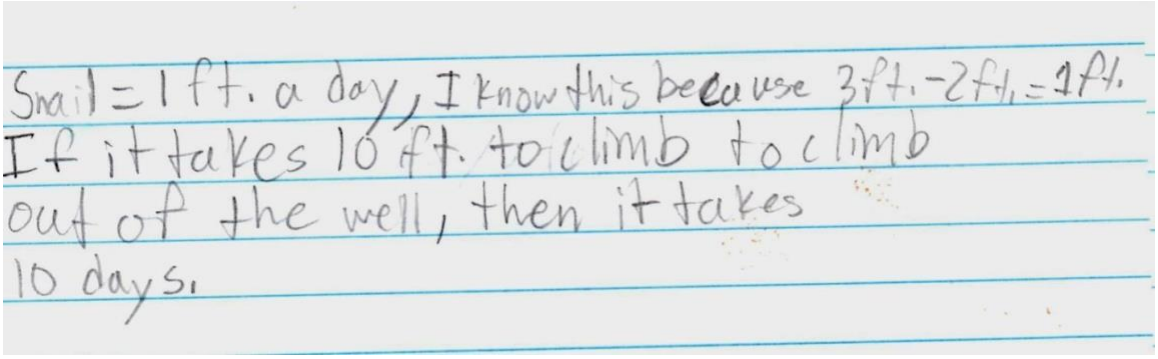
(2) Together in your group, agree on the best method for completing the problem.

(3) Produce a poster, showing a joint solution to the problem.

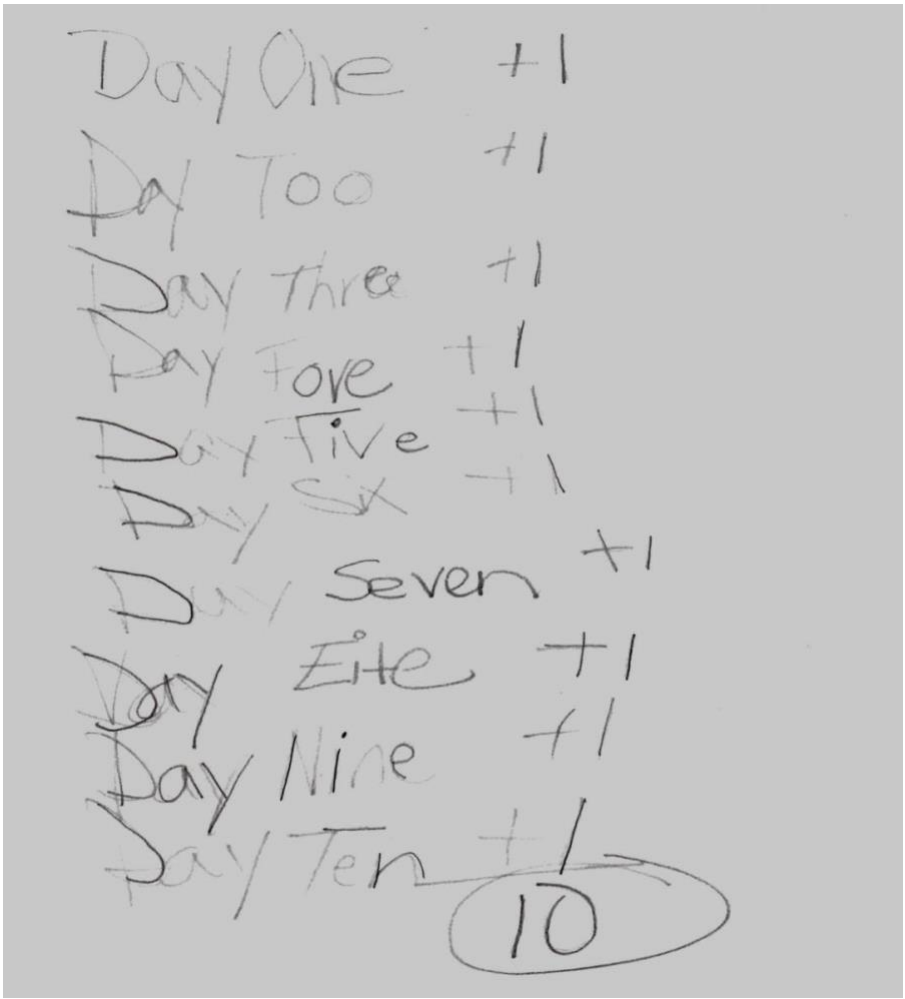
(4) Make sure that everyone in the group can explain the reasons for your chosen method, and describe any assumptions you have made.

(5) Check your work.

