# MATHEMATICS TEACHING PRACTICE 6: Build procedural fluency from conceptual understanding 

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

## Strategy and Process for Students with Disabilities

Using the Concrete, Semi-Concrete and Abstract (CSA) approach to mathematics

## Concrete:

- In the concrete stage, students use physical manipulatives to explore, build and experience math.
- In the early years, this includes students understanding quantities and foundational numeracy topics.
- The concrete tools are used so students can literally show their thinking.


## Semi-Concrete:

- In this stage, students translate their thinking to drawings or pictures instead of using concrete tools.
- For example, instead of using counters, students may draw circles or tallies to help them solve problems.

Abstract:

- Students who have a solid foundational understanding of a math idea in the concrete and semi-concrete stages move to the abstract stage.
- Students in this stage are ready to work with numbers, symbols and equations because they have developed a clear understanding of what each of those abstract symbols mean.
Implementing number talks
- Implement in a whole group setting.
- Students should not use any tools or written expression during Number Talks to keep the time brief and encourage mental computation.
- Develop and use quiet signals for communication.
- Use talk moves to increase engagement.
- Record thinking in a shared, accessible location for students to revisit.


## Digital Learning Experience

Regardless of the setting, students need explicit instruction in how to use manipulatives, whether they be concrete or virtual tools. Teachers must provide clear expectations for using math tools and model appropriateness.

Virtual manipulatives are available in a variety of programs and platforms:

- Kentucky Center for Math Virtual Instruction Resources
- Virtual Manipulatives

Teachers may create video lessons to show students how to access and use these virtual math manipulatives, embedding how-to videos in assignments to help students select and use the correct tools.

Take-home math kits also might be beneficial for students, particularly for young learners. Class kits can be created so teachers know each student has the same math tools, helping streamline the conversation about which tools to use and when to use them.

Whiteboards (virtual and physical) are an effective tool for students to quickly draw their representational thinking and show their board either on camera or by sharing or submitting a virtual white board.
Digital learning considerations:

- Pose a number talk image or computation and ask students to reply to one or more classmates' thinking using video or voice capture tools.
- Use reaction buttons in place of "quiet signals."
- Use breakout rooms to allow students to share how many and their strategy, then collect responses verbally in the whole group or via submission using a virtual tool.
- The instructor's most important job during virtual learning is to record student thinking as accurately as possible

Visit the Kentucky Center for Mathematics for additional number talk resources.

Implementing meaningful flash cards

- Connect the abstract math fact to a variety of representations, including:
- Dice, domino or 10 frame dot patterns;
- Finger patterns;
- Tally marks;
- "Groups of" images;
- Arrays;
- Area models; or
- Situations/contexts.
- To use the cards, ask students to do one or more of the following activities:
- Matching;
- Pull a card, build it, draw it, write it, say it (the fact); or
- Pull a card and use tools, drawings or symbols to record or say the fact family or number bonds.
(without editing) and to ask prompting and clarifying questions to support students in making their thinking visible to others.
For digital learning experiences:
- Consider replacing physical activities with virtual alternatives.
- Use breakout rooms for small group work.
- Submit student thinking using written, video or voice responses.

Sample math card sets:

- Numerical Representation Cards by Kentucky Numeracy Project
- Rekenrek (bead rack) Cards

Contact your special education regional cooperative for more information on using virtual tools and additional resources.

## Reflection Questions

1. Am I providing all students with Concrete, Semi-Concrete and Abstract (CSA) experiences to develop their conceptual understanding with the important underlying mathematics concepts?
2. Have I monitored students during the CSA phases to know when they can demonstrate a foundational understanding?
3. Have I focused intentionally on developing conceptual understanding before moving to algorithms and procedures?
4. Have I anticipated when students will need additional support during the CSA phases?