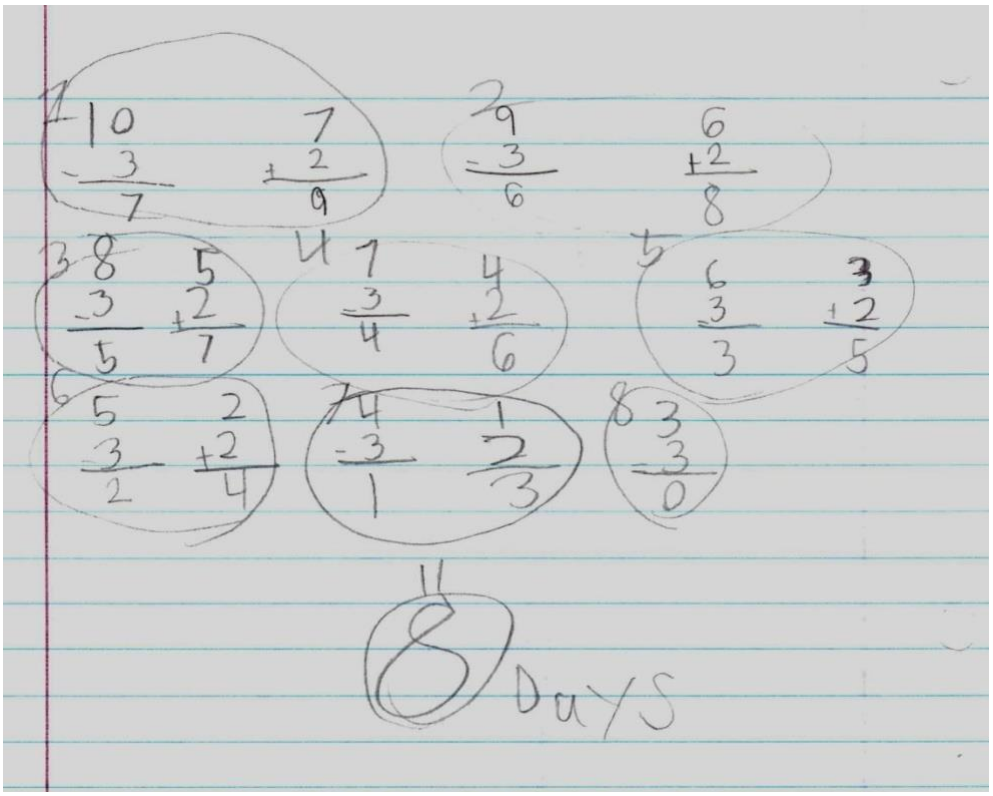
Will's Solution



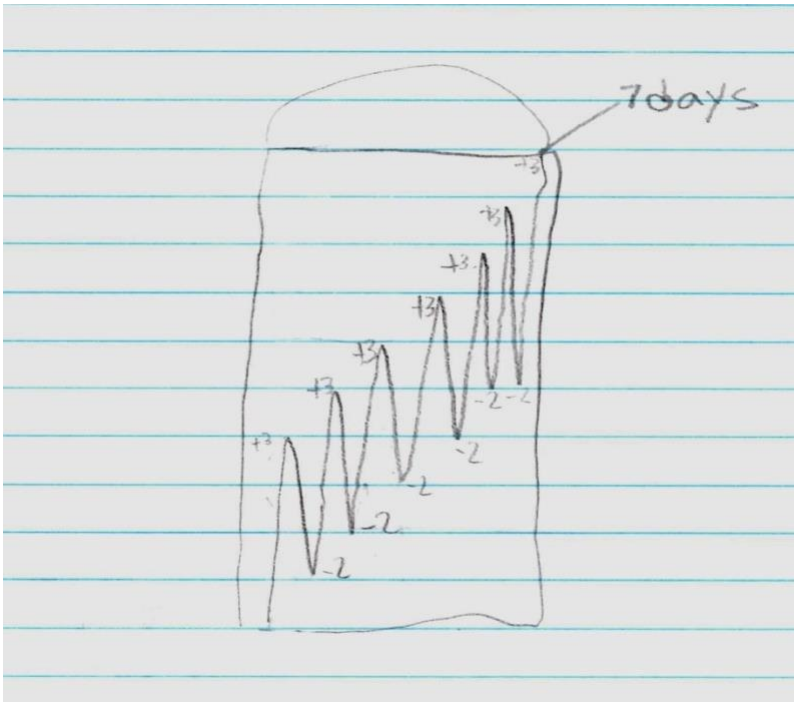
Whitney's Solution



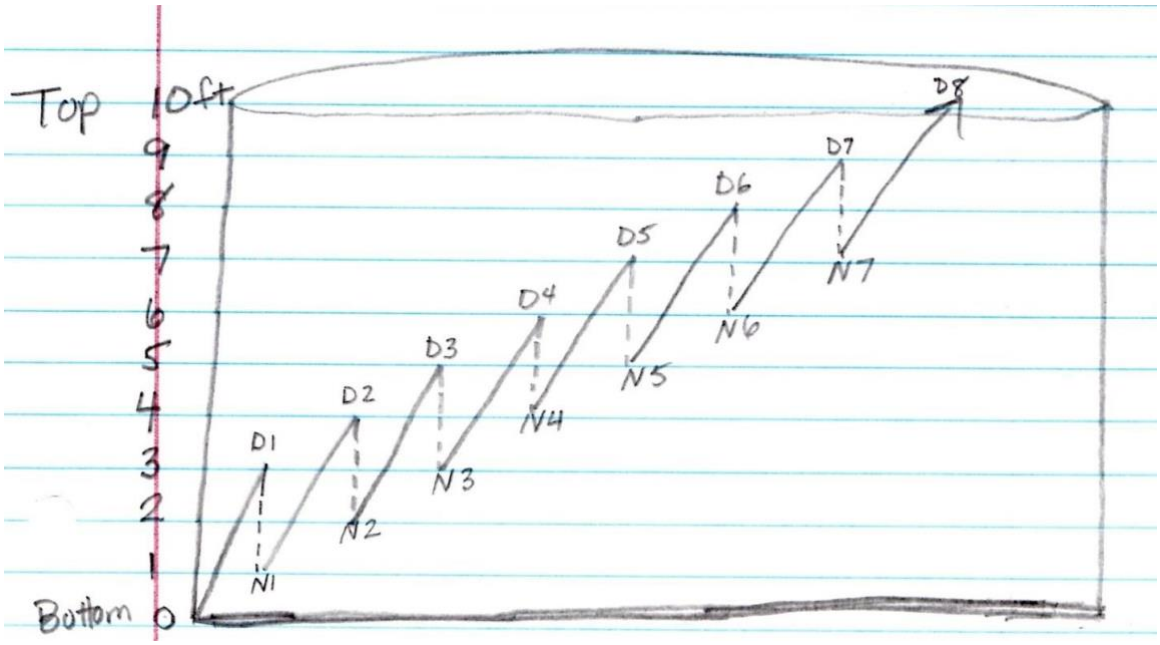
Chuck's Solution



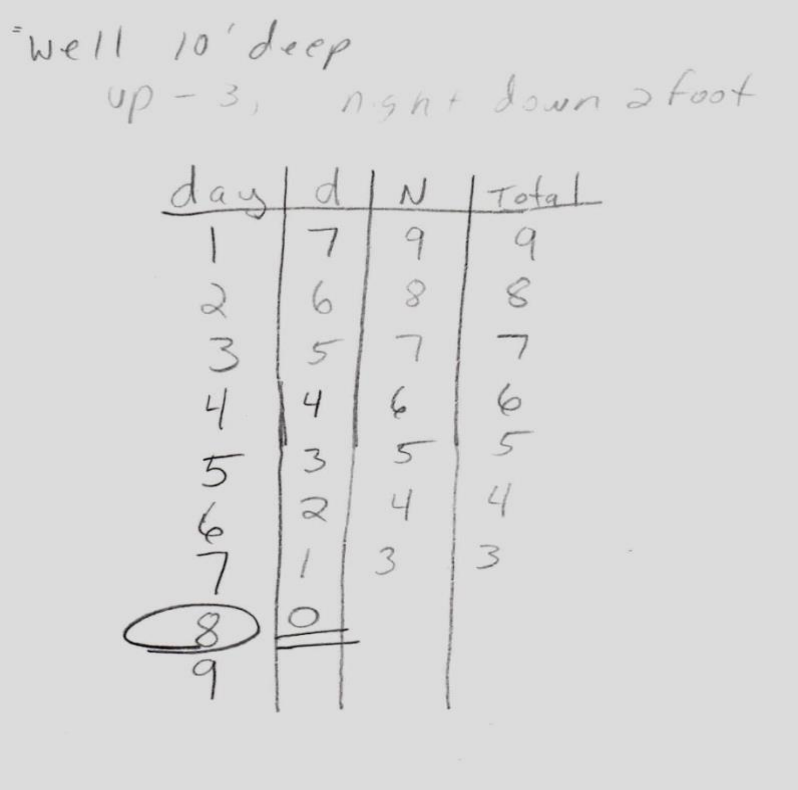
Tim's Solution



Denise's Solution



Bill's Solution



"well 10' deep
up - 3, night down 2 foot

day	d	N	Total
1	7	9	9
2	6	8	8
3	5	7	7
4	4	6	6
5	3	5	5
6	2	4	4
7	1	3	3
8	0		
9			